Carbon Capture and Storage with Enhanced Recovery

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Abstract

The use of carbon capture and storage (CCS) is an appealing option to meet the ambitious objectives of the Paris Agreement. CO_2 emissions can also be injected in active oil fields to perform enhanced oil recovery (EOR). We study a dynamic model of CCS and EOR of an economy subject to a cap on the admissible atmospheric CO_2 concentration. CCS can occur in inert reservoirs or in oil fields implementing EOR. We show that if the economy implements EOR it must do so at the beginning of the planning period.

Keywords: carbon pollution; carbon capture and storage; enhanced oil recovery; non-renewable resources; renewable resources.

JEL classifications: Q30, Q35, Q42, Q54.

1 Introduction

In order to attain the objectives of an at most $+2^0C$ increase of the mean temperature set up by the COP 21, or the Paris (pseudo) Agreement, the consumption path of fossil fuels should be drastically downgraded and a considerable stock of polluting fossil fuels should be left underground.[1]

On the oil extraction side it is well known that already large part of the exploited oil fields will be left underground due to the progressive decline of the pressure in the reservoirs, because of the decreasing volume of oil within the reservoirs and the gas leaks, both generated by the extraction process itself. The resistance to the move of oil within the reservoirs toward the extraction wells depends upon the geological properties of the fields. A first formal expression of this resistance has been given by d'Arcy (1856) in another context.[2] Recently Mason and Van 't Veld (2013) have proposed a new model of the oil industry based on what is known as the d'Arcy Law.[3] According to some main oil companies only 30% to 40% of the oil reserves of the exploited fields are extracted in the end.

To restore the pressure and capture *in fine* a larger part of the reserves a simple idea is to inject some fluid into the reservoir that is maybe inert from a chemical point of view, or triggering an expansion of the gas remaining within the reservoir by chemical reactions.[4] But the process has some cost and had not been developed on a large scale before the advent of the first oil shock. Figure 1 records the number of Enhanced Oil Recovery (EOR) projects in operation globally since the beginning of the seventies.

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[1]Although estimating the amount of available fossil resources is an intrinsically hazardous exercise (Mc Glade, 2012), the percentage of fossil fuels having to be left unburned could be as high as 60% to 80 %. More precisely, according to Mc Glade and Ekins (2014, 2015), given the objective of a maximum $+2^0$ temperature increase not to be crossed at whatever time, the percentage of coal, gas an oil having not to be yet exploited at 2050 would amount respectively to 80% for coal, 50% for gas and 33% for oil. See Rezai and Van der Ploeg (2017-a, 2017-b and 2018) for more on the subject, and (2017-c) for the financial implications.

[2]D'Arcy was in charge of the water fountains of the city of Dijon, France.

[3]See also Anderson *et al.*, 2018.

[4]On the different methods used to enhance the oil recovery see for example Mischenko (2001).

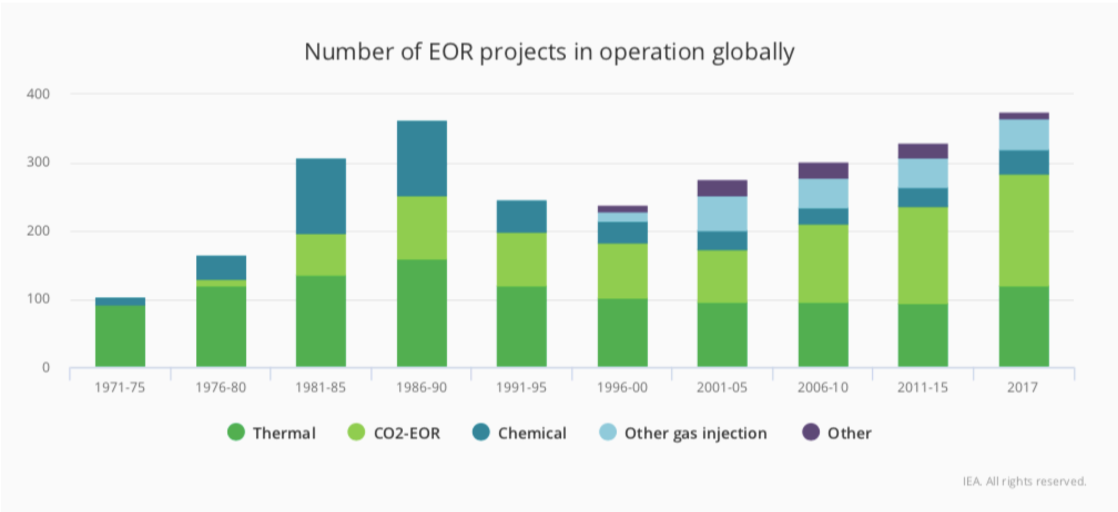

Figure 1: **EOR projects trend at a global scale.**

Figure 2 illustrates how the oil extraction rate can be boosted by the use of such enhancement methods in the field of Weyburn (Saskatchewan, Canada).[5]

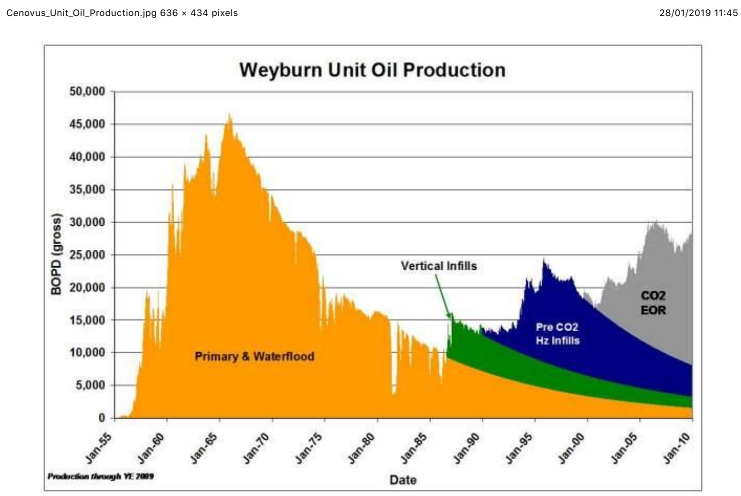

Figure 2: **Oil recovery rates from the Weyburn project.**

[5]See Muggeride *et al.* (2014, Figure 8, p 13) for a less spectacular increase of the extraction rate induced by the introduction of EOR.

Early models of dynamic control of the injection flow at the firm level are the model of Amit (1986) and later, in the same spirit, the model of Cairns and Davis (2001). Roughly at the same time the climate change problem was emerging and was becoming more and more acute. Hence why not kill two birds with the same stone by injecting CO_2 rather than other fluids? Nobody has ever considered that injecting CO_2 into the oil reservoirs could supply all the storage capacities required by a Carbon Capture and Sequestration (CCS) policy sufficiently strong to significantly improve the climate conditions.[6] But that can still help. Simultaneously, an active taxation policy of the CO_2 emissions, or, alternatively, an active subsidy policy of negative emissions, increases the profitability of the EOR process which was estimated rather low by Leach *et al.* (2009) for the field they studied twenty years ago.[7] However, it seems that such a device is becoming more attractive now since oil companies like Total recently announced five EOR projects to start within the next two years.

