Disaster risks, disaster strikes and economic growth: the role of preferences

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Abstract:

This paper studies the role of preferences on the link between disasters and growth. An endogenous growth model with disasters is presented in which one can derive closed-form solutions with non-expected utility. The model distinguishes disaster risks and disaster strikes and highlights the numerous mechanisms through which they may affect growth. It is shown that separating aversion to risk from the elasticity of inter-temporal substitution bears critical qualitative implications that enable to better understand these mechanisms. In a calibration of the model, it is shown that for standard parameter values, the additional restriction imposed by the time-additive expected utility can also lead to substantial quantitative bias regarding optimal risk-mitigation policies and growth. The paper thus calls for a wider use of non-expected utility in the modeling of disasters, in particular with respect to environmental disasters and climate change.

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1 Introduction

Risk is more than ever an essential concern for economic policies. The renewal of interest for the study of risk in macroeconomic models is not only the result of the 2008 economic and financial crisis, but also reflects the growing concerns around environmental risks such as climate change and environmental disasters. The risk of rare catastrophic events bears critical welfare implications not only as disasters hurt the economy, but also as their anticipation may affect agents economic decisions. This view has been first introduced by Rietz (1988) in an attempt to explain the equity premium puzzle (Mehra and Prescott (1985)). Since, his idea that a low subjective probability of a catastrophic event may drive agents investment decisions has gained momentum with a development of new theoretical frameworks (e.g. Barro (2006) and (2009), Gabaix (2012)) supported by empirical evidences on the history of catastrophic events (e.g. Barro and Ursua (2008)). More recently, some authors have adopted similar frameworks to analyze the macroeconomic impacts of environmental disasters in endogenous growth frameworks (Ikefuji and Horii (2012), Akao and Sakamoto (2016), Barro (2015), Müller-Fürstenberger and Schumacher (2015), Bakkensen and Barrage (2016), Bretschger and Vinogradova (2017)). As pointed out by Bakkensen and Barrage (2016), if these disasters reduce output or production means when they strike, they also affect consumption and savings decisions in an ambiguous way, resulting in potentially important long-term impacts. Among the many challenges implied by disasters, one of the key issues is therefore to understand to what extent they may affect the long-term through economic growth.

The empirical literature on the link between disasters and growth points towards contrasted evidences. Skidmore and Toya (2002) conclude that higher frequencies of climatic disasters may foster growth, possibly through more accumulation of human capital and a technological update of the physical capital. While Cavallo et al (2013) find no significant impact of disasters on growth both on the short and long run, Sawada et al (2011) find significant negative effects of disasters in the short run, but positive effects on long-run growth. Strobl (2011) studies hurricanes in the US coastal counties and finds evidences of negative effects with a very partial recovery, but the macroeconomic impact of these local catastrophes appears to be negligible. Other studies point towards the negative impacts of disasters on growth. Noy (2009) shows evidences of negative but heterogeneous impacts between countries, with more developed countries being less exposed. Hsiang and Jina (2014) study hurricanes, and find a strong negative long run effect of disasters on output and long-run growth with no evidence of a rebound effect in the twenty years following a catastrophe. There is therefore an ambiguous relationship between disasters and growth, arguably because multiple mechanisms are at play. Thus, developing a theoretical framework to disentangle these mechanisms is needed in order to better understand the long term economic impacts of catastrophes.

Some previous attempts have been recently made to understand these diverging empirical evidences. Ikefuji and Horri (2012) stress the role of human capital as a substitute to physical capital to sustain growth when physical capital pollutes. They also study the optimal trajectory of a pollution tax that reduces the risks of disasters, and show the ambiguous effect of this instrument on growth. Bakkensen and Barrage (2016) try to reconcile the heterogeneous empirical findings by disentangling hurricanes *strikes* and hurricanes *risks*. They show that while the former may persistently reduce output, the second may foster growth through more accumulation due to precautionary savings. They argue that the contradictory results found in empirical studies might partly be explained by different methodologies that either capture the effect of disaster strikes or disaster risks. Akao and Sakamoto (2016) study exogenous disasters and discuss the role of human capital and technology. As Bakkensen and Barrage (2016), they emphasize the key role of the elasticity of the utility function for disaster risks to foster growth through precautionary savings. Although they do not focus directly on growth, Müller-Fürstenberger and Schumacher (2015) and Bretschger and Vinogradova (2017) both analyze the effect of risk on capital accumulation in a Ramsey type of model where risk can be mitigated through abatement activities. Their results also support the idea that disasters may accelerate capital accumulation through precautionary savings depending on the elasticity of the utility function.

This paper adds to the literature by investigating further what appears to be a critical aspect of the link between disasters and growth: preferences. A large literature has emphasized that the traditional time-additive expected utility framework, widely used in the disaster literature, was overly restrictive. As shown by Bansal and Yaron (2004) using evidences from asset pricing, these preferences fail to explain agents valuation of risk in dynamic frameworks. When using a coefficient of relative risk aversion high enough to explain the equity premium, they provide odd predictions such as a price-dividend ratio increasing with uncertainty and decreasing with the mean growth rate. Building on Kreps and Porteus (1978) non-expected utility theory, Epstein and Zin (1989) and Weil (1990) have proposed an alternative approach which enables to distinguish aversion to risk from the inter-temporal elasticity of substitution.

These preferences have since been widely used in the macro-finance literature and more recently in models of environmental disasters. Barro (2015) extends the previous disaster model of Barro (2009) to disentangle environmental disasters from other types of catastrophes. In a different set-up, Bansal and Ochoa (2011) study market returns with non-expected utility and temperature driven disasters. They assume exogenous growth as in Barro's model, but affected by persistent economic fluctuations and transitory temperature-related shocks. Soretz (2007) builds an endogenous growth model with Epstein-Zin-Weil preferences in the situation where production is affected by pollution whose impact is modeled as a Wiener process. The model provides close-form solutions but does not fit to study catastrophic risks. van der Ploeg and de Zeeuw (2018) use the same preferences to study precautionary savings as a reaction to an endogenous climate tipping point. They characterize savings responses to the tipping depending on its impact delay and on the distance of the economy from its steady-state. However, the model does not provide closed-form solutions and does not enable to study repeated catastrophes. Other papers using numerical methods have introduced Epstein-Zin-Weil preferences in climate economy models, such as DSGE models (see van den Bremer and van der Ploeg (2018)) and Integrated Assessment Models (see Crost and Traeger (2014), Jensen and Traeger (2014), Cai et al (2015)).

Yet, to my knowledge there exists currently no framework to study analytically the relationship between endogenous growth and endogenous disasters in which agents display non-expected utility. This paper intends to fill this gap and proposes an endogenous growth model with disasters and non-expected utility that can be fully solved analytically. As in Barro (2015), the model can distinguish between exogenous disasters, and environmental disasters that can be mitigated through risk-mitigation policies. The paper intends to highlight the implications of non-expected utility both on the optimal risk-mitigation policy, and on the consumption-savings decisions. Also, by confronting the impact of disaster risks and disaster strikes as Bakkensen and Barrage (2016), the paper analyzes the implications for long-run growth. Relative to the case of standard time-additive expected-utility, the use of non-expected utility will prove to have critical implications and improve our understanding of the link between disasters and growth.

In order to start from a simple benchmark, the model is first solved in the case of exogenous disasters. Several preliminary intuitions are derived in this situation. I then turn to the case of disasters whose probability can be mitigated through a risk-mitigation policy. In appendix the model is also solved for multiple types of disasters including catastrophes of endogenous intensity. I focus in the main text on the case of a unique type with endogenous probability as it already captures the most interesting results. It follows from the model that the optimal shares of output consumed, saved, and spent in risk-mitigation are all constant on the optimal path. The effects of the model's parameters are studied and in particular the role of the preference parameters are emphasized. While risk and risk aversion (RRA) drive the decision to mitigate risk, the inter-temporal elasticity of substitution (IES) plays no role in this decision. However, it appears to be critical in the risk sensitivity of the consumption/savings decision. When the risk of disasters increases, current consumption is partly transferred to the future through savings when the IES is below unity. Interestingly, if the *sign* of this effect solely depends on the IES, its *magnitude* depends on the RRA. While a high aversion to fluctuations unambiguously leads to more precautionary savings, a high aversion to risk may increase either precautionary savings or precautionary consumption. This result shows that it is essential to depart from the standard time-additive utility function as aversion to risk and aversion to fluctuations end up having very different effects on the optimal solution. A second result of importance is that, when introducing an instrument to mitigate disasters, an increase in risk also generates a transfer from savings to risk-mitigation spending. As a result, and contrary to what has been emphasized so far in the literature, an IES below unity is a necessary but insufficient condition to guarantee a net positive response of savings to risk.

From the law of capital accumulation, one can compute analytically the stochastic growth rate of the economy as well as the average long-run growth rate. Most interestingly, one can look at the effect of disasters on the latter. Following the terminology used by Bakkensen and Barrage (2016) I distinguish the impact of disaster *risks* from the one of disaster *strikes*. If damages from catastrophes reduce expected growth, their anticipation has an ambiguous effect through the sensitivity of capital accumulation to risk. For realistic parameter values - i.e. unless the crowding out of risk-mitigation spending over savings is too high - disasters foster average long-run growth if aversion to risk and to fluctuations are both large enough. Since the existence of disasters necessarily reduces welfare, there are therefore situations in which growth and welfare are inversely linked.

In order to illustrate quantitatively the analytic findings of the paper, the model is then calibrated. From this exercise we reach two important conclusions. First, we see that if a positive impact of disasters on long-run growth is theoretically possible in this framework, it would require values for aversion to risk and fluctuations rather high relative to common estimates in the literature, and at odds with standard calibrations of asset pricing models. Second, if non-expected utility already proved to be critical for the qualitative results, the calibration shows that relative to non-expected-utility, time-additive expected utility also leads to quantitatively substantial bias. Under the null-hypothesis that our calibration of the Epstein-Zin-Weil utility function correctly represents agents preferences, a power-utility function either under-estimates the need to mitigate risk, or over-estimates the need to accumulate capital. Thus, these results argue in favor of a broader use of non-expected utility in numerical models of disasters, in particular climate models. Despite the additional complexity it may bring, the choice of the utility function has first order quantitative implications on the model's output and should therefore be taken carefully.

The rest of the paper is organized as follows. Section 2 presents the general framework. Section 3 considers the case of exogenous disasters as a benchmark to highlight the first intuitions of the model. Section 4 turns to endogenous disasters whose probability can be reduced through some risk-mitigation activities. Section 5 provides a calibration of the model and some quantitative implications for the link between disasters and growth, and the importance of using non-expected utility over other types of preferences. Section 6 concludes. Computations are reported to the appendix, where the model is also extended to the case of multiple types of disasters including catastrophes of endogenous intensity.

