Optimal climate and fiscal policy in an OLG economy

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Efficient policies price externalities through Pigouvian taxes, but these must be adjusted when there are other distortionary taxes in the economy. I develop a two-period overlapping generations climate-economy model to study integrated capital, labor, and carbon taxes. I derive five primary results. First, the optimal carbon tax in an economy with distortionary fiscal policy equals the market costs of carbon, but not always attains its Pigouvian level. Second, under weak separability in preferences over consumption and leisure, age-dependent labor income taxes allows the optimal carbon tax to achieve its first-best. Third, even if age-dependent taxes are available, non-separability in preferences and a decreasing labor supply over the life cycle leads to positive capital income taxes and an optimal price on carbon emissions that falls short of the Pigouvian tax. Fourth, an exogenous capital income tax rate implies an optimal carbon price that differs from both the market costs of carbon and its Pigouvian level. Fifth, the existence of an exogenous carbon price provides a firm rationale for positive taxes on capital.

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1. Introduction

A fundamental observation in the literature about pricing externalities e.g., the setting of Pigouvian taxes, is that those prices should be adjusted in the presence of other distortions in the economy. This is particularly relevant in the context of environmental regulations, since recent economic research has suggested that optimal prices for pollution control respond differently to, for instance, non-observable private information (Jacobs and de Mooij, 2015; Kaplow, 2012; Tideman and Plassmann, 2010), financial frictions or productivity shocks (van den Bijgaart and Smulders, 2017; Hoffmann et al., 2017), the existence of exogenous positive capital taxes (Barrage, 2018), time-inconsistency problems (Gerlagh and Liski, 2017b; Schmitt, 2014), distributional issues (Jacobs and de Mooij, 2015; Chirole-Assouline and Fodha, 2011, 2014) or differences between private and social discounting (Barrage, 2017; Belfiori, 2017).

Using a two-period overlapping generations model with endogenous labor supply and a climate-module structure based on Howarth and Norgaard (1995), Howarth (1998), Iqbal and Turnovsky (2008) and Gerlagh and Liski (2017b), this paper specifically studies the interaction between climate and fiscal policies when the government has no access to individualized lump-sum taxes (or transfers) and relies on the implementation of distortionary taxes on capital and labor income to finance an exogenous stream of expenditures and inherited debt. I use this approach to inform about the conditions under which the market costs of carbon may differ from the social costs of carbon (the Pigouvian tax) in a second- and third-best world.

First, I show that a non-zero optimal capital income tax drives a wedge between how the market values future marginal output losses due to current carbon emissions and how the society values those losses, in comparison to the first-best allocation.[1] Second, I find that under the assumption of weak separability in preferences over consumption and leisure, the availability of age-dependent labor income taxation allows the government to rely on a carbon price and labor income taxes to fulfill its spending requirements, and to avoid intertemporal distortions by imposing a non-zero capital income tax rate. Notice that although a labor tax affects intratemporal decisions, those do not impede that the carbon price attains its Pigouvian level, since the marginal rate of substitution for consumption is not affected.

[1] In the presence of no distortions in the economy, it has been shown that the marginal rate of substitution for consumption between two periods equals the opportunity cost of capital, so both valuations in equilibrium are the same. For a discussion about the role of intergenerational transfers in this context, see for instance, Howarth and Norgaard (1995, 1993).

Third, I show that even if age-dependent taxes are available, non-separability in preferences and a decreasing labor supply over the life-cycle leads to an optimal price on carbon emissions that falls short of the Pigouvian tax. In this case, a labor income tax also influences consumption-savings decisions due to the complementarity between consumption and leisure, and the government finds optimal to tax capital income using a non-zero tax rate to offset the distortion on consumption optimal paths by changes in labor supply, since the government cannot tax directly leisure levels (Erosa and Gervais, 2002); importantly, the sign of this tax rate is determined by the optimal allocation of labor over the life-cycle and the intertemporal elasticity of substitution. It is worth noting that, however, in the presence of no age-dependent taxation, regardless of assumptions about preferences, in general the optimal carbon price does not correspond to the Pigouvian tax because the government could use capital income taxes to mimic the allocations under age-dependent taxes (Conesa et al., 2009; Gervais, 2012), and that third-best policy would distort the marginal rate of substitution for consumption.