Here we present a simplified exploratory model of an oil industry which can resort to EOR processing in a society subject to a cap on the stock of carbon pollution. To keep the model tractable we occasionally use a linear-quadratic specification of the model. The model is laid down in Section 2 and the optimality conditions of the social planner problem are stated in Section 3. Next, we compare EOR and no-EOR policies in case of a non-binding ceiling in Section 4. In Section 5 we examine CCS policies with or without EOR in an economy facing an effective carbon budget constraint. The last Section 6 concludes.

2 Notation and model assumptions.

The economy can produce useful energy from two energy sources. The first one is a polluting fossil non-renewable resource (oil). Let $X(t)$ be the available fossil resource stock at time t and let $x(t)$ denote the extraction rate of fossil fuel, so that:

$$\dot{X}(t) = -x(t). \tag{2.1}$$

Let $X^0 = X(0)$ be the initial oil endowment. The other source is renewable and non-polluting (solar). Let $y(t)$ be the production rate of solar energy. We assume a constant unit production cost of solar energy, c_y, thus a total cost $c_y y$. At the end-user stage useful energies produced from any source

[6]See Herzog (2009) for a first account of the CCS option, without reference to the CCS-EOR coupling.

[7]See Van 't Veld *et al.* (2013) for a pessimistic view of the importance of CCS-EOR.

are perfect substitutes and assuming that energy is not storable we have $q(t) = x(t) + y(t)$, where $q(t)$ denotes the consumption rate of useful energy.

Let $U(q)$ be the gross surplus function. We assume that it is twice continuously differentiable and increasing, with $U'(q) > 0$. At some point we will adopt a specific quadratic form: $U(q) = \beta q - \gamma q^2/2$, so that $U'(q) = \beta - \gamma q$. For this form to make sense we must assume that $q < \beta/\gamma$. In order that solar energy be able to meet all energy demand, we also assume that $c_y < \beta$.

Carbon pollution

Burning fossil fuels $x(t)$ to produce useful energy generates greenhouse gases, $\zeta x(t)$, in particular CO_2. The fossil energy transformation industry has access to an abatement technology able to capture these gases before they are released into the atmosphere. We denote by $a(t)$ the abatement rate of greenhouse gases. Let $Z(t)$ denote the atmospheric carbon stock at time t and $Z^0 = Z(0)$ the initial atmospheric stock inherited from the past. If, as we assume, carbon pollution in the atmosphere does not decay, the motion of the CO_2 stock is given by:

$$\dot{Z}(t) = \zeta x(t) - a(t). \tag{2.2}$$

Assuming that capturing CO_2 directly from the atmosphere is prohibitively costly, the captured flow is bounded from above by the potential emission flow, that is $x(t)$ and $a(t)$ are subject to the constraint; $\zeta x(t) - a(t) \geq 0$.

As in Chakravorty *et al.*, (2006), we assume that if some critical atmospheric concentration threshold $\bar{Z}$ is crossed by the accumulated CO_2, the climate conditions on earth become catastrophic. Thus, society should stick to the ceiling constraint, $\bar{Z} - Z(t) \geq 0$. Below the ceiling we assume that climate damages are negligible. In order that the model makes sense, we assume that $Z^0 < \bar{Z}$: Initially, the carbon constraint is not binding.

Abatement and carbon sequestration

Let c_a denote the constant carbon capture unit cost, thus a total capture cost $c_a a$. There exist two possible carbon sinks. Captured gases can be stored at no cost in inert reservoirs that are sufficiently large to never face storage limits.[8] Let $b(t)$ be the captured gas flow stored in these carbon reservoirs. The other carbon sinks are provided by the oil extractive industry which injects the captured gases into the oil wells at a rate denoted by $s(t)$. Full

[8]See Lafforgue *et al.* for a model with a carbon ceiling and storage limits in carbon sinks.

sequestration of captured gases implies $a(t) - s(t) - b(t) = 0$ so that (2.2) can be written as:

$$\dot{Z}(t) = \zeta x(t) - b(t) - s(t). \tag{2.3}$$

Let $S(t)$ denote the sequestered carbon emissions stock in the oil wells at time t and let $S^0 = S(0)$ denote the initial gas stock in the oil wells. The motion of the gas stock in the oil wells obeys:

$$\dot{S}(t) = s(t). \tag{2.4}$$

We assume a constant unit sequestration cost c_s, thus a total sequestration cost, $c_s s(t)$.

Oil extraction costs

The gas injected in the oil wells contributes to an increase of the pressure in the wells, easing the oil extraction process. To formalize this idea we assume that the unit extraction cost decreases with the matter content of the oil well. Assume that oil and CO_2 can mix perfectly in the reservoir.[9] Let $\bar{R}$ denote the size of the reservoir. Then the ratio $(X + \alpha S)/\bar{R}$ is a measure of the pressure in the oil field, α being a conversion parameter. Normalize the size of the reservoir to unity so that $X + \alpha S$ measures the pressure. Let $H(X + \alpha S)$ denote the unit cost function as a function of the pressure, thus a total extraction cost $H(X + \alpha S)x$. The unit cost function $H(.)$ is twice continuously differentiable and decreasing, with $H'(.) < 0$. In what follows we occasionally specify the model by considering a linear form for the $H(.)$ function: $H(B) = \psi - \delta B$, so that H is now bounded from below by $\psi - \delta(X^0 + \alpha S^0)$. To have $H \geq 0$, we must then also assume that $X + \alpha S \leq \psi/\delta$. We assume throughout that $\alpha\zeta < 1$ in order to avoid the possibility that extraction may become cheaper over time by excessively increasing the pressure in the well.

3 The social planner's problem

The planner must determine an oil extraction policy, a solar energy production policy, a carbon sequestration policy in inert reservoirs and a carbon emissions injection policy in the oil wells maximizing the social welfare, that

[9]See Mischenko (2001) for a description of the CO_2 injection process in petroleum fields.

is, solve the following $(S.P.)$ problem:

$$\begin{array}{ll}
\max\limits_{x,y,b,s} & \displaystyle\int_0^\infty \{U(x+y) - H(X+\alpha S)x - c_y y - c_a b - (c_s + c_a)s\}\, e^{-\rho t} dt \\
s.t. & (2.1)\ ,\ (2.3)\ ,\ (2.4), \\
& Z(t) - \bar{Z} \geq 0, \\
& \zeta x - b - s \geq 0, \\
& x \geq 0\ ,\ y \geq 0\ ,\ a \geq 0\ , b \geq 0\ , s \geq 0\ .
\end{array}$$