2 General framework

The model features essentially two ingredients. One is the stochastic process driving catastrophes. The other is the representation of preferences. We assume utility is derived from the consumption of a unique good C. The central planner's preferences are defined recursively as first proposed by Epstein and Zin (1989) and Weil (1990), and extended to continuous time by Svensson (1989) and Duffie and Esptein (1992). These preferences can be represented by the following utility function:

$$(1-\gamma)U_t = \left[C_t^{\frac{\epsilon-1}{\epsilon}} dt + e^{-\rho dt} \left((1-\gamma)\mathbb{E}U(t+dt)\right)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}} \right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}} \tag{1}$$

where ρ is the pure rate of time preferences, γ the coefficient of relative risk aversion (RRA), and ϵ the inter-temporal elasticity of substitution (IES), so that $1/\epsilon$ can be understood as aversion towards inter-temporal fluctuations. In the specific case where $\gamma = 1/\epsilon$ we obtain the standard time-additive power utility function widely used in the literature. In the even more special case where this parameter tends to one, the power utility converges towards a logarithmic utility. The recursive form of the function defined in equation (1) yields the following Hamilton Jacobi Bellman (HJB) equation:

$$(1-\gamma)V(K_t) = \max\left[C_t^{\frac{\epsilon-1}{\epsilon}} dt + e^{-\rho dt}\left((1-\gamma)\mathbb{E}V(K_{t+dt})\right)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}} \tag{2}$$

Now, let's consider an economy facing disasters, i.e. catastrophic events that may happen with small probability and destroy part of the capital stock. As Martin and Pindyck (2015), we consider multiple types of catastrophes and keep the specification general enough so that these events may include but are not limited to environmental disasters. Although they are rare events, their effect is long lasting: once capital is destroyed, it takes time to re-build. These disasters are assumed endogenous to some risk-mitigation activities, and are taken to be uninsurable. We denote τ the share of output spent to mitigate disasters. The central planner must therefore allocate production (Y) between consumption (C), risk-mitigation activities (τY) and savings (S). Assuming there are n types of disasters and m types of risk-mitigation technologies, the law of capital accumulation is defined as:

$$dK_t = [Y_t - \sum_{j=1}^{m} \tau_{j,t} Y_t - C_t]dt + \sigma_{w,t} dz - \sum_{i=1}^{n} \sigma_{i,p,t} dq_{i,t} \tag{3}$$

where dz is a Wiener process with scaling parameter σ_w, and $dq_{i,t}$ a Poisson process with scaling parameter $\sigma_{i,p}$. The Wiener process models small fluctuations around the trend, while the Poisson process models rare catastrophic events. The use of the Poisson process in the modelling of agents' optimal consumption and savings decisions has been introduced by Wälde (1999) and later used in the study of natural disasters by Müller-Fürstenberger and Schumacher (2015) and Bretschger and Vinogradova (2017), and in a slightly different set-up by Ikefuji and Horii (2012). As Müller-Fürstenberger and Schumacher (2015), we will assume the Poisson process to be endogenous possibly both through its intensity and its probability, and to depend on risk-mitigation spending. The probability of a shock is assumed to be of the form $\mathbb{E}dq_{i,t} = \lambda_i f_i dt$ with λ_i a constant and f_i a function of abatement activities τ_j, $j = 1, ..., m$ to be defined. We also denote $\tilde{K}_i$ the stock of capital after a shock of the i^{th} process occurred, with $\forall i$, $0 < \tilde{K}_{i,t} < K_t$, so that the size of a shock for the process i at time t is $\sigma_{i,p,t} = K_t - \tilde{K}_{i,t}$. Bretschger and Vinogradova (2017) also consider the case of an endogenous variance for the Wiener process. Although possible in this model, for the sake of simplicity we keep the Wiener process exogenous as this feature does not bear critical implications.

The objective of the central planner is to maximize its utility (1) subject to the stochastic law of capital accumulation (3). The solution method is detailed in appendix. It makes use of useful contributions in the resolution of stochastic problems in continuous time (e.g. Merton (1971), Wälde (1999), Sennewald and Wälde (2006)) and how it applies to Epstein-Zin-Weil preferences in an endogenous growth model (see Epaulard and Pommeret (2003)). It is shown in appendix that if we define:

$$X(K, C, \tau) = V_k[(1 - \sum_{j=1}^{m} \tau_{j,t})Y - C] + \frac{1}{2}V_{kk}\sigma_w^2 + \sum_{i=1}^{n} \lambda_i f_i \left(V(\tilde{K}_i) - V(K) \right) \tag{4}$$

with $V_k = \partial V(K)/\partial K$ and $V_{kk} = \partial^2 V(K)/\partial K^2$, then the Hamilton-Jacobi-Bellman equation of this problem can be expressed as:

$$\rho \frac{\epsilon(1-\gamma)}{\epsilon - 1} V(K_t) = \max \left[\frac{\epsilon}{\epsilon - 1} \frac{C_t^{\frac{\epsilon-1}{\epsilon}}}{[(1-\gamma)V(K_t)]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}} + X(K, C, \tau) \right] \tag{5}$$

and the associated first order conditions with respect to C and τ_j are:

$$\frac{C_t^{-\frac{1}{\epsilon}}}{[(1-\gamma)V(K_t)]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}} + X_C = 0$$

$$X_{\tau_j} = 0 \qquad \forall j$$

with X_C and X_{τ_j} the derivatives of X with respect to C and τ_j, hence:

$$C_t^{-\frac{1}{\epsilon}} = V_k \left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1} \tag{6}$$

and:

$$\text{for } j = 1, ..., m \quad : \quad YV_k = \sum_{i=1}^{n} \lambda_i \left[f_i \frac{\partial V(\tilde{K}_i)}{\partial \tilde{K}_i} \frac{\partial \tilde{K}_i}{\partial \tau_j} + \frac{\partial f_i}{\partial \tau_j} \left(V(\tilde{K}_i) - V(K) \right) \right] \tag{7}$$

The previous equations highlight the trade-off between the different uses of resources. Equation (6) gives the optimal arbitrage between the benefits and the opportunity cost of consumption. Equation (7) simply states that at the optimum the marginal cost of risk-mitigation spending (on the left hand side) should be equal to the marginal benefits from reducing disasters' frequency and intensity (on the right hand side). This framework remains flexible and enables to study a large variety of risks in different economic settings. In the next sections we will specify both production and the processes driving disasters in order to fully solve the model. The next section starts with the benchmark case of exogenous disasters (i.e. no risk-mitigation activity) to present in the simplest way the mechanisms driving the link between disasters and growth and how they depend on preferences. Then, the case of disasters of endogenous probability is examined. A more comprehensive set-up with both disasters of endogenous probability and intensity is presented in appendix. In all these specifications, closed-form solutions are derived which enables to precisely identify the mechanisms at play.

3 Benchmark: exogenous disasters

3.1 Specification

The purpose of this model is to analyze the mechanisms through which disasters may affect economic growth, and how agents should optimally respond not only through their consumption/savings decisions, but also through risk-mitigation spending. Still, it is useful as a benchmark to start from a simpler version of the model where disasters are assumed exogenous. In this section, we consider a unique process ($n = 1$), and take the probability of a disaster as fixed ($m = 0$, i.e. no risk-mitigation instrument available), and their intensity as a constant fraction of the capital stock. Specifically, we take $f = 1 + \delta$, i.e. $\mathbb{E}(dq_t) = \lambda(1 + \delta)dt$, and $\tilde{K} = \omega K$ with $\omega \in [0; 1]$ a constant. The variance of the Wiener process is assumed to linearly depend on the level of the capital stock, with $\sigma_w = \sigma K$, so that fluctuations remain proportional to the size of the economy. Finally, we assume production follows from an AK technology. This last assumption is made for two reasons. First, it is technically convenient as it will prove to provide sufficient linearity to the problem to obtain closed-form solutions. Second, the AK specification is relevant in our setting as it captures the "no-rebound" effect observed empirically for natural disasters. As shown by Hsiang and Jina (2014) using the example of hurricanes, natural disasters cause permanent output losses that are not compensated by higher growth rates in the aftermath, nor in two following decades. These evidences therefore suggest that the AK specification is best fitted to model the effect of disasters on long-run growth.

3.2 Optimal resources allocation

The shape of the problem leads to the following guess for the value function (Weil (1990), Epaulard and Pommeret (2003)):

$$V(K) = \psi^{\frac{1-\gamma}{1-\epsilon}} \frac{K^{1-\gamma}}{1-\gamma} \tag{8}$$

with ψ a constant to be determined. Substituting the guess (8) into the first order condition with respect to C (6) derived in the previous section gives:

$$C^* = \psi K \tag{9}$$

and going back to the HJB equation (5) we can solve for ψ, the optimal share of capital consumed:

$$\psi = \rho\epsilon + (1-\epsilon)\left(A - \frac{\gamma\sigma^2}{2} - \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right) \tag{10}$$

and from the law of capital accumulation defined by equation (3) we can determine the optimal saving rate $s^* = S^*/Y$:

$$s^* = \frac{1}{A}\left[\epsilon(A-\rho) + (1-\epsilon)\left(\frac{\gamma\sigma^2}{2} + \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right)\right] \tag{11}$$