Fourth, following Barrage (2018), when the government is constrained to set a non-optimal capital income tax rate, I find that the prescription which indicates that the optimal carbon price should be equal to the market costs of carbon does not hold anymore. The intuition behind this result relies on the fact that since the government cannot choose optimally the optimal level of aggregate capital due to the exogenous capital tax rate, this affects the net returns on capital, and therefore, how to discount future marginal damages from current carbon emissions. Likewise, I show that restrictions on climate change policy provides a novelty role for the existence of positive capital income taxes. For instance, suppose that the carbon price is set below the market costs of carbon. Since that price does not internalize completely the damages from current emissions, the economy is producing at a higher level than the one corresponding to the efficient allocation. The government finds optimal to reduce the level of aggregate capital, and thus cutting emissions, by taxing the returns on capital. Moreover, I also show that the path of labor income taxes should be adjusted to correct for this inefficiency.

This paper relates to distinct strands of literature. Firstly, one of most critical issues in climate change policy has to do with the decision about which discount rate a policy maker should use to calculate the net present value of future production damages due to the emission of one unit of CO_2 today, in order to determine a carbon price that accounts for those production losses (Giglio et al., 2018; Gollier and Hammitt, 2014; Greenstone et al., 2013). In this matter, for instance, Nordhaus (2008) and Stern (2007) provide somehow different recommendations. Nordhaus (2008) argues that current investments in climate change mitigation should earn the same return that other investments in the economy, e.g., the market interest rate. Stern (2007) suggests to follow an ethics-based approach and recommends to use a very 'low' rate of pure time preference. Such assumption, however, would imply higher savings rates than the ones observed in the data (Belfiori, 2017).

In the same vein, Schneider et al. (2012), Goulder and Williams (2012), Weisbach and Sunstein (2009), and Dasgupta (2008) discuss the reasons behind these differences and point to the concepts attached to social discounting in each prescription as the cause of disagreement. I add to this literature by considering explicitly the difference between the market costs of carbon and the social costs of carbon as suggested in Goulder and Williams (2012). These concepts differ with respect to the discount rate used to evaluate future marginal damages to production by current pollutant activities. While the first one uses the market interest rate or the return on capital, the second one employs the consumption discount rate which is given by the marginal rate of substitution for consumption between two periods of successive generations. In this context, I consider thus the situations and the causes in which their valuations may be different.

Recent studies has also pointed to the importance of understanding the relationship between the existence of capital income taxes and the setting of climate policies, e.g. the carbon price, in dynamic climate-economy models. For instance, using a infinitely-lived agent model as in Golosov et al. (2014) and a climate structure as in Nordhaus (2008), Barrage (2018) shows that when climate change only has impacts in the production of the final good, a zero capital income tax does not distort the optimal carbon price and, therefore, it equals the social costs of carbon. It is well known that in infinitely-lived representative agent (ILA) frameworks, since a capital income tax distorts the consumption-savings decisions of households, the government finds optimal to fully rely on labor income taxes, in absence of lump-sum taxation, because that fiscal policy is welfare improving; that is, the optimal capital income tax should be zero in the long run (Chamley, 1986; Judd, 1985).[2] However, when the government faces an exogenous constraint implying a positive capital income tax, Barrage (2018) finds that the optimal carbon prices should be set below its Pigouvian level. In the same line, Schmitt (2014) proposes a dynamic model based on Bovenberg and de Mooij (1994) without commit-

[2] Straub and Werning (2014) indicate, however, that this result is no longer valid whenever the elasticity of intertemporal substitution is below one. See also Albanesi and Armenter (2012) for an analysis of the effects of intertemporal distortions in ILA models.

ment technologies, characteristic that generates endogenously positive capital income tax rates, and finds out that governments set optimal carbon taxes below Pigouvian levels.