Here $\rho > 0$ is the constant social discount rate. We omit the time index t when there is no danger of confusion. Let λ_s, λ_x and λ_z denote the co-state variables associated with S, X and Z, respectively. The present value Hamiltonian of the $(S.P.)$ problem reads:

$$\begin{aligned}
\mathcal{H} &= \{U(x+y) - H(X+\alpha S)x - c_y y - c_a b - (c_a + c_s)s\}\, e^{-\rho t} \\
&\quad + \lambda_s s - \lambda_x x + \lambda_z(\zeta x - b - s)\ .
\end{aligned}$$

Denote by γ's the Lagrange multipliers associated with the positivity constraints on x, y, b and s and by μ the Lagrange multiplier associated with the full abatement constraint, $\zeta x - a \geq 0$. Let ν_z denote the multiplier associated to the carbon budget constraint, $\bar{Z} - Z \geq 0$. The Lagrangian of the $(S.P.)$ problem reads:

$$\begin{aligned}
\mathcal{L} &= \mathcal{H} + \gamma_x x + \gamma_y y + \gamma_b b + \gamma_s s \\
&\quad + \mu(\zeta x - b - s) + \nu_z(\bar{Z} - Z)\ .
\end{aligned}$$

A first set of necessary conditions is:

$$e^{-\rho t}U'(x) = H(X+\alpha S)e^{-\rho t} + \lambda_x - \zeta\lambda_z - \zeta\mu - \gamma_x, \tag{3.1}$$
$$e^{-\rho t}U'(x) = c_y e^{-\rho t} - \gamma_y, \tag{3.2}$$
$$-\lambda_z = c_a e^{-\rho t} + \mu - \gamma_b, \tag{3.3}$$
$$\lambda_s - \lambda_z = (c_a + c_s)e^{-\rho t} + \mu - \gamma_s, \tag{3.4}$$

together with the usual complementary slackness conditions. The co-state variables satisfy:

$$\dot{\lambda}_x = e^{-\rho t}H'(X+\alpha S)x, \tag{3.5}$$
$$\dot{\lambda}_s = \alpha e^{-\rho t}H'(X+\alpha S)x, \tag{3.6}$$
$$\dot{\lambda}_z = \nu_z. \tag{3.7}$$

Lastly, the following transversality condition holds:

$$\lim_{t\uparrow\infty} [\lambda_x X + \lambda_s S + \lambda_z Z] = 0\ . \tag{3.8}$$

The interpretation of the necessary conditions is straightforward. Suppose there is an interval of time $\mathcal{T}$ with positive extraction, $x(t) > 0$, for all $t \in \mathcal{T}$. Then there is a present value marginal benefit associated with oil extraction equal to $e^{-\rho t}U'(q)$. Equation (3.1) states that this marginal benefit must balance the full oil use marginal cost. The marginal cost associated with extraction consists of the sum of the direct extraction cost, $e^{-\rho t}H(X + \alpha S)$, the marginal cost of extraction now rather than in the future, λ_x, and the marginal cost of accumulating CO_2, $-\zeta\lambda_z$. If the economy implements full abatement of emissions, i.e., $\zeta x = a$, the planner would like to extract less, which brings along a cost of violating the condition $\zeta x - b - s \geq 0$, i.e., the willingness to pay for its relaxation, $\zeta\mu$. If renewables are used, equation (3.2) states that their marginal benefit equals their marginal cost c_y. Capturing gases for storage in the inert reservoirs brings a marginal benefit $-\lambda_z$ in terms of avoided shadow cost of pollution. Equation (3.3) states that this benefit must cover the carbon capture present value marginal cost, possibly augmented by the scarcity rent on available gas in case of full abatement of emissions. Injecting captured emissions in the oil wells brings along an additional benefit λ_s, the gas rent from EOR, in addition to the avoided carbon pollution shadow cost, $-\lambda_z$. This total benefit must balance the sum of the present value capture and injection cost, and, in case of full abatement, the scarcity rent on available gas. This is the meaning of (3.4). The last condition (3.7) states that the shadow cost of pollution, $-\lambda_z > 0$, or, equivalently, the optimal carbon tax, is constant in present value as long as the carbon cap constraint does not yet bind and decreases when it is finally binding.

We first show that simultaneous supply of fossil and renewables is suboptimal.

Lemma 1.

There is no interval of time $\mathcal{T}$ with with $x(t) > 0$ and $y(t) > 0$ for $t \in \mathcal{T}$.

Proof

Suppose, to the contrary, the existence of such an interval. Then $U'(x(t)+y(t)) = c_y$. Two cases present themselves: $s(t) > 0$ and $s(t) = 0$ for $t \in \mathcal{T}$.

If $s(t) > 0$ then $b(t) = a(t) + s(t) > 0$ and $\gamma_b(t) = 0$ so that with (3.4) in (3.1) we have

$$e^{-\rho t}(c_y - H(X(t) + \alpha S(t)) - \zeta(c_a + c_s)) = \lambda_x - \zeta\lambda_s, \quad t \in \mathcal{T} \ .$$

Differentiating with respect to time yields

$$-\rho(c_y - H(X+\alpha S) - \zeta(c_a + c_s)) = H'(X+\alpha S)(\alpha s - \alpha\zeta x).$$

The right-hand side is non-negative since $\zeta x(t) \geq a(t) \geq s(t)$. The left-hand side is negative, so that we obtain a contradiction.

If $s(t) = 0$, the following possibilities arise:

i. $\zeta x(t) > a(t) = s(t) = 0$.

ii. $\zeta x(t) > a(t) > s(t) = 0$.

iii. $\zeta x(t) = a(t) > s(t) = 0$.

If i holds, then $\mu(t) = 0$ and $\lambda_z(t)$ is a constant. Equation (3.1) becomes

$$e^{-\rho t}(c_y - H(X(t) + \alpha S(t))) = \lambda_x(t) - \zeta\lambda_z(t).$$

Differentiation with respect to time taking into account (3.5) yields $c_y = H(X(t) + \alpha S(t))$. But $X(t) + \alpha S(t)$ is not constant. So, i cannot hold.

If ii holds then $\mu(t) = 0$ and $\lambda_z(t)$ is a constant. Moreover, $b(t) = a(t) + s(t) > 0$ so that $\gamma_b(t) = 0$. These facts contradict (3.3).

If iii holds then $b(t) = a(t) + s(t) > 0$ so that $\gamma_b(t) = 0$. Hence $-c_a e^{-\rho t} = \lambda_z(t) + \mu(t)$. Equation (3.1) becomes

$$e^{-\rho t}(c_y - H(X(t) + \alpha S(t)) - \zeta c_a) = \lambda_x(t).$$

Differentiation with respect to time taking into account (3.5) yields $c_y = H(X(t) + \alpha S(t)) + \zeta c_a$. But $X(t) + \alpha S(t)$ is not constant. So iii cannot hold. Q.E.D.

We now show that in the case of carbon capture, $b > 0$ and $s > 0$ cannot be optimal over a non-degenerate time interval, that is the economy should not store CO_2 simultaneously in the inert reservoirs and the oil wells.

Lemma 2

$b(t) = a(t) - s(t) > 0$ if and only if $s(t) = 0$.