Consumption and savings are therefore constant fractions of capital and output on the optimal path. Interestingly, the consumption share is decreasing with risk - i.e. higher σ or λ, lower ω - and risk aversion - higher γ - *if and only if* $\epsilon < 1$. Symmetrically, when $\epsilon < 1$ the saving rate is increasing with risk and risk aversion. This situation can be interpreted as *precautionary savings*, while the opposite one ($\epsilon > 1$) can be interpreted as *precautionary consumption*. The arbitrage between precautionary savings and consumption depends on the relative importance of an income and a substitution effect caused by an increase in risk. When the IES (ϵ) takes a low-value, aversion to inter-temporal fluctuations ($1/\epsilon$) is high, in which case a higher risk of a catastrophe (and therefore a higher risk of being poorer) in the future incentivizes some transfers from current to future consumption. This income effect can be more than compensated by a substitution effect when agents are little averse to fluctuations. In this second situation, when capital is more at risk, the incentives to consume rather than accumulate are higher and an increase in risk leads to more consumption in the present at the expense of savings. The role of the inter-temporal elasticity of substitution in determining the link between risk and consumption/savings decisions has been early emphasized by Leland (1968) and Sandmo (1970), and more recently in the case of natural disasters by Müller-Fürstenberger and Schumacher (2015), Akao and Sakamoto (2016), Bakkensen and Barrage (2016) and Brestchger and Vinogradova (2017). However, because they use a time-additive power utility function, these papers cannot disentangle the effect of risk aversion from aversion to fluctuations. The use of non-expected utility enables to clarify these previous results and better identify the role of each parameter. As illustrated by the following comparative statics, we see that the *sign* of the effect of risk on consumption and savings only depends on the value of ϵ relative to 1:

$$\frac{\partial \psi}{\partial \lambda} = -A\frac{\partial s^*}{\partial \lambda} = -(1-\epsilon)(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\begin{cases} < 0, & \text{if } \epsilon < 1. \\ \geq 0, & \text{otherwise.} \end{cases}$$

$$\frac{\partial \psi}{\partial \omega} = -A\frac{\partial s^*}{\partial \omega} = (1-\epsilon)\lambda(1+\delta)\omega^{-\gamma}\begin{cases} > 0, & \text{if } \epsilon < 1. \\ \leq 0, & \text{otherwise.} \end{cases}$$

while the *magnitude* of this effect positively depends on the risk aversion coefficient γ since (see proof # 1 in appendix):

$$\forall \gamma \neq 1, \quad \frac{\partial \frac{1-\omega^{1-\gamma}}{1-\gamma}}{\partial \gamma} = \frac{ln(\omega)\omega^{1-\gamma}(1-\gamma) + (1-\omega^{1-\gamma})}{(1-\gamma)^2} > 0$$

$$\frac{\partial \omega^{-\gamma}}{\partial \gamma} = -ln(\omega)\omega^{-\gamma} > 0$$

Thus, if a low IES implies that precautionary savings dominate over precautionary consumption, a high value of the RRA simply magnifies this effect but does not play on its sign.

3.3 Optimal growth and the effects of disasters

The previous results suggest that the effect of disaster *risks* on growth is ambiguous. In some situations, higher risk can foster capital accumulation, and thus economic growth. However, even in this case it remains unclear what is the long-run aggregate impact of disaster *risks* and *strikes* on growth. To examine this issue, we first compute the stochastic growth rate of the economy from the law of capital accumulation as stated by equation (3):

$$\begin{aligned}\frac{dC^*}{C} &= (A-\psi)dt + \sigma dz - (1-\omega)dq_t \\ &= \left[\epsilon(A-\rho) + (1-\epsilon)\frac{\gamma\sigma^2}{2} + \frac{1-\epsilon}{1-\gamma}\lambda(1+\delta)(1-\omega^{1-\gamma})\right]dt + \sigma dz - (1-\omega)dq_t\end{aligned} \tag{12}$$

The first term in dt is the trend growth rate, and σdz represents the fluctuations around this trend. When the economy is hit by a shock, consumption decreases by $(1-\omega)\%$. Note that in a deterministic model without shocks, we obtain the standard Keynes-Ramsey formula where A is the marginal return on capital :

$$\frac{dC^{*det}}{C} = \epsilon\left[A-\rho\right]dt$$

Finally, because $\mathbb{E}(dz) = 0$ and $\mathbb{E}(dq_t) = \lambda(1+\delta)dt$, the expected growth rate of this economy, which is also the average long-run growth rate is :

$$\mathbb{E}\left(\frac{dC^*}{C}\right) = \left[\epsilon(A-\rho) + (1-\epsilon)\frac{\gamma\sigma^2}{2} + \frac{1-\epsilon}{1-\gamma}\lambda(1+\delta)(1-\omega^{1-\gamma}) - \lambda(1+\delta)(1-\omega)\right]dt \tag{13}$$

The previous formula enables to disentangle the effect on growth of disaster *risks,* i.e. the mechanisms through which the anticipation of disasters may affect economic decisions, from the effect of disaster *strikes* captured by the last term of the right-hand side of equation (13). The sensitivity of the expected growth rate to disasters can be analyzed by looking at the following comparative statics:

$$\frac{\partial\,\mathbb{E}\left(\frac{dC^*}{C}\right)}{\partial\lambda} = (1+\delta)\left[(1-\epsilon)\frac{1-\omega^{1-\gamma}}{1-\gamma} - (1-\omega)\right]dt \tag{14}$$

$$\frac{\partial\,\mathbb{E}\left(\frac{dC^*}{C}\right)}{\partial\omega} = \lambda(1+\delta)\left[1-(1-\epsilon)\omega^{-\gamma}\right]dt \tag{15}$$

From these results, the effects of disaster risks and strikes on growth appear clear. Disaster strikes have an obvious negative effect on average long-run growth: when the probability or the intensity of disasters increase - higher λ, lower ω - the expected drop in output due to shocks is larger and so expected growth declines. However, this effect must be weighted against the ambiguous impact of disaster risks on growth. This effect is driven by precautionary savings or consumption, and is therefore positive when $\epsilon < 1$ and negative otherwise. In both cases, it is magnified for higher values of risk aversion γ.

When $\epsilon < 1$, since precautionary savings need to compensate for the losses caused by disasters strikes, our results show that disasters and growth can be positively linked in the long-run *if and only if* ϵ is sufficiently small and γ is sufficiently large, i.e. if the economy displays both high risk aversion and high aversion to inter-temporal fluctuations. These results give theoretical support to the empirical findings of Sawada et al (2011) who found negative effects of disasters on short-run growth, but positive effects in the long-run. Indeed, when precautionary savings dominate, despite their negative immediate impact disasters may encourage capital accumulation and thus promote growth in the long-run. As noted by Bakkensen and Barrage (2016), whether cross-sectional or panel analysis are used to assess empirically the impact of disasters may affect the results as these methods will essentially capture different effects. While cross-sectional studies identify the potentially positive effect of disaster risks on growth, studies using panel data with fixed effect identify the negative effect of disaster strikes.

Two last comments deserve attention. First, it should be noted that disasters generate large transfers between generations. These transfers are due both to the impact of disaster risks - that either favor consumption or savings - on the deterministic pattern of growth and to the stochastic realization of disasters. Second, although higher risk may in some situations improve the average long-run growth rate, it unambiguously reduces welfare. This result holds even ignoring the impact of disasters on human lives, and considering only their effect on the stock of capital. Thus, and as pointed out by Akao and Sakamoto (2016) and Bakkensen and Barrage (2016), there are cases in which growth and welfare vary with opposite signs as a response to risk. This last result is important to stress as a positive link between disasters and growth *should not* be interpreted as disasters being welfare-improving.

4 Disasters of endogenous probability

4.1 Specification

In this section we turn to the situation in which resources can be allocated to reduce the risk of disasters through a unique instrument τ (i.e. $m = 1$). In particular, we assume risk-mitigation spending can reduce the *probability* of disasters. The specification is the same as in the previous section, except for f that we now assume to be a function of τ such that $f = 1 + \delta - \tau^{\alpha}$ with $0 < \alpha < 1$ the inverse of the efficiency of risk-mitigation spending[1]. This specification therefore assumes that the probability of

[1]Since $0 < \tau < 1$, a lower value of α means more mitigation can be performed with less resources

a catastrophe depends on the share of output spent in risk-mitigation. If the entire output was spent to mitigate risk, the probability of a shock would fall to $\lambda\delta$, the probability to face a non-avoidable catastrophe. Absent any abatement activity, the probability would go up to $\lambda(1+\delta)$. If the model remains general with respect to the type of disasters considered, one can understand $\lambda(1-\tau^{\alpha})$ as the probability of an environmental disaster, while $\lambda\delta$ corresponds to the probability of non-environmental disasters such as a stock market collapse, a pandemic or a war. Disasters of endogenous probability have been extensively used in the literature, including in several papers by Barro (2009) and (2015) and Ikefuji and Horii (2012). As in the previous section, damages will be assumed to be a constant fraction of the capital stock. I show in appendix that the model can alternatively be solved for disasters of endogenous intensity as done by Müller-Fürstenberger and Schumacher (2015) and Bretschger and Vinogradova (2017), as well as for multiple disasters and multiple instruments. Since these specifications yield similar intuitions regarding the mechanisms linking disasters and growth, I focus here on the simplest scenario. The resolution method applied is similar to the one of the previous section and can be found in appendix.

4.2 Optimal resource allocation

Applying the new specification, the two first order conditions (6) and (7) together with the HJB equation (5) yield:

$$C^* = \psi K \tag{16}$$

and:

$$\tau^* = \left(\frac{(1-\omega^{1-\gamma})\lambda\alpha}{A(1-\gamma)}\right)^{\frac{1}{1-\alpha}} \tag{17}$$

with:

$$\begin{aligned}\psi &= \epsilon\rho + (1-\epsilon)\left((1-\tau^*)A - \frac{\gamma\sigma^2}{2} - \lambda f^* \frac{(1-\omega^{1-\gamma})}{1-\gamma}\right)\\ &= \epsilon\rho + (1-\epsilon)\left[A - \frac{\gamma\sigma^2}{2} - \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma} + (\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})\left(\frac{\lambda(1-\omega^{1-\gamma})}{A^{\alpha}(1-\gamma)}\right)^{\frac{1}{1-\alpha}}\right]\end{aligned} \tag{18}$$

and consequently the saving rate $s^* = S^*/Y$ is:

$$s^* = 1 - \tau^* - \frac{1}{A}\left[\epsilon\rho + (1-\epsilon)\left((1-\tau^*)A - \frac{\gamma\sigma^2}{2} - \lambda f^* \frac{(1-\omega^{1-\gamma})}{1-\gamma}\right)\right] \tag{19}$$

As in the previous section, the IES appears to be the critical determinant in the arbitrage between precautionary savings and consumption. Aversion to risk again plays on the magnitude of these effects, but the link now also depends on the effect of risk on risk-mitigation spending. With respect to risk-mitigation, total spending are found to be a constant share of output on the optimal path. This share τ^* is strictly increasing with disaster risk (higher λ, lower ω) and risk aversion (γ), but aversion to fluctuations plays no role. The only ambiguous effect is the one of the risk-mitigation efficiency parameter α. As shown in appendix (see proof # 2) for low values α has a positive effect on τ^*, but above a certain threshold $\bar{\alpha}$ its effect becomes negative. This non-monotonic relationship can be interpreted as a trade-off between more incentives to spend resources in mitigation when it is more efficient (substitution effect) against the possibility to mitigate more with less resources as the efficiency increases (level effect).