In contrast, a different strand of literature has suggested that there is space for positive capital income taxes in overlapping generations (OLG) models due to life-cycle characteristics no present in ILA models, such as differences in labor supply or productivity profiles, tax instruments available to the government, and preferences modeling (Garriga, 2017; Peterman, 2013; Conesa et al., 2009; Iqbal and Turnovsky, 2008; Erosa and Gervais, 2002, 2001; Garriga, 2001). Since not so much effort has been done to analyze the setting of optimal carbon prices in OLG models with distortionary taxation,[3] I contribute to this literature by providing a set of additional results in terms of preferences modeling and tax instruments available to the government.[4]

Finally, this paper also relates to the literature that evaluate the role of age-dependent taxation in the setting of fiscal policy as in Bastani et al. (2013), Weinzierl (2011), and Blomquist and Micheletto (2008). I complement these studies by showing that the introduction of labor income taxes which can be conditioned by age, at least under the assumption of weak separability in preferences over consumption and leisure, leads to a zero capital income tax and to an optimal carbon price that attains its Pigouvian level. The intuition behind this result is straightforward. When the government has access to more fiscal instruments to finance spending, it is optimal to choose the ones that avoid or reduce intertemporal distortions.

The organization of this paper is as follows. Section 2 lays out the main characteristics of the model, provides some definitions about the costs of carbon and describes first-best allocations. Section 3 presents the Ramsey problem and derives optimal taxes using the primal approach. Section 4 studies the role of age-related income taxation and characterizes optimal taxes. Section 5 and Section 6 provide a discussion about the role of constant capital and carbon tax rates, respectively. Section 7 concludes.

[3]Rausch and Abrell (2014) provide a characterization of capital-carbon tax interactions in an OLG framework. However, they do not discuss the consequences of distinct preferences specification, the role of age-dependent taxation in those interactions, and restrictions on both fiscal and climate policy, as this paper does. Fried et al. (2016) study the introduction of a revenue-neutral carbon tax policy in a life-cycle model with distortionary taxes and quantify their distributional effects. They do not derive optimal carbon prices and do not consider the implications of existence of age-dependent taxation. A similar analysis can be found in Dao and Dávila (2014), nevertheless, the authors only consider the cases of exogenous labor supply and no restrictions on policy instruments.

[4]Since I assume no population growth, notice that I do not consider other topics typical of OLG-climate-economy models such as demographic change (Gerlagh et al., 2017) and political economy features (Karp and Rezai, 2014).

2. The model

I consider a two-period overlapping generations model with endogenous labor supply based on Howarth and Norgaard (1993), Howarth (1998), and Iqbal and Turnovsky (2008), and add a climate-module structure as in Gerlagh and Liski (2017b) to derive optimal fiscal and climate policies in: i) a first-best world; ii) when the government has no access to individualized lump-sum taxes to finance an exogenous stream of government spending and inherited debt (that is, a second-best setting), and iii) when I impose some restrictions on the second-best policy instruments available to the government (a third-best scenario).

2.1. Household's problem

Each generation lives only two periods. Time is discrete and runs to infinity. Households supply labor in both periods and there is no population growth. I assume a constant population normalized to 1 and full capital depreciation. Each household is endowed with one unit of time per period. The time-separable utility function U_t is strictly increasing, strictly concave, twice continuosly differentiable and satisfies the usual Inada conditions. In period t, each household solves the following problem taking as given the path for prices, fiscal policy and initial asset holdings, which I assume to be zero since I do not consider any form of altruism:[5]

$$\max_{\{C_{1,t}, C_{2,t+1}, L_{1,t}, L_{2,t+1}, K_{t+1}, B_{t+1}\}} W_t \equiv U(C_{1,t}, L_{1,t}) + \beta U(C_{2,t+1}, L_{2,t+1}) \tag{1}$$

subject to,

$$C_{1,t} + K_{t+1} + B^D_{t+1} = (1 - \tau^L_{1,t})\phi_1 w_t L_{1,t} + T_{1,t} \tag{2}$$

$$C_{2,t+1} = (1 - \tau^L_{2,t+1})\phi_2 w_{t+1} L_{2,t+1} + (1 - \tau^K_{t+1}) r_{t+1} K_{t+1} + R_{t+1} B^D_{t+1} + T_{2,t+1} \tag{3}$$

where $C_{1,t}$ and $C_{2,t+1}$ denote consumption at young and old age, respectively; $\beta \in (0,1)$ is the subjective utility discount factor; $L_{1,t}$ and $L_{2,t+1}$ are the fractions of time allocated to work in each period; ϕ_1 and ϕ_2 identify labor productivities at each age; K_{t+1} represents savings; $T_{1,t}$ and $T_{2,t+1}$ are lump-sum transfers from the government which, for the moment, I do not restrict to be non-negative;[6] w_t and w_{t+1} describe wage

[5]A general formulation of this household's problem, with generations living more than two periods, can be found in Erosa and Gervais (2002) and Garriga (2017).