Proof

Suppose $b(t) > 0$ and $s(t) > 0$ along some interval of time $\mathcal{T}$. Then $\gamma_b(t) = \gamma_s(t) = 0$ along $\mathcal{T}$.

If $\zeta x(t) > a(t)$, and therefore $\lambda_z(t)$ is constant, we get a contradiction in (3.3) because then $\mu(t) = 0$.

So, $\zeta x(t) = a(t) > s(t) > 0$. Then $\lambda_s(t) = e^{-\rho t}c_s$. Consequently $\alpha H'(X(t) + \alpha S(t))x(t)$ is constant. Moreover, since $\gamma_b(t) = 0$, we have

$$e^{-\rho t}[U'(x(t)) - H(X(t) + \alpha S(t)) - \zeta c_a] = \lambda_x(t).$$

If $\lambda_x(t) = \frac{1}{\alpha}\lambda_s(t)$ we get a contradiction because then $x(t)$ and $X(t) + \alpha S(t)$ are both constant, which cannot be the case. So, let us first assume $\lambda_x(t) > \frac{1}{\alpha}\lambda_s(t)$. Then

$$U'(x) - H(X + \alpha S) - \zeta c_a > \frac{c_s}{\alpha}.$$

Some oil will be left unextracted because $\lambda_x(T_y) > \frac{1}{\alpha}\lambda_s(T_y) \geq 0$. Hence, extract a marginal unit now ($dx > 0$), take care that $\zeta x = a$ by taking $da = \zeta dx$ and make sure that $X + \alpha S$ is unaltered ($ds = -\frac{1}{\alpha}dx$). We see that the benefits are larger than the cost. This contradicts optimality.

Let us then assume $\lambda_x(t) < \frac{1}{\alpha}\lambda_s(t)$. Hence, $\lambda_s(T_y) > 0$, but this contradicts $S(T_y) > 0$. Q.E.D.

We now show that the alternative to EOR ($b > 0$) will only be used after the ceiling has been reached.

Lemma 3.

$b(t) = a(t) - s(t) > 0$ implies $\zeta x(t) = a(t)$.

Proof

Suppose $\zeta x(t) > a(t)$ along some interval of time. Then, along that interval, $\mu(t) = 0$ and $\lambda_z(t)$ is constant. Hence $\gamma_b(t) \neq 0$ and $b(t) = 0$. Q.E.D.

The following Proposition P.1 sums up our results so far:

Proposition P. 1 ***Energy and CCS policies***

a. The economy never consumes oil and renewables simultaneously.

b. If the economy decides to abate carbon emissions, and uses the inert fields, it must fully capture the potential pollution flow.

c. The economy stores captured emissions either in the oil wells or in the inert reservoirs..

It should be noted that maybe at first glance one can argue that $\lambda_s(t) = \alpha\lambda_x(t)$. However, a simple example shows that this need no be the case. Suppose that at some instant of time the resource stock is exhausted, but gas has been inserted in the past. Then it is of no use to increase the inserted gas stock, yielding a zero shadow price, whereas increasing the resource stock would yield a benefit, if the cost would be lower than the cost of renewables.

We assume now, and later prove, that there is a final phase with use of renewables only. This phase starts at a time T_y. Let T_Z be the time at which the carbon cap constraint begins to be active, $Z(T_Z) = \bar{Z}$, or, equivalently, the 'carbon budget', $\bar{Z} - Z^0$, be exhausted. Since after T_y the stock of atmospheric CO_2 remains constant, emissions being zero, we must have $T_Z \leq T_y$, for the carbon cap constraint to be ever active. That the shadow cost of carbon, $-\lambda_z(t)$, decreases during the ceiling phase $[T_Z, T_y)$ means that with the economy coming closer and closer of the final transition to renewable energy, the opportunity cost of the climate burden, the cost of abating all the polluting emissions, should decrease. Before T_Z, the shadow cost of carbon is the value of delaying as much as possible the time at which the economy faces the climate constraint.

If the oil stock is exhausted at time T_Z, then $T_Z = T_y$ and the economy enters the green energy regime at the time it exhausts its carbon budget. It may also be the case that $X(T_Z) > 0$ in which case the economy performs full abatement of carbon emissions during a time interval $[T_Z, T_y)$ until the oil reserves deplete and the economy enters the green economy regime.

4 The market economy

Whether or not in case of a ceiling CCS is needed, depends on how the market economy performs. By the definition we employ here, in the market economy there is no concern for climate and therefore for the accumulation of atmospheric CO_2. But otherwise the market economy functions efficiently and, hence, satisfies the necessary conditions derived in the previous section, disregarding the accumulation of CO_2 altogether. Whether or not the market economy performs optimally depends on the resulting path of accumulated carbon in the atmosphere. The carbon constraint is irrelevant, if $\bar{Z} > \zeta X^0$, since then even without any capture of emissions, the carbon budget is never exhausted by the market economy. Looking into the path of atmospheric CO_2 allows us to see whether the budget constraint is binding or not. In the market economy, EOR is the only motivation for carbon capture. Moreover, $a(t) = s(t)$ for all t. The reason is simple. If $a(t) > s(t) \geq 0$ then $\gamma_b(t) = 0$. We also have $\lambda_z(t) = 0$. Then we have a contradiction in (3.3). Thus there is no CCS in inert reservoirs, $b(t) = 0$, and either $s(t) = a(t) = 0$ or $\zeta x(t) = a(t) = s(t) > 0$.

Our strategy is to first consider paths with no CCS at all. Since $S(0) = S^0$, this means that $S(t) = S^0$ throughout. We then see whether the proposed program satisfies all the necessary conditions, as well as the sufficient conditions. Next we consider full CCS.

4.1 The no-EOR policy

The first period is the period up to T_y. In this period there is no CCS: $\zeta x > s = 0$. After T_y only renewables are used. The Hamiltonian for the first period reads

$$\mathcal{H} = e^{-\rho t}[U(x) - H(X + \alpha S^0)x] + \lambda_x[-x].$$

Necessary conditions include

$$e^{-\rho t}[U'(x) - H(X + \alpha S^0)] - \lambda_x = 0, \tag{4.1}$$

$$-\dot{\lambda}_x = -e^{-\rho t}H'(X + \alpha S^0)x \ . \tag{4.2}$$

This yields the following differential equation:

$$-\rho(U'(x) - H(X + \alpha S^0)) + U''(x)\dot{x} = 0. \tag{4.3}$$

Define $X_y \equiv X(T_y)$. Let us assume for the moment that the equation $c_y = H(B)$ has a positive solution, B_y, smaller than $X^0 + \alpha S^0$. This is for example the case if U satisfies the Inada conditions $\lim_{q\downarrow 0} U'(q) = \infty$ and $\lim_{q\uparrow\infty} U'(q) = 0$ together with $\lim_{z\downarrow 0} H(z) = \infty$ and $\lim_{z\uparrow\infty} H(z) = 0$. Then the boundary conditions for this second-order differential equation in X are $X(0) = X^0$ and $X_y = B_y - \alpha S^0$. Also, $\lambda_x(T_y) = 0$ when $X_y > 0$ and $\lambda_x(T_y) > 0$ if $X_y = 0$. There are two alternatives. One is where $c_y < H(X_y + \alpha S^0)$ for all $X_y \leq X^0$. In this case fossil fuel will never be used and $T_y = 0$. The second alternative is $c_y > H(X_y + \alpha S^0)$ for all $X_y \leq X^0$. Then $X_y = 0$.