4.3 Optimal growth and the effects of disasters

The law of capital accumulation in equation (3) enables again to compute the stochastic growth rate:

$$\frac{dC^*}{C} = [(1-\tau^*)A - \psi]dt + \sigma dz - (1-\omega)dq_t \tag{20}$$

and thus the expected growth rate (which is also the average long-run growth rate) of this economy:

$$\mathbb{E}\left(\frac{dC^*}{C}\right) = [(1-\tau^*)A - \psi - \lambda f^*(1-\omega)]dt \tag{21}$$

This formula provides some novel intuitions relative to the one of the previous section. To better understand the new mechanisms at play, one can decompose the effect of disasters on the average long-run growth rate. Differentiating the expected growth rate with respect to λ, we have:

$$\frac{1}{dt}\frac{\partial\,\mathbb{E}\left(\frac{dC^*}{C}\right)}{\partial\lambda} = \underbrace{-A\frac{\partial\tau^*}{\partial\lambda}}_{<0} \underbrace{- \frac{\partial\psi}{\partial\lambda}}_{?} \underbrace{- f^*(1-\omega)}_{<0} \underbrace{- \lambda(1-\omega)\frac{\partial f^*}{\partial\lambda}}_{>0} \tag{22}$$

and similarly with respect to ω:

$$\frac{1}{dt}\frac{\partial\, \mathbb{E}\left(\frac{dC^*}{C}\right)}{\partial \omega} = \underbrace{-A\frac{\partial \tau^*}{\partial \omega}}_{>0} \underbrace{-\frac{\partial \psi}{\partial \omega}}_{?} \underbrace{+\lambda f^*}_{>0} \underbrace{-\lambda(1-\omega)\frac{\partial f^*}{\partial \omega}}_{<0} \tag{23}$$

What do we learn from these comparative statics? All terms in equations (22) and (23) are detailed in appendix. For both equations, the first two terms can be associated with the effect of disaster *risks*, while the last two correspond to the effect of disaster *strikes*. In the following we focus on the second equation, the derivative of expected growth with respect to ω, the share of capital remaining after a catastrophe. This derivative therefore captures the effect on expected growth of a *reduction* in disasters intensity. Similar intuitions can alternatively be derived from the comparative static with respect to λ.

First, when ω increases, disaster strikes become less harmful to the economy as a smaller share of capital $1-\omega$ is destroyed. This effect is captured by the term $\lambda f^* > 0$ in equation (23). How much this effect matters solely depends on the frequency of catastrophes. For more frequent disasters, a reduction of their intensity has larger positive effects on expected growth through this damages term. However, the reduction of disasters intensity has a second, indirect effect on expected growth through expected damages. Indeed, as ω increases, less efforts are performed to mitigate risks. As a result, the equilibrium frequency of disasters λf^* increases and so do expected damages. This second effect is captured by the last term in equation (23), $-\lambda(1-\omega)\frac{\partial f^*}{\partial \omega} < 0$. A higher value of ω has therefore an ambiguous impact on expected damages since less intense catastrophes also lead to less stringent mitigation policies and thus more frequent disasters. In particular, an increase in ω will reduce expected damages from disaster strikes if and only if $f^* > (1-\omega)\partial f^*/\partial \omega$. Contrary to the previous section with exogenous disasters, allowing for the possibility to mitigate catastrophes therefore leads to less obvious results as more intense disasters will drive more careful policies and could *in fine*, for some parameter values, reduce expected damages.

Turning to disaster risks, we first see - as in the previous section - that disasters intensity may either favor or dampen growth through the consumption savings decision. This effect is captured by the term $-\partial\psi/\partial\omega$ that, for realistic parameter values, is positive if and only if $\epsilon > 1$. This result again says that when the IES is above unity, aversion to fluctuations is low and agents are willing to increase their savings when risk is lowered (and alternatively increase current consumption when risk increases). But in addition to the consumption-savings effect, disaster risks now also affect expected growth through the trade-off between risk-mitigation and savings, given by the term $-A\frac{\partial \tau^*}{\partial \omega} > 0$. Indeed, as τ^* is strictly decreasing in ω, for less intense catastrophes less resources are spent to reduce their probability, which leaves more for savings. Thus, while in the case of exogenous disasters risk was fostering growth if and only if $\epsilon < 1$, this condition is not sufficient anymore when mitigation is possible. Since higher risk now also leads to a transfer from savings to risk-mitigation, a net increase in savings due to risk becomes possible under slightly more restrictive conditions over ϵ. Thus, the standard result of the disaster literature that takes $\epsilon < 1$ as a sufficient condition for disasters to foster capital accumulation is not robust to the introduction of endogenous risk-mitigation policies.

Overall, the introduction of an instrument to reduce disasters' probability has an ambiguous effect on growth. If some resources are shifted from capital accumulation to risk-mitigation, in the long run this negative effect might be compensated by the reduction of expected damages from disasters. In a different set-up, Ikefuji and Horii (2012) also found an ambiguous effect on growth of introducing a pollution tax to reduce disasters probability. The underlying mechanisms in this model are different than theirs, but these results bring new evidences that the impact of risk-mitigation policies on growth is ambiguous, even though they positively impact welfare.

4.4 The qualitative implications of non-expected utility

An interesting property of the class of non-expected utility used in this paper is that they generalize more standard forms of utility functions. In particular, the time-additive power utility function emerges as a particular case of Epstein-Zin-Weil utility when $\gamma = 1/\epsilon$, i.e. when aversion to risk and fluctuations are taken to be equivalent. In the even more special case where γ and ϵ both converge towards 1, the Epstein-Zin-Weil utility corresponds to the popular logarithmic utility used in numerous applications in the study of environmental disasters (e.g. Müller-Fürstenberger and Schumacher (2015), Golosov et al (2014)). As such, this framework can be used to compare the results with these more restrictive specifications.

The assumption made by the logarithmic utility that aversion to risks and fluctuations are both equal to unity is very problematic. First, because it simply makes these two parameters disappear and together with them most of the interesting effects linking disasters and growth. Precautionary savings and consumption just balance each other out, and given common estimates for the value of risk aversion, the optimal effort to mitigate risk is necessarily under-estimated. When preferences are instead represented by a time additive power-utility function, the free parameter gives more flexibility than the logarithmic utility. In static models with uncertain outcomes, the parameter is interpreted and calibrated as the relative risk aversion coefficient (RRA). In dynamic deterministic settings, it is alternatively interpreted as the inverse of the inter-temporal elasticity of substitution (IES). However, in dynamic stochastic models, the constraint becomes problematic since it is not clear anymore what the parameter actually captures. As we have seen, in this model risk aversion and aversion to fluctuations end up playing very different roles. The restriction imposed by the time-additive expected utility therefore leads to a mis-interpretation of the effect of preferences on the relationship between risk and growth. In particular, if the IES determines whether individuals will respond to risk through an increase in current consumption or savings, the RRA has no effect on the *sign* of this mechanism. However, the RRA is critical to determine its *magnitude*, independently of its sign. A high aversion to risk could therefore as well lead to strong precautionary savings or strong precautionary consumption, while a high aversion to fluctuations will necessarily lead to more precautionary savings.

We thus see that non-expected utility - and in particular Epstein-Zin-Weil preferences - is essential to better understand the impact of disaster risks on agents decisions, and as such on the optimal path of an economy dealing with catastrophic events. The next question one can ask is whether or not these qualitative differences may translate into substantial quantitative errors in numerical analysis. In the next section, I calibrate the model as specified in this section, and assess the bias that the parameter restrictions imposed by time-additive power utility and log-utility may imply for optimal risk-mitigation policies, optimal consumption/savings decisions and optimal growth.

5 Calibration

The calibration of the model should essentially answer two questions. First, if our analytic findings have shown that both a positive and a negative effect of disasters on expected growth were *possible*, one can wonder how *plausible* are each of these two scenarios. In particular, we will try to assess to what extent should individuals have a strong distaste for risk and fluctuations to perform enough precautionary savings to cover the expected output losses from disaster strikes. Second, this calibration should evaluate to what extent using log-utility and time-additive power utility functions can bias the quantitative assessment of optimal risk-mitigation policies on the one hand, and optimal consumption/savings decisions on the other hand, relative to the null hypothesis of non-expected utility with standard calibrations.

5.1 Calibration choices

We calibrate the model presented in section 4. The baseline values of the parameters used in the calibration are given in Table I. The efficiency of the risk-mitigation technology is taken to be $\alpha = 1/4$ such that cutting by two the risk of a catastrophic event would cost around 6% of GDP. As we know relatively little about this parameter, this is of course subject to debate but it should serve as a starting point for our analysis. All other parameters are taken from Barro (2015), that itself builds on the empirical assessment of disasters in Barro and Ursua (2008). This choice is one among many possibilities, but it has been preferred to other options for several reasons. First, because Barro (2015) also distinguishes environmental disasters from other types of catastrophes, and suggests calibrations for each type of event. Second, because although subject to debate, these parameters enable to explain financial market data - and in particular the equity premium - for a reasonable value of risk aversion. Thus, even though the values assigned to disasters impacts and disasters probability rely on past events that may arguably have little to do with the future, they still enable to explain how agents take their investment decisions. The same can be said of the rate of time preferences that does not represent the ethical discount rate discussed in the climate literature, but rather the value that best explains market data. Finally, these estimates are rather common in the literature, and can thus be seen as a convenient common benchmark for calibration. For all the previous reasons, I believe this calibration offers a good starting point to discuss quantitatively the mechanisms previously highlighted in the paper. In any case, the closed-form solutions enable to easily replicate the exercise for other parameter values.

5.2 How likely is it that disasters foster economic growth?

Taking the parameters' values in Table I, one can compute the variables of interest, including the optimal risk-mitigation spending, optimal consumption and long run growth. The results are reported in Table II. We obtain that around three quarters of production should be consumed at each period, and 0.315% spent in risk-mitigation. The effect of such investment is to decrease the probability of an environmental disaster by around one quarter, from 1% to 0.76%. The resources left after consumption and risk-mitigation spending are saved, and lead to an annual trend growth rate (i.e. the expected growth rate absent any catastrophe) of 1.49%.

Table I: Parameters used in the calibration

Parameter	Notation	value
Risk aversion coefficient	γ	3.3
Intertemporal elast. of subst.	ϵ	2
Rate of time preferences	ρ	0.029
Gross return from capital	A	0.059
Damages from disasters	$1-\omega$	21%
Ex ante probability of a natural disaster	λ	1%
Probability of an exogenous disaster	$\lambda\delta$	3%
St. dev. of normal shocks per year	σ	2%
Inverse of technology efficiency	α	0.25

Lecture: In the baseline calibration, the risk aversion coefficient is assumed to be equal to 3.3. These parameters' values are used to compute the model's variables of interest.