[6]Following Hipsman (2018), and assuming the existence of complete markets, I could not consider age-

payments; r_{t+1} is the return to capital investments; B^D_{t+1} is the demand for government bonds which have one-period maturities and a return R_{t+1}; households pay labor and capital income taxes $\{\tau^L_{1,t}, \tau^L_{2,t+1}, \tau^K_{t+1}\}$, accordingly. Solving the household's problem, the first-order conditions imply:

$$\frac{U_{C_{1,t}}}{U_{C_{2,t+1}}} = \beta(1-\tau^K_{t+1})r_{t+1} \tag{4}$$

$$-\frac{U_{L_{1,t}}}{U_{C_{1,t}}} = (1-\tau^L_{1,t})\phi_1 w_t \tag{5}$$

$$-\frac{U_{L_{2,t+1}}}{U_{C_{2,t+1}}} = (1-\tau^L_{2,t+1})\phi_2 w_{t+1} \tag{6}$$

$$R_{t+1} = (1-\tau^K_{t+1})r_{t+1} \tag{7}$$

where $U_{X_{i,t}}$ is the derivative of the utility function U_t with respect to $X_{i,t}$. Condition (4) is the usual Euler equation which relates marginal rates of substitution for consumption between two periods to the discounted after-tax returns on capital. Conditions (5-6) define intratemporal marginal rates of substitution over consumption and labor relatively to after-tax labor income weighted by age-specific productivities. The last equation, (7), corresponds to the no-arbitrage condition which establishes that in equilibrium government bonds and capital should earn the same net return, and implies that government debt and capital are perfect substitutes (Ludwig and Reiter, 2010).[7]

2.2. Firms

Following Howarth and Norgaard (1995), each period, under perfect competition a representative firm employs a technology that exhibits constant returns to scale to produce aggregate output Y_t, which depends on productivity A_t, capital K_t, aggregate labor L_t, energy E_t, and a climate change damage function Ω_t. This function depends on the stock of pollution Z_t in a particular point of time as a result of previous carbon emissions, which affects output through changes in global mean temperature with respect to the pre-industrial level. As in Gerlagh and Liski (2017a,b), I assume that temperature reacts to current emissions according to a response function, θ_i, which depends on carbon cycle and temperature adjustment parameters. Thus, let $\Omega(Z_t)$ be total damages

dependent transfers since young households can bring anticipated transfers during their old age to the present.

[7]For a detailed analysis of the implication of public debt in OLG economies with endogenous labor supply see, for example, Lopez-Garcia (2008).

due to past carbon emissions, and $Z_t = \sum_{i=1}^{\infty} \theta_i E_{t-i}$ the history of emissions weighted by the response function θ_i, respectively.[8] Under these conditions, I consider a general formulation as follows:

$$\Omega(Z_t) = \exp(-Z_t) \tag{8}$$

The production function F_t is strictly concave, twice continuously differentiable and satisfies the usual Inada conditions:

$$Y_t = F_t(A_t, K_t, L_t, E_t, Z_t) = \Omega(Z_t) K_t^{\alpha} [A_t(E_t, L_t)]^{1-\alpha} \tag{9}$$

where $\alpha \in (0, 1)$ and the composite energy-labor input, $A_t(E_t, L_t)$, has constant returns to scale along the lines of Gerlagh and Liski (2017b). Taking $\Omega(Z_t)$ as exogenous, the firm's problem is then as follows:

$$\max_{\{K_t, L_t, E_t\}} Y_t - w_t L_t - r_t K_t - \tau_t^E E_t \tag{10}$$

The first-order conditions are given by:

$$r_t = \alpha \frac{Y_t}{K_t} \tag{11}$$

$$w_t = (1-\alpha) Y_t \frac{A_{L,t}}{A_t} \tag{12}$$

$$\tau_t^E = (1-\alpha) Y_t \frac{A_{E,t}}{A_t} \tag{13}$$

As usual inputs are paid their marginal productivities. It is important to note that since the firm does not fully internalize the social cost of emitting one unit of carbon at period t, the emissions price τ_t^E (a carbon tax) has to be selected (optimally) by the government in order to correct this inefficiency, that is, without intervention $\tau_t^E = 0$. Finally, effective labor supply is the weighted sum of age-dependent labor profiles:

$$L_t = \phi_1 L_{1,t} + \phi_2 L_{2,t} \tag{14}$$

2.3. The government

To finance an exogenous stream of expenditures $\{G_t\}_{t=0}^{\infty}$, transfers, and inherited debt B_0, the government can issue one-period maturity bonds B_t^S, impose proportional taxes

[8] I assume implicitly that energy use maps one to one with emissions.

on labor and capital income as in Garriga (2017), and set an excise tax on carbon emissions τ_t^E along the lines of Barrage (2018). For simplicity, I assume full commitment. The government's budget constraint is:

$$R_t B_t^S + G_t + T_{1,t} + T_{2,t} = B_{t+1}^S + \tau_{1,t}^L \phi_1 w_t L_{1,t} + \tau_{2,t}^L \phi_2 w_t L_{2,t} + \tau_t^K r_t K_t + \tau_t^E E_t \tag{15}$$

In this case, for $t \geq 0$, the intertemporal constraint can be written as follows:

$$B_t^S = \sum_{i=1}^{\infty} (I_{t+i} - G_{t+i} - T_{1,t+i} - T_{2,t+i}) / \prod_{i=1}^{\infty} R_{t+i} \tag{16}$$

where $I_t = \tau_{1,t}^L \phi_1 w_t L_{1,t} + \tau_{2,t}^L \phi_2 w_t L_{2,t} + \tau_t^K r_t K_t + \tau_t^E E_t$ describes government revenues.

2.4. Competitive equilibrium

A competitive equilibrium for this economy can thus be defined as follows:

Definition 1. *Given a set of policies $\{\tau_{1,t}^L, \tau_{2,t+1}^L, \tau_t^K, \tau_t^E, B_{t+1}^S\}_{t=0}^{\infty}$, initial debt B_0 and an exogenous stream of expenditures $\{G_t\}_{t=0}^{\infty}$, a competitive equilibrium in this economy consists of relative prices $\{r_t, w_t, R_t\}_{t=0}^{\infty}$, allocations for the firm $\{K_t, L_t, E_t\}_{t=0}^{\infty}$ and the households $\{C_{1,t}, C_{2,t+1}, L_{1,t}, L_{2,t+1}, K_{t+1}, B_{t+1}^D, T_{1,t}, T_{2,t+1}\}_{t=0}^{\infty}$ such that:*

1. *The allocations for the households solve (4-7),*
2. *The allocations for the firm solve (11-13),*
3. *The intertemporal budget constraint for the government (16) is satisfied, subject to the transversality condition: $\lim_{i \to \infty} \frac{B_i^S}{\prod_{i=1}^{\infty} R_{t+i}} = 0$.*
4. *Market clearing conditions are satisfied:*

$$\begin{aligned} C_{1,t} + C_{2,t} + K_{t+1} + G_t &= Y_t \qquad (17) \\ L_t &= \phi_1 L_{1,t} + \phi_2 L_{2,t} \qquad (18) \\ B_{t+1}^D &= B_{t+1}^S \qquad (19) \end{aligned}$$