Note that since marginal utility has to be continuous over time, $q(t)$ is continuous over time as well. This implies that $y(t)$ jumps up from a nil level before T_y to the positive level $\tilde{y}$, defined as the unique solution of $U'(y) = c_y$, at the left-hand limit at time T_y. On the other hand $x(t)$ must jump down to zero from the level $\tilde{y}$ at the right-hand limit at time T_y.

A necessary condition for a no-EOR policy to be an equilibrium is also that it is suboptimal to use full CCS. Hence, with $\lambda_z(t) = 0$ and $\mu(t) = 0$ for all $t \geq 0$ we need

$$\lambda_s(t) \leq e^{-\rho t}(c_a + c_s) \ .$$

In the case at hand, with $X_y > 0$ by assumption, we have $\lambda_x(T_y) = 0$. Since $\lambda_s(T_y) \geq 0$ it follows that $\lambda_x(t) \leq \lambda_s(t)/\alpha$. Hence, the following must hold:

$$e^{-\rho t}[U'(x) - H(X + S^0)] = \lambda_x = \lambda_s/\alpha \leq e^{-\rho t}(c_a + c_s)/\alpha.$$

Equivalently:

$$U'(x) - H(X + \alpha S^0) \leq (c_a + c_s)/\alpha. \tag{4.4}$$

It is not clear beforehand when this condition holds. Of course it holds for t just before T_y and also for large enough abatement and injection cost. But marginal utility is increasing, since extraction decreases, and marginal extraction costs are increasing. Hence, if $U'(x) - H(X + \alpha S^0)$ is monotonically decreasing and

$$U'(x(0)) - H(X_0 + \alpha S^0) \leq (c_a + c_s)/\alpha, \tag{4.5}$$

then the proposed program is optimal. But if, on the other hand, $U'(x) - H(X + \alpha S)$ is monotonically decreasing and

$$U'(x(0)) - H(X_0 + \alpha S^0) > (c_a + c_s)/\alpha, \tag{4.6}$$

then there must be an initial phase with full CCS. Finally, we cannot neglect the possibility of non-monotonicity of $U'(x) - H(X + \alpha S^0)$.

4.2 EOR policies for specific functional forms.

In order to gain more insight we therefore perform an analysis with the aid of our specific functional forms. The first thing to note is that employing a linear form for the unit extraction cost function has the consequence that the curves $\lambda_s(t)$ and $e^{-\rho t}(c_a + c_s)$ can cross only once. To see this, suppose that for an initial interval of time $\lambda_s(t) < e^{-\rho t}(c_a + c_s)$. If the curves intersect at $t_1 > 0$, then

$$\frac{\dot{\lambda}_s(t_1)}{\lambda_s(t_1)} = \frac{-\alpha\delta x(t_1)}{c_a + c_s}.$$

If they intersect again, say at t_2, in such a way that λ_s is steeper than $e^{-\rho t_2}(c_a + c_s)$ then

$$\frac{\dot{\lambda}_s(t_2)}{\lambda_s(t_2)} = \frac{-\alpha\delta x(t_2)}{c_a + c_s}.$$

But $x(t_2) < x(t_1)$ by construction, which is incompatible with λ_s being steeper at t_2 than at t_1. Hence if it is optimal to start with zero CCS ($a = s = 0$) it is optimal to go on with zero CCS forever, because if the path would switch to full CCS, the two curves would have to cross again which cannot be optimal because otherwise $\lambda_s(T_y) = \alpha\lambda_x(T_y) > 0$, contradicting that $X_y > 0$.

We may conclude that if $X_y > 0$ any optimal path before T_y consists either of a unique extraction phase without EOR only or of a sequence of an initial EOR phase followed by a no EOR phase when the path approaches T_y. It remains to describe the structure of the optimal path when $X_y = 0$. We characterize the optimal paths in the two cases of incomplete depletion and complete depletion of the oil reserves.

Incomplete depletion of the oil stock, $X_y > 0$.

To go to the core of the argument, assume that $S^0 = 0$. For our functional forms the relevant differential equation then reads

$$-\rho[\beta - \gamma x - (\psi - \delta X)] - \gamma\dot{x} = 0.$$

In this case we have

$$\tilde{y} = \frac{\beta - c_y}{\gamma} \text{ and } X_y = \frac{\psi - c_y}{\delta}.$$

So, we need:

$$\beta > c_y \text{ and } 0 < \frac{\psi - c_y}{\delta} < X_0.$$

We posit

$$X(t) = K_0 + K_1 e^{z_1 t} + K_2 e^{z_2 t}.$$

Then

$$\begin{aligned} \dot{X}(t) &= z_1 K_1 e^{z_1 t} + z_2 K_2 e^{z_2 t}, \\ \ddot{X}(t) &= z_1^2 K_1 e^{z_1 t} + z_2^2 K_2 e^{z_2 t}. \end{aligned}$$

Inserting this in the differential equation yields

$$-\rho[\beta + \gamma(z_1 K_1 e^{z_1 t} + z_2 K_2 e^{z_2 t}) - (\psi - \delta X)] + \gamma(z_1^2 K_1 e^{z_1 t} + z_2^2 K_2 e^{z_2 t}) = 0,$$

which is equivalent to:

$$K_1e^{z_1t}(\gamma z_1^2-\rho\gamma z_1-\rho\delta)+K_2e^{z_{21}t}(\gamma z_2^2-\rho\gamma z_2-\rho\delta)-\rho\delta K_0=\rho(\beta-\psi).$$

We solve z from the characteristic polynomial:

$$\gamma z_1^2-\rho\gamma z_1-\rho\delta=0,$$

giving

$$\begin{aligned} z_1 &= \frac{\rho}{2}\left(1-\sqrt{1+4\frac{\delta}{\rho\gamma}}\right), \\ z_2 &= \frac{\rho}{2}\left(1+\sqrt{1+4\frac{\delta}{\rho\gamma}}\right). \end{aligned}$$

The integration constants K_i $(i=0,1,2,)$ and T_y are solution of the following system:

$$\begin{aligned} &K_0=-\frac{\beta-\psi}{\delta}, \\ &X^0=K_0+K_1+K_2, \\ &-z_1K_1e^{z_1T_y}-z_2K_2e^{z_2T_y}=\tilde{y}=\frac{\beta-c_y}{\gamma}, \\ &K_1e^{z_1T_y}+K_2e^{z_2T_y}=X_y-K_0=\frac{\psi-c_y}{\delta}+\frac{\beta-\psi}{\delta}=\frac{\beta-c_y}{\delta}. \end{aligned}$$