Note: The value of 0.029 for the rate of time preferences corresponds to the effective rate of time preferences calculated by Barro and Ursua (2008) to match evidences on the risk-free rate of return of assets.

Table II: Variables computed at parameters' baseline value

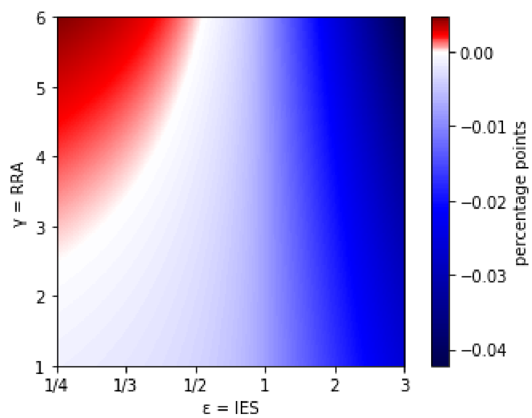
Variable	Notation	value
Share of production consumed	ψ/A	74.5%
Share of production in risk-mitigation	τ^*	0.314%
Reduction in prob. of an env. disaster	$(\tau^*)^\alpha$	23.7%
Trend growth rate	$\frac{dC}{C}^{*,trend}$	1.49%

Lecture: From the expressions computed in Section 4 and using parameters' values reported in Table I, we obtain that the optimal share of production spent in risk-mitigation should be 0.314%.

I then investigate whether one might expect disasters to foster long-run growth. Figure 1 plots for different values of aversion to risk (γ) and inter-temporal elasticity of substitution (ϵ) the effect on growth of introducing disasters to the model. That is, it computes the difference between the expected growth rate of the model as calibrated in Table I, and the one of the same model with $\lambda = 0$ (or $\omega = 1$).

The red area is associated with a net positive impact of disasters on expected growth, while the blue area signals a negative impact. It appears from these figures that, given the parameter values defined in Table I, disasters may foster average long-run growth for high values of risk aversion and aversion to fluctuations (i.e. low elasticity of inter-temporal substitution). For instance, when $\epsilon = 0.5$, the presence of disasters will increase the expected growth rate if and only if $\gamma > 5.633$. For a lower IES $\epsilon = 1/3$, $\gamma > 3.413$ is sufficient for disasters to foster growth, and for $\epsilon = 1/4$ the impact becomes positive for any $\gamma > 2.462$.

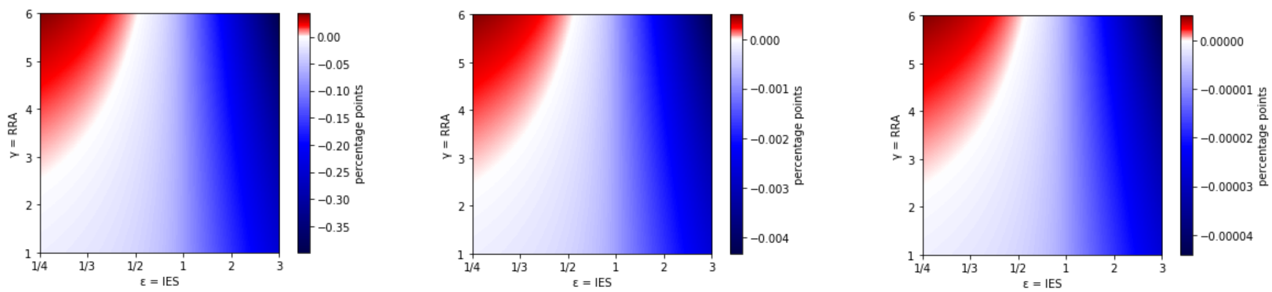
Figure 1: Difference between long run growth in a disaster vs. disaster free economy



Lecture: When all parameters are calibrated following Table I except for γ and ϵ, the expected long-run growth rate is higher in the disaster than in the disaster free economy if and only if γ and ϵ lie in the red area.

Interestingly, Figure 2 shows that these results are barely sensitive to the calibration of disasters' frequency, although the difference in growth rates is exacerbated in both directions for more frequent events. Expected growth being linear in λ, this parameter affects the relative importance of risk for growth, but quantitatively it has no remarkable effect on the link between preferences and expected growth. However, as shown by Figure 3 the results critically depend on the value assigned to disasters intensity. In particular, for very intense disasters (e.g. $1 - \omega > 40\%$), the range of parameter values for which disasters may positively affect growth is much broader than for less intense events. On the contrary, for disasters of small magnitude (e.g. $1 - \omega < 5\%$) the values of γ and ϵ leading to a positive effect of disasters on growth are far beyond what is commonly admitted as plausible in the literature. Intuitively, this effect of ω is due to the concavity of the value function which exacerbates the response to disaster risks relative to the impact of disaster strikes for high expected damages. The effects of λ and ω can be most easily understood in the case of exogenous disasters by looking at the comparative statics in equations (14) and (15). In particular, if the derivative of expected growth with respect to ω will always be relatively close to zero because of the term λ factoring the expression, its sign is very sensitive to the value of risk and risk aversion, hence the highly non-linear effect of disasters intensity on precautionary savings.

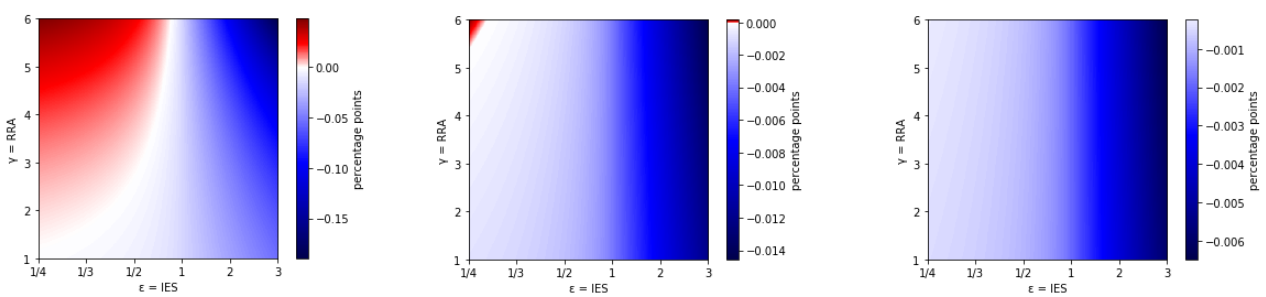
Figure 2: Difference between long run growth in a disaster vs. disaster free economy for various disasters frequencies



$\lambda = 10\%$ $\lambda = 0.1\%$ $\lambda = 0.001\%$

Lecture: These three figures replicate Figure 1 for different values of λ, i.e. for more or less frequent events.

Figure 3: Difference between long run growth in a disaster vs. disaster free economy for various disasters intensities



$1 - \omega = 40\%$ $1 - \omega = 10\%$ $1 - \omega = 5\%$

Lecture: These three figures replicate Figure 1 for different values of ω, i.e. for more or less intense events.

From the previous results, one can wonder how likely it is that an economy lie in the red area. The literature has not reached a clear consensus on the *true* value of the RRA and the IES. The finance literature unambiguously stresses the high value of the RRA. In an attempt to explain the equity premium puzzle, Mehra and Prescott (1985) argue that a reasonable upper bound for the relative risk aversion coefficient is 10. Barro (2009) shows that within a model displaying rare catastrophic events, a value between 3 and 4 is enough to explain the equity premium, and closer to micro evidences. In his framework applied to natural disasters, Barro (2015) takes $\gamma = 3.3$ and shows that given the observed past catastrophic events it enables to explain financial market data. With respect to the IES, the value is even more debated and there exists contrasted evidences on whether it should be taken as above or below unity. The finance literature suggests that the IES is above one. It has been shown by Bansal and Yaron (2004) that in order to explain numerous properties of asset pricing one needs to have simultaneously $\gamma > 1$ and $\epsilon > 1$, which is at odds with expected utility, and in our case suggests that *precautionary consumption* should be favored in front of higher risks on capital. Our baseline calibration taken from Barro (2015) assumes $\epsilon = 2$ which again enables to match evidences from the financial market, and rejects the possibility of a positive link between disasters and growth. Yet, most studies on micro data argue that $\epsilon < 1$ better represents people's preferences (for a recent survey, see Attanasio and Weber (2010)).

This paper does not intend to fix this debate, but the model highlights the implications that bears the value of the IES when studying rare catastrophic events within an endogenous growth framework. If one believes $\gamma = 3.3$ is a good estimate for the RRA, it follows that in the baseline calibration $\epsilon < 0.324$ is necessary in order to get a positive effect of disasters on average long-run growth. This value is far from being implausible in the literature but still leaves us with a certain number of asset pricing puzzles (see Bansal and Yaron (2004)). If agents anticipate disasters potentially less frequent but of higher magnitude than 21% of GDP, then a positive relationship between disasters and growth is possible for higher values of ϵ. The probability of an event however appears to have little effect on the admissible range of parameter values. Interestingly, although we have considered so far the objective and subjective risks to be equivalent, these need not to be the same. A straightforward generalization of the model could distinguish the two and allow for distortions in agents anticipation of future catastrophic events. In that case, if agents over-estimate disasters' intensity, when $\epsilon < 1$ they will over-save relative to the optimum, which would foster growth at the expense of current consumption. On the contrary, in the situation where people under-estimate the risk of disasters, such a positive effect becomes less likely as precautionary savings are low relative to what they should optimally be, and disaster strikes are more likely to offset the positive effects of disaster risks on expected growth. Another possibility is that disaster strikes affect the subjective beliefs about disaster risks. This idea is the source of a growing literature. In particular, several empirical studies (e.g. Akerlof et al (2013), Deryugina (2013), Kaufmann et al (2017)) have shown that weather fluctuations lead people to update their beliefs about climate change. This feature would complicate our framework and come at the expense of analytic solutions. However, it could be interesting to investigate its implications in further studies using numerical methods.

A precise quantitative assessment of the effect of disasters on expected growth is made difficult by the large uncertainty surrounding most parameters. The baseline calibration proposed by Barro (2015) unambiguously rejects the possibility of a positive effect of disasters in this theoretical framework. Whether these parameter values are adequate is debated, but the closed form solutions proposed in the previous section should enable the reader to easily replicate the exercise for any parameter value, and judge the likelihood of a positive link between disasters and growth in this set-up.