2.5. First-best allocations

In order to understand the implications of distortionary fiscal policy in the setting of carbon prices, I begin by describing the set of first-best allocations when age-dependent lump-sum taxes are available, that is, $T_{1,t}, T_{2,t+1} < 0$, and the other taxes are equal to zero, except the tax rate on carbon emissions. Thus, if there were not other distortions in the economy, except the climate externality, a benevolent government who has access to individualized lump-sum taxes for financing government spending and inherited debt would seek to maximize a social welfare function, taking K_0 and B_0 as given, to solve the following problem:

$$\max_{\{C_{1,t},C_{2,t+1},L_{1,t},L_{2,t+1},K_{t+1},E_t\}_{t=0}^{\infty}} \gamma^{-1}U_0 + \sum\nolimits_{t=0}^{\infty} \gamma^t W_t \tag{20}$$

subject to the set of technological and resource constraints described above. W_t is the utility function of generation t (see equation 1) and $1 > \gamma > 0$ is the intergenerational discount factor.[9] It is worth mentioning that the choice of a social welfare function (SWF) in an OLG framework is not straightforward. I assume that the SWF is the discounted sum of individual lifetime utilities as in Garriga (2017), Conesa et al. (2009), Ludwig and Reiter (2010),and Erosa and Gervais (2002). Let $\gamma^t\mu_t$ denote the Lagrange multiplier associated to the resource constraint (17). The first-best allocations can be then derived from the optimality conditions which are given by:

$$\begin{aligned}
U_{C_{1,t}} &= \mu_t & (21)\\
\frac{\beta}{\gamma}U_{C_{2,t+1}} &= \mu_{t+1} & (22)\\
U_{L_{1,t}} &= -\mu_t\phi_1 F_{L_t} & (23)\\
\frac{\beta}{\gamma}U_{L_{2,t+1}} &= -\mu_{t+1}\phi_2 F_{L_{t+1}} & (24)\\
\frac{1}{F_{K_{t+1}}} &= \gamma\frac{\mu_{t+1}}{\mu_t} & (25)\\
\sum\nolimits_{i=1}^{\infty}\gamma^i\frac{\mu_{t+i}}{\mu_t}\underbrace{\frac{\partial Y_{t+i}}{\partial\Omega_{t+i}}\frac{-\partial\Omega_{t+i}}{\partial E_t}}_{\theta_i Y_{t+i}} &= F_{E_t} & (26)
\end{aligned}$$

where F_{X_t} is the derivative of the production function F_t with respect to X_t. This problem is similar to the one described in Howarth and Norgaard (1995), and Howarth (1998). Here, I extend their framework by considering endogenous labor supply and a distinct climate-module structure. The conditions (21-25) characterize consumption and labor paths at young and old age when there are no distortionary taxes in the econ-

[9]For the case when welfare weights are chosen such that the government does not redistribute income between generations see Gerlagh et al. (2017).

omy. Equation (26) relates the marginal benefits of emitting one unit of CO_2 at period t (right-hand side) to the discounted marginal future damages (left-hand side). Equation (25) deserves a special discussion. First, notice that different values for the intergenerational discount factor (that is, the constant welfare weight for current and future generations) imply a distinct set of efficient allocations. Second, the first-order condition for capital in the social planner's problem relates the welfare weights to the market interest rate (the marginal productivity of capital).[10] In this sense, as it is shown in Ludwig and Reiter (2010), given a certain set of policy instruments available, by choosing a particular discount factor γ, the social planner can achieve a specific competitive equilibrium allocation for capital (or even, if necessary, rule out any dynamic inefficiency).

2.5.1. Carbon policies

Before describing and discussing the results from the social planner's problem under a distortionary fiscal policy scheme, I provide two key definitions taking into account the set of first-best allocation defined above:

Definition 2. *The Pigouvian tax in this economy denotes the net present value of marginal output losses due to one unit of energy consumption at period t evaluated at the optimal allocation and valued at the successive generations' marginal rates of substitution for consumption:*

$$\tau_t^{PIGOU} = \sum\nolimits_{i=1}^{\infty} \beta^i \prod_{j=1}^{i} \frac{U_{C_{2,t+j}}}{U_{C_{1,t+j-1}}} \theta_i Y_{t+i} \tag{27}$$

Definition 2 can be interpreted as the net present value of future marginal damages from climate change using a social discount rate and it is derived from combining conditions (21-22) and (26). This discount rate reflects how each generation values consumption between two consecutive periods. The definition follows closely the ones provided in Howarth and Norgaard (1995) for an OLG model and Barrage (2018) for an infinitely-lived representative agent framework. Likewise, by using conditions (25) and (26), I can define the market costs of carbon as follows:

Definition 3. *The market costs of carbon emissions in this economy is defined as the net present value of future marginal damages evaluated at the market interest rate:*