The condition

$$U'(x)-H(X)\le(c_a+c_s)/\alpha \tag{4.7}$$

can be written as

$$\beta-\gamma x-(\psi-\delta X)\le(c_a+c_s)/\alpha. \tag{4.8}$$

This condition is satisfied if

$$\beta-\gamma x(0)-(\psi-\delta X^0)\le(c_a+c_s)/\alpha. \tag{4.9}$$

or

$$\beta+\gamma(z_1K_1+z_2K_2)-(\psi-\delta X^0)\le(c_a+c_s)/\alpha. \tag{4.10}$$

Since α, c_a and c_s do not appear in the left-hand side, the condition is satisfied for small enough α and large enough c_a and c_s, as expected in general. We also observe that for large X^0 (e.g., $X^0=\psi/\delta$) the inequality holds for $x(0)$ large enough. However, numerical calculations are needed to verify what large enough means. Define $k_i=K_ie^{z_iT}$, $i=1,2$. Then k_1 and k_2 can be solved from

$$\begin{aligned} -z_1k_1-z_2k_2 &= \frac{\beta-c_y}{\gamma}, \\ k_1+k_2 &= \frac{\beta-c_y}{\delta}. \end{aligned}$$

It follows that

$$\begin{aligned} k_1 &= -\frac{\beta - c_y}{z_1 - z_2}\left(\frac{1}{\gamma} + \frac{z_2}{\delta}\right), \\ k_2 &= \frac{\beta - c_y}{z_1 - z_2}\left(\frac{1}{\gamma} + \frac{z_1}{\delta}\right). \end{aligned}$$

We can now solve T_y from

$$X^0 = -\frac{c_y - \beta}{\delta} + k_1 e^{-z_1 T_y} + k_2 e^{-z_2 T_y}.$$

The derivative of X_0 with respect to T_y is

$$\begin{aligned} \frac{dX_0}{dT_y} &= (z_1)\frac{\beta - c_y}{z_1 - z_2}(\frac{1}{\gamma} + \frac{z_2}{\delta})e^{-z_1 T_y} + (-z_2)\frac{\beta - c_y}{z_1 - z_2}(\frac{1}{\gamma} + \frac{z_1}{\delta})e^{-z_2 T_y} \\ &= \frac{\beta - c_y}{z_1 - z_2}\{\frac{z_1}{\gamma}e^{-z_1 T_y} - \frac{z_2}{\gamma}e^{-z_2 T_y}\} + \frac{\beta - c_y}{z_1 - z_2} z_1 z_2 \frac{1}{\delta}\{e^{-z_1 T_y} - e^{-z_2 T_y}\}. \end{aligned}$$

Since $z_1 < 0$ and $z_2 > 0$ it is easily seen that T_y increases as X^0 increases.

Complete depletion of the oil reserves

Let is now consider the alternative, namely that $c_y > H(X_y)$ for all $X_y \leq X^0$. For our specific functional forms this means that

$$c_y > \psi - \delta X_y = \psi.$$

The second-order differential equation that we need to consider is the same as before and the solution can be written as

$$\begin{aligned} X(t) &= K_0 + K_1 e^{z_1 t} + K_2 e^{z_2 t}, \\ \dot{X}(t) &= z_1 K_1 e^{z_1 t} + z_2 K_2 e^{z_2 t}, \\ \ddot{X}(t) &= z_1^2 K_1 e^{z_1 t} + z_2^2 K_2 e^{z_2 t}. \end{aligned}$$

with

$$\begin{aligned} z_1 &= \frac{\rho}{2}\left(1 - \sqrt{1 + 4\frac{\delta}{\rho\gamma}}\right), \\ z_2 &= \frac{\rho}{2}\left(1 + \sqrt{1 + 4\frac{\delta}{\rho\gamma}}\right). \end{aligned}$$

As before

$$K_0 = -\frac{\beta - \psi}{\delta},$$

and

$$X_0 = K_0 + K_1 + K_2,$$

$$-z_1 K_1 e^{z_1 T_y} - z_2 K_2 e^{z_2 T_y} = \bar{y} = \frac{\beta - c_y}{\gamma},$$

The main difference lies in full exhaustion. We now have

$$K_0 + K_1 e^{z_1 T_y} + K_2 e^{z_2 T_y} = 0.$$

It follows that

$$k_1 = \frac{\beta - c_y}{z_1 - z_2}(-\frac{1}{\gamma} - \frac{z_2}{\delta}\frac{\beta - \psi}{\beta - c_y}),$$

$$k_2 = \frac{\beta - c_y}{z_1 - z_2}(\frac{1}{\gamma} + \frac{z_1}{\delta}\frac{\beta - \psi}{\beta - c_y}).$$

Upon insertion into

$$X^0 = -\frac{\beta - \psi}{\delta} + K_1 + K_2,$$

we find a new T_y. In the case at hand the condition that $X^0 > X_y = 0$ is always satisfied. Moreover, $\alpha\lambda_s(T_y) = 0$ because it is of no use to insert if the stock has been exhausted.

Hence, for the market economy it makes a big difference for depletion of the fossil fuel stock whether the marginal extraction cost of fossil fuel is larger or smaller than the renewables cost. But in both cases it is well possible to have zero CCS throughout. This is likely to occur with a high initial fossil fuel stock, and, hence, low marginal extraction cost.

Let us next consider the case where the proposed equilibrium cannot be realized. This happens for example if $\lambda_s(0) > c_a + c_s$. Then there must be an initial interval of time with full CCS. Indeed, then it is profitable to use CCS in a market economy. The shadow value of gas inserted is now higher that its cost. One may ask the question whether it is possible to have full CCS until the moment where renewables take over. The answer is simple. If $X_y = 0$ then $\lambda_s(T_y) = 0$ and there will be no CCS in a final phase with fossil fuel extraction. If $X_y > 0$ and $S(T_y) > 0$ then $\lambda_x(T_y) = \lambda_s(T_y) = 0$. But then there is no CCS at the end either. We conclude that the final phase before the transition to renewables always has no CCS in the market economy. So the market equilibrium can be characterized by an initial (possibly degenerate) phase with CCS followed by a phase with no CCS, leading to full or partial exhaustion, before renewables take over.

For our functional forms we can be more specific. In the market economy $\lambda_z = 0$. Consider an initial interval of time with $\zeta x > a = s > 0$. Then

$\dot{\lambda}_s(t) = -\rho(c_a + c_s)e^{-\rho t} = -\alpha\beta e^{-\rho t}x(t)$. Therefore x is constant:

$$x(t) = x^* = \frac{\rho(c_a + c_s)}{\alpha\beta}$$

Using this in (3.1) and (3.5) yields a differential equation for s. Its solution is decreasing towards zero. To find the optimum w need to determine the initial s. Since $\lambda_s(0) = c_a + c_a$ and $\lambda_s(T_y) = 0$ we can determine T_y from $\dot{\lambda}_s(t) = -\alpha\beta e^{-\rho t}x^*$. Then we need to check whether or not $X_y = X^0 - x^* T_y$ is positive. If $X_y < 0$ then th proposed program is suboptmal. Otherwise we can now determine the optimal $s(0)$.