5.3 Does using Epstein-Zin-Weil preferences matters quantitatively?

A last issue that the model's calibration can address is to what extent the use of non-expected utility may matter quantitatively. Table III reports the optimum share in production of consumption and risk-mitigation, together with the economy's trend growth rate for alternative specifications of the utility function. It shows that restricting the value of the RRA and the IES to be the same does not impact the optimal risk-mitigation spending as long as this parameter is calibrated as the RRA. However, in this situation it leads to substantial deviations from our baseline results with respect to the optimal consumption savings decisions and thus with respect to the growth rate. In particular, under the null hypothesis that the Epstein-Zin-Weil utility as calibrated in Table I correctly represents individuals preferences, using a time-additive power utility with a parameter calibrated as the RRA leads to over-estimate the trend growth rate by a factor of around 2.7, from 1.49% to 4.02%. Indeed, relative to our baseline calibration, this specification calls for strong precautionary savings and therefore overestimates the economy's optimal growth rate. Turning to the even more restrictive case of a logarithmic utility, it appears that the results deviate from the baseline not only with respect to the trend growth rate, but also with respect to the risk-mitigation policy. Under our null hypothesis, the restriction imposed by the logarithmic utility leads to overestimate the trend growth rate by a factor of 2, and to underestimate the optimal share of production spent in risk-mitigation by around a third.

Table III: Variables computed for alternative utility functions

Variable	EZW utility	CRRA utility	log-utility
Calibration	$\gamma = 3.3$, $\epsilon = 2$	$\gamma = 1/\epsilon = 3.3$	$\lim \gamma = 1/\epsilon \to 1$
ψ/A	74.5%	31.5%	49.2%
τ^*	0.314%	0.314%	0.215%
$\frac{dC}{C}^{*,trend}$	1.49%	4.02%	2.99%

Lecture: When using Epstein-Zin-Weil utility as calibrated in Table I, the optimal share of production spent in risk-mitigation (τ^*) is 0.314%. Everything else being equal, using a time-additive power utility when RRA is also 3.3 gives the same value of τ^*. When log-utility is used instead, τ^* falls to 0.215%.

Overall, it appears that using the restrictive class of time-additive power utility in dynamic stochastic models of disasters may lead not only to qualitative mis-interpretations, but also to large quantitative errors. In particular, when calibrating the free parameter of the utility as the RRA, one largely overestimates the economy's optimal saving rate, and thus the optimal growth rate. On the other hand, when taking lower values to better match evidences regarding the IES, it leads to largely underestimate the optimal effort that should be performed to mitigate risks. These results confirm that our analytic evidences matter quantitatively. They also bring support to previous studies that introduced Epstein-Zin-Weil utility in Integrated Assessment Models (IAMs) of the climate literature (see Crost and Traeger (2014), Jensen and Traeger (2014), Cai et al (2015)), and showed numerically that it implied a higher carbon price. Although the use of non-expected utility may require intensive computations in these models, the present results suggest that the choice of the utility function should be taken seriously as the effect on the model's output are of first order.

6 Conclusion

This paper proposed a stylized model of endogenous growth with disasters in a framework where individuals exhibit recursive preferences. The model is fully solved analytically and closed-form solutions are given for optimal consumption, savings, risk mitigation policies and growth. The numerous mechanisms through which disasters may affect growth were discussed with an emphasis on how they each depend on preferences for risk on the one side, and inter-temporal fluctuations on the other. The ability to disentangle these two concepts appeared to be critical as they end up playing very different roles. In a calibration of the model, it has then been shown that the qualitative differences implied by non-expected utility relative to more restrictive preferences (such as the standard time-additive power utility or the logarithmic one) translated into large quantitative differences. The choice of these more restrictive specifications does not only lead to incorrect interpretations of analytic results, but also to potentially large bias in numerical estimations.

This analysis should be taken as a first step towards a better understanding of the effect of preferences on the link between disasters and growth. To keep the model tractable and as intuitive as possible, a certain number of potentially relevant mechanisms have been left aside. In particular, the literature has shown that when facing disasters, the possibility to switch from physical to human capital could have important implications (see Akao and Sakamoto (2016), Ikefuji and Horii (2012), Bakkensen and Barrage (2016)). Disasters could also positively impact productivity through a "build back better" effect (Hallegate and Dumas (2009)). The model is also silent about the possibility for the economy to insure against some risks (as investigated by Ikefuji and Horii (2012) and Müller-Fürstenberger and Schumacher (2015)), or to trade with other economies that would not experience similar disasters at the same moment. Another interesting extension would be to consider disasters endogenous not only to risk-mitigation activities, but to a pollution stock to model some of the effects of climate change. All these fascinating elements should be seen as avenues for future research. Given the important welfare implications of disasters, I believe a lot of efforts are needed to improve our understanding of their link with the economy.

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Appendices

A General framework

We assume preferences from consumption can be represented by the following utility function:

$$(1-\gamma)U_t = \left[C_t^{\frac{\epsilon-1}{\epsilon}} dt + e^{-\rho dt}\left((1-\gamma)\mathbb{E}U(t+dt)\right)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}} \tag{24}$$

where ρ is the pure rate of time preferences, γ the coefficient of relative risk aversion, and ϵ the intertemporal elasticity of substitution. The recursive form of the utility yields the following Hamilton Jacobi Bellman (HJB) equation:

$$(1-\gamma)V(K_t) = \max\left[C_t^{\frac{\epsilon-1}{\epsilon}} dt + e^{-\rho dt}\left((1-\gamma)\mathbb{E}V(K_{t+dt})\right)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}} \tag{25}$$

The law of capital accumulation is defined as:

$$dK_t = [Y_t - \sum_{j=1}^{m}\tau_{j,t}Y_t - C_t]dt + \sigma_{w,t}dz - \sum_{i=1}^{n}\sigma_{i,p,t}dq_{i,t} \tag{26}$$

where dz is a Wiener process with scaling parameter σ_w, and $dq_{i,t}$ a Poisson process with endogenous parameter, i.e. $\mathbb{E}dq_{i,t} = \lambda_i f_i dt$ with λ_i a constant and f_i a function of abatement activities to be defined. Shocks are also supposed to be of endogenous size, and we denote $\tilde{K}_i$ the stock of capital after a shock from the i^{th} Poisson process occurred. From the stochastic law of capital accumulation, one can substitute for the expectation term in equation (25) using the change of variable formula and Îto's lemma, which yields:

$$\mathbb{E}V(K_{t+dt}) = V(K_t)+\mathbb{E}dV(K_t) = V(K_t)+V_k[(1-\sum_{j=1}^{m}\tau_{j,t})Y_t-C_t]+\frac{1}{2}V_{kk}(\sigma_w dz)^2+\sum_{i=1}^{n}\mathbb{E}\left(V(\tilde{K}_{i,t})-V(K_t)\right)dq_{i,t}$$

$$= V(K_t) + V_k[(1-\sum_{j=1}^{m}\tau_{j,t})Y_t - C_t]dt + \frac{1}{2}V_{kk}\sigma_w^2 dt + \sum_{i=1}^{n}\lambda_i f_i\left(V(\tilde{K}_{i,t}) - V(K_t)\right)dt$$

Substituting back into the HJB equation (25) gives:

$$(1-\gamma)V(K_t) = \max\left[C_t^{\frac{\epsilon-1}{\epsilon}}dt + e^{-\rho dt}\left((1-\gamma)V(K_t) + (1-\gamma)V_k[(1-\sum_{j=1}^{m}\tau_{j,t})Y - C]dt + (1-\gamma)\frac{1}{2}V_{kk}\sigma_w^2 dt \right.\right.$$
$$\left.\left. + (1-\gamma)\sum_{i=1}^{n}\lambda_i f_i\left(V(\tilde{K_{i,t}}) - V(K_t)\right)dt\right)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}} \tag{27}$$

Then, following the strategy used by Epaulard and Pommeret (2003), we denote :

$$X(K,C,\tau) = V_k[(1-\sum_{j=1}^{m}\tau_{j,t})Y - C] + \frac{1}{2}V_{kk}\sigma_w^2 + \sum_{i=1}^{n}\lambda_i f_i\left(V(\tilde{K_{i,t}}) - V(K_t)\right)$$

where τ is the vector of all τ_j, $j = 1, ..., m$. Making use of two approximations when dt is small enough, $e^{-\rho dt} \simeq 1 - \rho dt$ and $(1 + xdt)^a \simeq 1 + axdt$, we have:

$$(1-\gamma)V(K_t) = \max\left[C_t^{\frac{\epsilon-1}{\epsilon}}dt + (1-\rho dt)\left((1-\gamma)V(K_t)\left[1 + \frac{X(K,C,\tau)dt}{V(K_t)}\right]\right)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}}$$

$$\Leftrightarrow \quad (1-\gamma)V(K_t) = \max\left[C_t^{\frac{\epsilon-1}{\epsilon}}dt + (1-\rho dt)\left((1-\gamma)V(K_t)\right)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\left(\left[1 + \frac{\epsilon-1}{\epsilon(1-\gamma)}\frac{X(K,C,\tau)dt}{V(K_t)}\right]\right)\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}}$$

$$\Leftrightarrow \quad (1-\gamma)V(K_t) = \max\left[C_t^{\frac{\epsilon-1}{\epsilon}}dt + (1-\rho dt)\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\right.$$
$$\left. + (1-\rho dt)\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\frac{\epsilon-1}{\epsilon(1-\gamma)}\frac{X(K,C,\tau)dt}{V(K_t)}\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}}$$

and because $dt^2 = 0$, we can simplify the expression:

$$(1-\gamma)V(K_t) = \max\left[\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\right.$$
$$\left. + \left(C_t^{\frac{\epsilon-1}{\epsilon}} - \rho\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}} + \frac{\epsilon-1}{\epsilon(1-\gamma)}\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\frac{X(K,C,\tau)}{V(K_t)}\right)dt\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}}$$

$$\Leftrightarrow \quad (1-\gamma)V(K_t) = \max(1-\gamma)V(K_t)$$
$$\times\left[1 + \frac{\left(C_t^{\frac{\epsilon-1}{\epsilon}} - \rho\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}} + \frac{\epsilon-1}{\epsilon(1-\gamma)}\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}\frac{X(K,C,\tau)}{V(K_t)}\right)dt}{\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}}\right]^{\frac{\epsilon(1-\gamma)}{\epsilon-1}}$$

$$\Leftrightarrow \quad 0 = \max \frac{\epsilon(1-\gamma)}{\epsilon - 1} \frac{\left(C_t^{\frac{\epsilon-1}{\epsilon}} - \rho \left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}} + \frac{\epsilon-1}{\epsilon(1-\gamma)} \left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}} \frac{X(K,C,\tau)}{V(K_t)} \right)}{\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}}}$$

$$\Leftrightarrow \quad \rho \frac{\epsilon(1-\gamma)}{\epsilon-1} V(K_t) = \max \left[\frac{\epsilon}{\epsilon-1} \frac{C_t^{\frac{\epsilon-1}{\epsilon}}}{\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}} + X(K,C,\tau) \right] \tag{28}$$