[10] For instance, in steady state, the real interest rate r_{ss} equals the inverse of the intergenerational discount factor, $1/\gamma$. This condition can be seen as a modified golden rule for accumulation of capital when abstracting from productivity growth.

$$MCC_t = \sum\nolimits_{i=1}^{\infty} \frac{1}{\prod_{j=1}^{i} r_{t+j}} \theta_i Y_{t+i} \tag{28}$$

Notice that while the Pigouvian tax (27) values the net present value of marginal climate damages using the marginal rates of substitution between consumption today and tomorrow of successive generations that live only two periods, the market costs of carbon discount future damages using the market interest rate. It is well known that in a economy with no distortions, optimality implies that the marginal rate of substitution for consumption between two periods equals the real interest rate (the marginal rate of transformation), thus:

Proposition 1. *At the optimal allocation, in absence of any other distortion in the economy, it follows that in a first-best world the optimal carbon price equals both the Pigouvian tax and the market costs of carbon:*

$$\tau_t^E = \tau_t^{PIGOU} = MCC_t \tag{29}$$

Proof. Using the first-order conditions (21-22) and (25-26), and according to the previous definitions, we get the result. ■

The result pointed out in Proposition 1 resembles the analysis in Howarth (1998) which indicates that the social costs of carbon (The Pigouvian tax) corresponds one to one to the market costs of carbon. The intuition for this result is straightforward. Without additional distortions, the optimal carbon price that maximizes welfare is precisely the Pigouvian tax since the social discount rate equals the market discount rate. The next section, however, describes under which conditions, in terms of tax interaction effects, the optimal carbon price may differ from its Pigouvian level.

3. Optimal taxation in a second-best world

When I rule out the possibility of lump-sum taxation, the social planner should establish optimal tax rates for the policy instruments available, that is, labor, capital, and carbon taxes. In order to determine the path for optimal taxes I follow the primal approach as in Iqbal and Turnovsky (2008).[11] Thus, instead of solving for tax rates directly, I characterize optimal allocations which are compatible with a competitive equilibrium

[11] Similar results are derived in Erosa and Gervais (2001), Garriga (2001), Erosa and Gervais (2002) and Conesa et al. (2009).

and then derive prices and taxes that implement such allocations given the constraints imposed to the social planner's problem. The following lemma allows me to apply this approach:

Lemma 1. *Any competitive equilibrium which is a set of allocations for the firm $\{K_t, L_t, E_t\}_{t=0}^{\infty}$ and the household $\{C_{1,t}, C_{2,t+1}, L_{1,t}, L_{2,t+1}, K_{t+1}, B_{t+1}^{D}\}_{t=0}^{\infty}$, supported by a particular set of policies $\{\tau_{1,t}^{L}, \tau_{2,t+1}^{L}, \tau_t^{K}, \tau_t^{E}, B_{t+1}^{S}\}_{t=0}^{\infty}$, an exogenous stream of expenditures $\{G_t\}_{t=0}^{\infty}$ and initial debt B_0, satisfy:*

$$C_{1,t} + C_{2,t} + K_{t+1} + G_t = Y_t \tag{30}$$

$$L_t = \phi_1 L_{1,t} + \phi_2 L_{2,t} \tag{31}$$

$$U_{C_{1,t}} C_{1,t} + \beta U_{C_{2,t+1}} C_{2,t+1} + U_{L_{1,t}} L_{1,t} + \beta U_{L_{2,t+1}} L_{2,t+1} = 0 \tag{32}$$

$$U_{C_{2,0}} C_{2,0} + U_{L_{2,0}} L_{2,0} = U_{C_{2,0}} \left[(1 - \tau_0^{K}) F_{K_0} K_0 + R_0 B_0 \right] \tag{33}$$

Any allocation that satisfies (30)-(33), can be decentralized as a competitive equilibrium for a particular set of policies, prices, and asset holdings.