The Proposition P.2 sums up th$(c_a + c_s)$e results.

Proposition P. 2 ***CCS policies without a climate constraint***

In a market economy not subject to the climate constraint.

a. *There exists a critical value of the marginal cost of the renewable alternative such that below it, some part of the oil stock remains unexploited underground, whatever the CSS and EOR policy adopted by the extractive industry, and above it the oil reserves are exhausted in finite time.*

b. *The extractive industry may not perform EOR, and thus no CCS, if the injection cost is too high to justify using this option.*

c. *For lower levels of the injection cost, EOR may be profitable both in case of complete depletion and in case of incomplete depletion of the oil reserves.*

d. *With linear marginal extraction cost the phase to the transition to renewables is either composed of a unique extraction phase of fossils without CCS and EOR, or of a first phase of CCS and EOR, followed by a no EOR phase until the transition to renewables.*

e. *With linear marginal extraction cost the time length of the oil exploitation phase is an increasing function of the initial oil endowment.*

5 CCS and EOR under a carbon budget constraint

Figure 3 depicts the equilibrium in the market economy. Since we have shown that the market economy either never implements EOR, or performs EOR only during a limited time interval right from the beginning, the pollution stock will stay constant during a first time interval and next rise during the no-EOR phase. To have an interesting discussion we assume that the ceiling constraint, $Z \leq \bar{Z}$ is sufficiently stringent for the carbon budget $(\bar{Z} - Z^0)$ to be exhausted in finite time. Let T_Z be the time at which $Z(T_Z) = \bar{Z}$ and, as before, T_y the date at which occurs the transition to renewable energy.

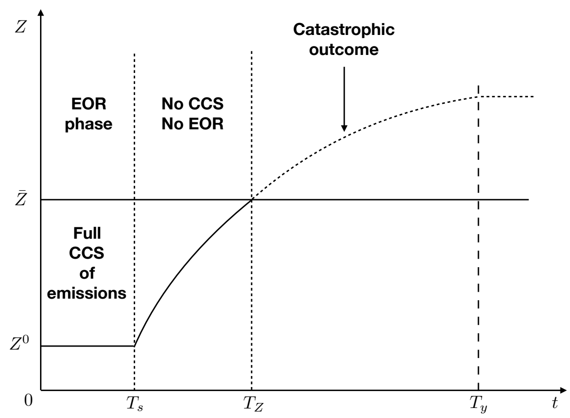

Figure 3: $Z(t)$ **in the Market Economy.**

A laissez-faire economy not internalizing the carbon constraint will perform insufficient abatement efforts before T_Z. The optimal regulation of the carbon problem thus requires to implement a carbon pricing scheme before T_Z, given by the optimal trajectory of $\lambda_z(t)$. The carbon pricing policy is expected to have two potential effects with respect to the laissez-faire situation. First, the economy should abate more emissions than what is implied by the gas demand of the oil industry wanting to perform EOR. Second the time T_Z at which the carbon budget is exhausted should be delayed by the regulation. The stranded oil stock (when relevant) can also change. EOR has the effect to reduce the oil exploitation cost, so the extractive industry could increase its production rate and may extract more, at least temporarily, thereby dampening the expected effects of the carbon regulation policy.

5.1 CCS without EOR

Let us start by considering the following policy: up to T_Z there is no CCS, $\zeta x(t) > a(t) = b(t) = s(t) = 0$. At time T_Z, the upper bound $\bar{Z}$ is reached;

from then on we have $\zeta x(t) = a(t)$ and $s(t) = 0$ until T_y. We will be more precise on the conditions that need to hold for the proposed CCS policy without EOR to be optimal. Under this policy the carbon budget can be calculated as follows. Integrating $\dot{Z}(t) = \zeta x(t) - a(t)$ over $[0, T_Z)$ yields:

$$Z(T_Z) - Z^0 = \zeta \int_0^{T_Z} x(t)dt - \int_0^{T_Z} a(t)dt = \zeta(X_0 - X(T_Z)) - \int_0^{T_Z} a(t)dt.$$

With $a(t) = s(t) = 0$ between time 0 and some time T_Z and $a(t) = \zeta x(t) > 0$ between T_Z and T_y we find

$$\begin{aligned} \bar{Z} - Z^0 &= \zeta \int_0^{T_y} x(t)dt - \int_{T_Z}^{T_y} a(t)dt = \zeta(X_0 - X(T_y)) - \zeta \int_{T_Z}^{T_y} x(t)dt \\ &= \zeta(X_0 - X(T_Z)). \end{aligned}$$

This describes the total amount of fossil fuel that can be extracted until the ceiling is reached. Note that insertion of CO_2 in the well only affects the extraction cost, but does not increase the total amount that can be extracted.

We first consider the period of time between 0 and T_Z. As long as the CO_2 stock is below the ceiling the shadow cost $\lambda_z^* = -\lambda_z$ is a positive constant. The maximization of the Hamiltonian with respect to b reads:

$$\max\ (\lambda_z^* - e^{-\rho t} c_a) b(t) \text{ subject to } \zeta x(t) \geq b(t) \geq 0\ .$$

Clearly during a first time interval, we must have $\lambda_z^* < e^{-\rho t} c_a$, because $b(t) = 0$ in that interval. Note also that once $\lambda_z^* > e^{-\rho t} c_a$, this will remain so, implying that $\zeta x(t) = b(t) = a(t)$. In the first interval we also have $\mu(t) = 0$ so that

$$e^{-\rho t}[U'(x) - H(X)] = \lambda_x - \zeta \lambda_z.$$

This yields the following second-order differential equation.

$$-\rho(U'(x) - H(X)) + U''(x)\dot{x} = 0.$$

The solution of this equation gives the entire path for the first interval, $[0, T_Z)$, since X^0 is given and $X(T_Z)$ is defined by the carbon budget as $X(T_Z) = X^0 - (\bar{Z} - Z^0)/\zeta$. A first condition for this path to be optimal is that renewables are not yet competitive at time T_Z, that is $U'(x(T_Z)) < c_y$. In addition, X^0 must be sufficiently large for $X(T_Z)$ be positive.

The Hamiltonian for the second interval $[T_Z, T_y)$, where by construction $s(t) = 0$ and $a(t) = \zeta x(t)$, reads

$$\mathcal{H} = e^{-\rho t}[U(x) - H(X)x - c_a \zeta x] + \lambda_x[-x].$$

Maximization with respect to x yields:

$$e^{-\rho t}[U'(x) - H(X) - c_a\zeta] = \lambda_x,$$

and we therefore get another differential equation:

$$-\rho(U'(x) - H(X) - c_a\zeta) + U''(x)\dot{x} = 0.$$

We have to consider two possibilities. The first one is where there exists $X(T_Z) > X_y > 0$ such that:

$$c_y - H(X_y) - c_a\zeta = 0.$$

Then, for a given T_Z, we know $X(T_Z)$ and X_y. We still need to determine T_Z and T_y. This can be done by using the condition that energy consumption should be continuous, at T_Z as well as at T_y. Continuity at T_y requires, as before, $\lim_{t\uparrow T_y} x(t) = \tilde{y}$. At T_Z, the capture rate in inert reservoirs, b, jumps upward from 0 to $\zeta x(T_Z)$. Then we must have $\lambda_z^* = e^{-\rho T_Z}c_a$ since if $\lambda_z^* > e^{-\rho T_z}c_a$, the economy should have started to abate emissions before T_Z, thus stopping the accumulation of carbon up to the cap, and in the reverse case, the abatement cost is higher than the shadow value of emissions, implying no abatement, which cannot be the case when the cap constraint is tight.