From the previous equation we obtain the following first order conditions with respect to C and τ_j:

$$\frac{C_t^{-\frac{1}{\epsilon}}}{\left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}} + X_C = 0 \tag{29}$$

$$X_{\tau_j} = 0 \qquad \forall j \tag{30}$$

with X_C and X_{τ_j} the derivatives of X with respect to C and τ_j, hence:

$$C_t^{-\frac{1}{\epsilon}} = V_k \left[(1-\gamma)V(K_t)\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}$$

and:

$$YV_k = \sum_{i=1}^{n} \lambda_i \left[f_i \frac{\partial V(\tilde{K}_i)}{\partial \tilde{K}_i} \frac{\partial \tilde{K}_i}{\partial \tau_j} + \frac{\partial f_i}{\partial \tau^j} \left(V(\tilde{K}_i) - V(K) \right) \right]$$

B Exogenous disasters

In this section we assume $n = 1$ and $m = 0$, $\tilde{K} = \omega K$ with ω constant, and $f = (1+\delta)$. We also assume $\sigma_w = \sigma K$ and $Y = AK$. The shape of the problem leads to the following guess for the value function:

$$V(K) = \psi^{\frac{1-\gamma}{1-\epsilon}} \frac{K^{1-\gamma}}{1-\gamma}$$

with ψ a constant to be determined. Substituting the guess into the first order condition derived in the previous section gives:

$$C^{-\frac{1}{\epsilon}} = \psi^{\frac{1-\gamma}{1-\epsilon}} K^{-\gamma} (1-\gamma)^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1} (\psi^{\frac{1-\gamma}{1-\epsilon}})^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1} (K^{1-\gamma})^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1} (1-\gamma)^{-\frac{\epsilon-1}{\epsilon(1-\gamma)}+1} = (\psi K)^{-\frac{1}{\epsilon}}$$

$$\Leftrightarrow \quad C^* = \psi K$$

In order to check our guess for the value function is correct, we substitute it into the HJB equation and determine the value of ψ that enables to solve the problem. Recall equation (28):

$$\rho\frac{\epsilon(1-\gamma)}{\epsilon-1}V(K_t) = \max\left[\frac{\epsilon}{\epsilon-1}\frac{C_t^{\frac{\epsilon-1}{\epsilon}}}{[(1-\gamma)V(K_t)]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}} + X(K,C)\right]$$

with $X(K,C) = V_k[AK-C] + \frac{1}{2}V_{kk}\sigma_w^2 + \lambda(1+\delta)\left(V(\tilde{K}) - V(K)\right)$ and $V(K) = \psi^{\frac{1-\gamma}{1-\epsilon}}\frac{K^{1-\gamma}}{1-\gamma}$, so that:

$$\begin{aligned} X(K,C) &= \psi^{\frac{1-\gamma}{1-\epsilon}}K^{-\gamma}[AK-\psi K] - \frac{\gamma\sigma^2}{2}\psi^{\frac{1-\gamma}{1-\epsilon}}\frac{K^{1-\gamma}}{1-\gamma} - \lambda(1+\delta)(1-\omega^{1-\gamma})\psi^{\frac{1-\gamma}{1-\epsilon}}\frac{K^{1-\gamma}}{1-\gamma} \\ &= \psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma}\left[A - \psi - \frac{\gamma\sigma^2}{2} - \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right] \end{aligned}$$

and:

$$\frac{C_t^{\frac{\epsilon-1}{\epsilon}}}{[(1-\gamma)V(K_t)]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}} = \frac{(\psi K)^{\frac{\epsilon-1}{\epsilon}}}{\left[\psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma}\right]^{\frac{\epsilon-1}{\epsilon(1-\gamma)}-1}} = \psi\psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma}$$

Hence, going back to the HJB:

$$\begin{aligned} \rho\frac{\epsilon(1-\gamma)}{\epsilon-1}\psi^{\frac{1-\gamma}{1-\epsilon}}\frac{K^{1-\gamma}}{1-\gamma} &= \max\left[\frac{\epsilon}{\epsilon-1}\psi\psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma} + \psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma}\left[A-\psi-\frac{\gamma\sigma^2}{2} - \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right]\right] \\ \Leftrightarrow \quad \rho\epsilon + (1-\epsilon)A - (1-\epsilon)\frac{\gamma\sigma^2}{2} &- (1-\epsilon)\lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma} = \epsilon\psi + (1-\epsilon)\psi = \psi \end{aligned}$$

So the only remaining unknown, that is the consumption share of capital on the optimal path, is:

$$\psi = \rho\epsilon + (1-\epsilon)\left(A - \frac{\gamma\sigma^2}{2} - \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right) \tag{31}$$

One can then use the law of capital accumulation defined by equation (26) to compute both the optimal saving rate $s^* = S^*/Y$ and the stochastic growth rate of the economy:

$$\begin{aligned} s^* &= \frac{Y-C^*}{Y} = 1 - \frac{\psi}{A} \\ &= \frac{1}{A}\left[\epsilon(A-\rho) + (1-\epsilon)\left(\frac{\gamma\sigma^2}{2} + \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right)\right] \end{aligned}$$

and:

$$\frac{dK^*}{K} = \frac{dC^*}{C} = (A-\psi)dt + \sigma dz - (1-\omega)dq_t$$
$$= \left[\epsilon(A-\rho) + (1-\epsilon)\frac{\gamma\sigma^2}{2} + \frac{1-\epsilon}{1-\gamma}\lambda(1+\delta)(1-\omega^{1-\gamma})\right] dt + \sigma dz - (1-\omega)dq_t$$

Finally, using the fact that $\mathbb{E}(dz) = 0$ and $\mathbb{E}(dq_t) = \lambda(1+\delta)dt$, one can easily recover the expected growth rate and the associated comparative statics with respect to risk and risk aversion. The sign of these expression can easily be determined except for the effect of risk aversion. Indeed, the overall effect of risk aversion on expected growth may be positive or negative depending on the value of the IES:

$$\frac{\partial\,\mathbb{E}\left(\frac{dC^*}{C}\right)}{\partial\gamma} = (1-\epsilon)\left(\frac{1}{2} + \lambda(1+\delta)\frac{ln(\omega)\omega^{1-\gamma}(1-\gamma) + (1-\omega^{1-\gamma})}{(1-\gamma)^2}\right) dt \quad \begin{cases} > 0, & \text{if } \epsilon < 1. \\ \leq 0, & \text{otherwise.} \end{cases}$$

Proof #1 : To show this, let's define $g(\gamma) = ln(\omega)\omega^{1-\gamma}(1-\gamma) + (1-\omega^{1-\gamma})$. First, notice that $g(1) = 0$. Then, if we take the derivative of this function, we have:

$$g'(\gamma) = ln(\omega)\left[-ln(\omega)\omega^{1-\gamma}(1-\gamma) - \omega^{1-\gamma}\right] + ln(\omega)\omega^{1-\gamma}$$
$$= -[ln(\omega)]^2\omega^{1-\gamma}(1-\gamma)$$

Thus, for $\omega > 0$, $g'(\gamma) < 0$ for $\gamma < 1$ and $g'(\gamma) > 0$ for $\gamma > 1$, hence $g(1)$ is a global minimum and $g(\gamma) > 0$ for $\omega > 0$ and $\gamma \neq 1$.

C Catastrophes of endogenous probability

In this section we turn to disasters of endogenous probability. We keep the assumption that ω is fixed, but we now take $m = 1$ (i.e. there exist one risk-mitigation instrument) and $f = 1 + \delta - \tau^\alpha$ with $0 < \alpha < 1$. Production still comes from an AK technology and the Wiener process is still scaled by a standard deviation $\sigma_w = \sigma K$. The general form of the problem being the same as in the previous section, we again make the following guess:

$$V(K) = \psi^{\frac{1-\gamma}{1-\epsilon}}\frac{K^{1-\gamma}}{1-\gamma}$$

Substituting the guess into the two first order conditions, and applying our new specification, we obtain:

$$C^* = \psi K$$

and:

$$AKV_k = \lambda\alpha\tau^{\alpha-1}V(K)(1-\omega^{1-\gamma})$$
$$\Leftrightarrow \quad \tau^* = \left(\frac{(1-\omega^{1-\gamma})\lambda\alpha}{A(1-\gamma)}\right)^{\frac{1}{1-\alpha}}$$

It is straightforward to show that τ^* is increasing with λ and γ (see proof #1 above) and decreasing with ω. The effect of α is less obvious, but one can show that τ^* is an increasing function of α if and only if α is below some threshold value $\bar{\alpha}$, and decreasing otherwise.

Proof #2 : Differentiating τ^* with respect to α we get:

$$\frac{\partial\tau^*}{\partial\alpha} = \left(\frac{(1-\omega^{1-\gamma})\lambda\alpha}{A(1-\gamma)}\right)^{\frac{1}{1-\alpha}} \frac{1}{(1-\alpha)^2}\left[ln\left(\frac{(1-\omega^{1-\gamma})\lambda}{A(1-\gamma)}\right) + \frac{1-\alpha}{\alpha}\right]$$

we can see that this derivative is negative if and only if $\frac{1-\alpha}{\alpha} < -ln\left(\frac{(1-\omega^{1-\gamma})\lambda}{A(1-\gamma)}\right)$, the right hand side being a positive constant since for credible parameters values the term contained in the log will be below 1. Then, as $0 < \alpha < 1$ it is obvious that for α close to 0 the derivative will be negative, while for α close to 1 it will be positive. Hence, we have a threshold $\bar{\alpha}$ such that:

$$\frac{\partial\tau^*}{\partial\alpha}\begin{cases} > 0 \text{ for } \alpha < \bar{\alpha} \\ < 0 \text{ otherwise}\end{cases}$$

We can then solve for ψ. The problem is the same as in the case of exogenous shocks except that now:

$$X(K,C,\tau) = \psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma}\left[(1-\tau)A - \psi - \frac{\gamma\sigma^2}{2} - \lambda(1+\delta-\tau^\alpha)\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right]$$