Finally, it needs to be checked that it is optimal to have $s(t) = 0$ throughout, in particular in the second interval. For optimality we have $\lambda_s(T_y) = 0$ because $\lambda_z(T_y) = 0$. So, in principle we can solve for $\lambda_s(t)$, $T_Z < t < T_y$. A necessary condition now is that $\lambda_s(T_Z) < e^{-\rho T_Z}c_s$.

Alternatively, it is also possible that nothing is left in the ground at T_y, $X_y = 0$. This gives another boundary condition. A similar procedure can be applied as for the preceding case, the only difference being that $\lim_{t\uparrow T_y} \lambda_X(t) > 0$.

It can be shwon numerically that an optimum without EOR exists in both cases, $X_y > 0$ and $X_y = 0$. A sufficient condition is that inserting gas in inert wells is large enough compared to the abatement cost.

5.2 CCS with EOR

We have shown above that it might be optimal to have no EOR at all. In this section we investigate the options for EOR in an optimum. We first show that once the economy keeps the atmospheric CO_2 stock constant, the stock will always be constant from then onwards.

Lemma 4

Suppose there exists t_1 with $\zeta x(t_1) = a(t_1)$ then $\zeta x(t) = a(t)$ for all $t \geq t_1$.

Proof

Suppose $\zeta x(t_1) = a(t_1)$ for some t_1 and $\zeta x(t_2) > a(t_2)$ for some $t_2 \geq t_1$. Abatement a solves max $(-e^{-\rho t} c_a + \lambda_z^*) a(t)$ subject to $\zeta x(t) \geq a(t) \geq s(t)$. Hence $-e^{-\rho t_1} c_a + \lambda_z^* \geq 0$ and $-e^{-\rho t_2} c_a + \lambda_z^* \leq 0$. This contradicts that λ_z^* is a constant. Q.E.D.

Moreover, lemma 3 excludes the possibility of $a(t) > s(t)$ in an interval of time with $\zeta x(t) > a(t)$. If initially, meaning with $\zeta x(t) > a(t)$, we have $s(t) = 0$ then $\lambda_s - \lambda_z \leq (c_a + c_s) e^{-\rho t}$, since $\mu(t) = 0$. But then it is impossible to have $s(t) > 0$ later in the interval of time with $\zeta x(t) > a(t)$.

The conclusion is that the alternative to the case studied in the previous subsection with no EOR at all, is to have three phases:

Phase 1: for $0 \leq t \leq T_Z$ we have $\zeta x(t) > a(t) = s(t) > 0$;

Phase 2: for $T_Z \leq t \leq T_s$ we have $\zeta x(t) = a(t) = s(t) = 0$;

Phase 2: for $T_s \leq t \leq T_y$ we have $\zeta x(t) = a(t) > s(t) = 0$;

Obviously, an exception occurs if $c_s = 0$. Then $s = a$ throughout. Note also that with our linear specification of the marginal extraction cost the extraction rate x is constant as long as $\zeta x > a = s > 0$. As in the sub-section 5.1, it is possible that some oil remains underground, the stranded oil stock, X_y being determined as in the previous sub-section or that the oil stock is depleted. The following Proposition P.3 sums up the results.

Proposition P. 3 ***CCS and EOR under a carbon budget constraint***

When subject to a carbon budget constraint, two possibilities arise:

a. *Either the economy never uses the EOR option and it does not capture carbon emissions at all. If oil exploitation is still more competitive than the renewables alternative, the economy applies CCS to the whole flow of emissions and sequestration into the inert reservoirs until renewable* *energy be competitive. At the energy transition time, the depletion of the oil reserves may be complete or incomplete.*

b. *Or the economy uses the EOR option at the beginning of oil exploitation but stops using it not before the ceiling is reached. When the carbon budget has been depleted, the economy eventually performs CCS in the inert reservoirs until the transition to renewables. The depletion of oil may be complete or incomplete.*

6 Concluding remarks

In the currently ongoing debates on combating climate change carbon capture and storage is an important issue. Many countries, such as the Netherlands, France, Canada and even the US (some States to be precise...), are seriously considering this option. Presently the cost is high but it is expected that technological progress will lead to a significant cost reduction. An additional option is to use the inserted greenhouse gases, CO_2 in particular, to enhance oil recovery by increasing the well's pressure. The aim of this paper has been to offer a preliminary investigation of the pros and cons by analysing a formal model of CCS and EOR. The model is very stylized. It assumes linear cost functions and an exogenously imposed upper bound on atmospheric CO_2 accumulation. The main feature of the model is the fact that the marginal extraction cost of fossil fuel is a decreasing function of the pressure in the well. This pressure can be increased by inserting CO_2, recovered from burning fossil fuel. On the one hand, CCS helps to reduce atmospheric CO_2 accumulation, at a cost. On the other hand, EOR leads to more fossil fuel being processed, which may lead to less fossil fuel left in situ. In a social optimum the two effects are both taken into account.

We have first considered the market outcome, where the economic agents do not take into account the ceiling on CO_2 accumulation. It is shown that a typical market equilibrium has an initial phase with full EOR. Later on EOR is no longer implemented and the atmospheric CO_2 stock increases. It is interesting to observe that the market economy will under some circumstances engage in EOR even without incentives provided by the government. Moreover, it need not be the case that the ceiling is reached, possibly thanks to the capacity to implement EOR. In reality, however, not reaching the ceiling (Paris agreement) purely as a consequence of CCS and EOR is not very likely.

We have also characterized the social optimum. Again there are two phases. In the most plausible case, it is necessary to start EOR and CCS immediately and maximally. EOR might still be in order when the ceiling is reached, namely when the insertion costs are low. But EOR will be abandoned before the transition to renewables takes place.

Much is still to be done. We are working with a three state variable problem (stocks of fossil fuel, atmospheric CO_2 and inserted gases). Analytically this is rather challenging. Numerical exercises are in order to get more feeling for e.g., when fossil fuel gets exhausted, and when the transition to renewables will take place. The linearity of the cost functions poses a problem as well, because it implies that there is never simultaneous supply of renewables and fossil fuel, although they are assumed perfect substitutes. It seems also worthwhile to work on a model with a damage function. This may smoothen transitions from full EOR to no EOR. Work on this is ongoing. In spite of some heroic assumptions we made, we are convinced that studying CCS in conjunction with EOR offers a fruitful perspective for future research.

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