Hence, going back to the HJB:

$$\rho\frac{\epsilon(1-\gamma)}{\epsilon-1}\psi^{\frac{1-\gamma}{1-\epsilon}}\frac{K^{1-\gamma}}{1-\gamma} = \frac{\epsilon}{\epsilon-1}\psi\psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma} + \psi^{\frac{1-\gamma}{1-\epsilon}}K^{1-\gamma}\left[(1-\tau^*)A - \psi - \frac{\gamma\sigma}{2} - \lambda f^*\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right]$$

$$\Leftrightarrow \quad \psi = \epsilon\rho + (1-\epsilon)\left((1-\tau^*)A - \frac{\gamma\sigma^2}{2} - \lambda(1+\delta-\tau^{*\alpha})\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right)$$

and finally, substituting for τ^* we get:

$$\psi = \epsilon\rho + (1-\epsilon)\left[A - \frac{\gamma\sigma^2}{2} - \lambda(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma} + (\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})\left(\frac{\lambda(1-\omega^{1-\gamma})}{A^\alpha(1-\gamma)}\right)^{\frac{1}{1-\alpha}}\right]$$

Lastly, we can compute the optimal saving rate and optimal growth rate of this economy starting from the stochastic law of capital accumulation defined by equation (26):

$$s^* = \frac{Y(1-\tau^*) - C^*}{Y} = 1 - \tau^* - \frac{\psi}{A} = 1 - \tau^* - \frac{1}{A}\left[\epsilon\rho + (1-\epsilon)\left((1-\tau^*)A - \frac{\gamma\sigma^2}{2} - \lambda f^*\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right)\right]$$

$$\frac{dK^*}{K} = \frac{dC^*}{C} = [(1-\tau^*)A - \psi]dt + \sigma dz - (1-\omega)dq_t$$

and so the expected growth rate is:

$$\mathbb{E}\left(\frac{dC^*}{C}\right) = [(1-\tau^*)A - \psi - \lambda f^*(1-\omega)]dt \tag{32}$$

We can then compute comparative statics to analyze the incidence of disasters. Differentiating with respect to λ yields:

$$\frac{1}{dt}\frac{\partial\,\mathbb{E}\left(\frac{dC^*}{C}\right)}{\partial\lambda} = -A\frac{\partial\tau^*}{\partial\lambda} - \frac{\partial\psi}{\partial\lambda} - f^*(1-\omega) - \lambda(1-\omega)\frac{\partial f^*}{\partial\lambda} \tag{33}$$

with:

$$-A\frac{\partial\tau^*}{\partial\lambda} = -A\frac{\lambda^{\frac{\alpha}{1-\alpha}}}{1-\alpha}\left(\frac{(1-\omega^{1-\gamma})\alpha}{A(1-\gamma)}\right)^{\frac{1}{1-\alpha}} < 0$$

$$-\frac{\partial\psi}{\partial\lambda} = (1-\epsilon)\left[(1+\delta)\frac{(1-\omega^{1-\gamma})}{1-\gamma} - \lambda^{\frac{\alpha}{1-\alpha}}\frac{(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})}{1-\alpha}\left(\frac{(1-\omega^{1-\gamma})}{A^\alpha(1-\gamma)}\right)^{\frac{1}{1-\alpha}}\right] \quad \begin{cases} > 0, & \text{if } \epsilon < 1. \\ \leq 0, & \text{otherwise.} \end{cases}$$

$$-f^*(1-\omega) = -(1-\omega)\left[1+\delta - \left(\frac{(1-\omega^{1-\gamma})\lambda\alpha}{A(1-\gamma)}\right)^{\frac{\alpha}{1-\alpha}}\right] < 0$$

$$-\lambda(1-\omega)\frac{\partial f^*}{\partial \lambda} = \frac{(1-\omega)\alpha}{1-\alpha}\left(\frac{(1-\omega^{1-\gamma})\lambda\alpha}{A(1-\gamma)}\right)^{\frac{\alpha}{1-\alpha}} > 0$$

and similarly with respect to ω:

$$\frac{1}{dt}\frac{\partial\,\mathbb{E}\left(\frac{dC^*}{C}\right)}{\partial \omega} = -A\frac{\partial \tau^*}{\partial \omega} - \frac{\partial \psi}{\partial \omega} + \lambda f^* - \lambda(1-\omega)\frac{\partial f^*}{\partial \omega} \tag{34}$$

with:

$$-\frac{\partial \tau^*}{\partial \omega}A = A\left(\frac{\lambda\alpha}{A}\right)^{\frac{1}{1-\alpha}}\frac{\omega^{-\gamma}}{1-\alpha}\left(\frac{(1-\omega^{1-\gamma})}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}} > 0$$

$$-\frac{\partial \psi}{\partial \omega} = -(1-\epsilon)\omega^{-\gamma}\left[\lambda(1+\delta) - \lambda^{\frac{1}{1-\alpha}}\frac{(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}})}{1-\alpha}\left(\frac{(1-\omega^{1-\gamma})}{A(1-\gamma)}\right)^{\frac{\alpha}{1-\alpha}}\right] \quad \begin{cases} < 0, & \text{if } \epsilon < 1. \\ \geq 0, & \text{otherwise.} \end{cases}$$

$$\lambda f^* = -\lambda(1+\delta) + \lambda\left(\frac{(1-\omega^{1-\gamma})\lambda\alpha}{A(1-\gamma)}\right)^{\frac{\alpha}{1-\alpha}} > 0$$

$$-\lambda(1-\omega)\frac{\partial f^*}{\partial \omega} = -\frac{1-\omega}{1-\alpha}\left(\frac{(1-\omega^{1-\gamma})\lambda\alpha}{A(1-\gamma)}\right)^{\frac{\alpha}{1-\alpha}-1}\frac{\lambda^2\alpha^2}{A}\omega^{-\gamma} < 0$$

D With multiple catastrophes of endogenous probability and endogenous magnitude

We now turn to the case where the capital stock is subject to shocks coming from two independent Poisson processes (i.e. $n = 2$) with different frequencies and intensities. As in section 4, the probability of a shock of type 1 is assumed endogenous to risk-mitigation activities τ_1, and $\mathbb{E}dq_t^1 = \lambda_1 f_1 dt$ with $f_1 = 1+\delta-\tau_1^{\alpha_1}$. Its intensity is again supposed to be a fixed proportion of the capital stock and $\tilde{K_1} = \omega_1 K_1$. However, we now have an additional process whose probability will be assumed exogenous and simply equal to $\mathbb{E}dq_t^2 = \lambda_2 dt$, but whose intensity will be endogenized. The specification of this second process roughly follows the one proposed by Bretschger and Vinogradova (2017). For simplicity, we abstract from the modelling of pollution as can be found in their paper, and simply assume shocks depend on some adaptation efforts τ_2 such that $\tilde{K_2} = K - (\nu - \alpha_2\tau_2)K$. We consider τ_2 as the share of production spent in adaptation policies as it enables to reduce the negative impact of disasters but does not reduce their likelihood. The share of capital that remains after a shock is denoted $\omega_2(\tau_2) = 1 - \nu + \alpha_2\tau_2$, and ν is therefore the share of capital destroyed by disasters absent any adaptation activity. For simplicity we consider the case without Brownian motion so that $\sigma_w = 0$. As in the previous section, production is derived from an AK technology. Making a similar guess as before, we have:

$$C^* = \psi K \tag{35}$$

$$\begin{aligned} AKV_k &= \lambda_1\alpha_1\tau_1^{\alpha_1-1}V(K)(1-\omega_1^{1-\gamma}) \\ \Leftrightarrow \quad \tau_1^* &= \left(\frac{(1-\omega_1^{1-\gamma})\lambda_1\alpha_1}{A(1-\gamma)}\right)^{\frac{1}{1-\alpha_1}} \end{aligned} \tag{36}$$

and:

$$\begin{aligned} AKV_k &= \lambda_2\frac{\partial V(\tilde{K_2})}{\partial \tilde{K_2}}\frac{\partial \tilde{K_2}}{\partial \tau_2} = \lambda_2\omega_2^{-\gamma}V_k\alpha_2 K \\ \Leftrightarrow \quad \omega_2^* &= \omega_2(\tau_2^*) = \left(\frac{\lambda_2\alpha_2}{A}\right)^{\frac{1}{\gamma}} \end{aligned} \tag{37}$$

hence:

$$\tau_2^* = \frac{\omega_2^* - (1-\nu)}{\alpha_2} \tag{38}$$

The expression of τ_1^* remains the same as in section 4. Interestingly, the adaptation policy τ_2^* solely depends on the efficiency of the technology α_2, and on the difference between the share of capital remaining after catastrophes at equilibrium, ω_2^*, relative to the case absent adaptation policies, $1-\nu$. The share of capital preserved at equilibrium depends positively on the probability of an adverse event λ_2, on the efficiency of adaptation technology α_2, and negatively on the interest rate A. Given that $0 < (\lambda_2\alpha_2)/A < 1$, risk aversion γ also plays positively on ω_2^*. Thus, as for the first instrument τ_1^*, risk and risk aversion positively affect the optimal instrument τ_2^*, but the efficiency of the instrument α_2 has an ambiguous effect.

Given the independence of the two catastrophes and of the two instruments, the share of output that should optimally be spent to mitigate each catastrophe is not affected by the existence of the other. Contrary to Martin and Pindyck (2015) who investigate the binary decision to undertake a project to avert or not a catastrophe when facing multiple types of disasters, standard cost-benefit analysis holds in this framework. For each catastrophe, the marginal cost of mitigation efforts should equate the marginal benefits of reducing this specific catastrophe. However, because each catastrophe impacts the trajectory of output, the amounts of resources spent in each instrument $\tau_1^* Y_t$ and $\tau_2^* Y_t$ depend on the *existence* and *realization* of other catastrophes as well. The full trajectory of output Y_t can be determined applying similar methods than the ones used in the previous specifications. With:

$$X(K, C, \tau) = \psi^{\frac{1-\gamma}{1-\epsilon}} K^{1-\gamma} \left[(1 - \tau_1 - \tau_2)A - \psi - \lambda_1 (1 + \delta - \tau_1^{\alpha_1}) \frac{(1 - \omega_1^{1-\gamma})}{1-\gamma} - \lambda_2 \frac{(1 - (\omega_2^*)^{1-\gamma})}{1-\gamma} \right]$$

we find:

$$\psi = \epsilon\rho + (1-\epsilon) \left((1 - \tau_1^* - \tau_2^*)A - \lambda(1 + \delta - (\tau_1^*)^{\alpha}) \frac{(1 - \omega^{1-\gamma})}{1-\gamma} - \lambda_2 \frac{(1 - (\omega_2^*)^{1-\gamma})}{1-\gamma} \right)$$

Once ψ is obtained, one can easily plug this result into the stochastic law of motion of capital and compute the stochastic and expected growth rate of this economy. The results provide similar intuitions to the one discussed in section 4.