Proof. In appendix A. ∎

The main advantage of the primal approach has to do with the fact that allows me to reduce the number of variables and equations needed to solve for optimal allocations, and then decentralize them in a transparent manner. For example, notice that by replacing out prices and taxes in the budget constraint for the households using their first-order conditions, we can get the implementability conditions (32).[12] This step assures that if condition (32) is satisfied, the same allocations also solve (4-7). Likewise, equations (30) and (31) are equivalent to the first two constraints that come from the market clearing conditions in definition 1. Finally, using the first-order conditions for both the household and the firm I can solve for prices and taxes.

According to the Lemma (1), the government thus maximizes social welfare, taking K_0 and B_0 as given, to solve again:

$$\max_{\{C_{1,t}, C_{2,t+1}, L_{1,t}, L_{2,t+1}, K_{t+1}, E_t\}_{t=0}^{\infty}} \gamma^{-1} U_0 + \sum\nolimits_{t=0}^{\infty} \gamma^t W_t \tag{34}$$

Let $\gamma^t \mu_t$, and $\gamma^t \lambda_t$ denote the Lagrange multipliers associated to the following constraints: (i) the resource constraint (30), and (ii) the implementability condition (32).

[12] The same argument applies for the implementability condition for the initial old, (33).

Notice that I substitute constraint (31) into (30), and that condition (33) is not needed to solve the problem for generation t. Before describing the optimality conditions, in order to facilitate the interpretation of the results, I provide two important definitions. First, in the spirit of Atkeson et al. (1999), I define general equilibrium elasticities for consumption and labor to measure how marginal utilities react to changes in consumption and leisure as a result of implementing distortionary taxes.

Definition 4. *For $i = \{1, 2\}$, let $\Theta^{C_{i,t}}$, $\Theta^{L_{i,t}}$ be general equilibrium elasticities, which account for interactions between consumption-labor marginal utilities.*

$$\Theta^{C_{i,t}} = \frac{C_{i,t}U_{C_{i,t}C_{i,t}} + L_{i,t}U_{L_{i,t}C_{i,t}}}{U_{C_{i,t}}} \tag{35}$$

$$\Theta^{L_{i,t}} = \frac{L_{i,t}U_{L_{i,t}L_{i,t}} + C_{i,t}U_{C_{i,t}L_{i,t}}}{U_{L_{i,t}}} \tag{36}$$

Second, as previous literature has shown, the marginal cost of public funds can be defined as the welfare costs associated to raise an additional unit of fiscal revenues, see for example, Barrage (2018) and Jacobs and de Mooij (2015). Thus,

Definition 5. *For $X = \{C, L\}$, let $\Lambda^{X_{1,t}}$ and $\Lambda^{X_{2,t+1}}$ denote the marginal cost of public funds (MCF), which can be used to measure the costs of using distortionary taxation, that is, the costs associated of transferring a marginal unit of private consumption at each age to the government.*

$$\Lambda^{X_{1,t}} = 1 + \lambda_t \left(1 + \Theta^{X_{i,t}}\right) \tag{37}$$

$$\Lambda^{X_{2,t+1}} = 1 + \lambda_t \left(1 + \Theta^{X_{i,t+1}}\right) \tag{38}$$

According to the previous definition, notice that in the case of lump-sum taxation, the marginal cost of public funds is one. Since the implementability condition (32) is not binding when the government has access to lump-sum taxes, the Lagrange multiplier associated to that constraint, λ_t, is zero, and I get the result. Once the government relies on labor and capital income taxes, the marginal cost of public funds is larger than one as long as $\Theta^{C_{i,t}}$, $\Theta^{L_{i,t}} > -1$.

Following the previous definitions, the optimality conditions are thus given by:

$$
\begin{aligned}
U_{C_{1,t}} \cdot \Lambda^{C_{1,t}} &= \mu_t && (39)\\
\frac{\beta}{\gamma} U_{C_{2,t+1}} \cdot \Lambda^{C_{2,t+1}} &= \mu_{t+1} && (40)\\
U_{L_{1,t}} \cdot \Lambda^{L_{1,t}} &= -\mu_t \phi_1 F_{L_t} && (41)\\
\frac{\beta}{\gamma} U_{L_{2,t+1}} \cdot \Lambda^{L_{2,t+1}} &= -\mu_{t+1} \phi_2 F_{L_{t+1}} && (42)\\
\frac{1}{F_{K_{t+1}}} &= \gamma \frac{\mu_{t+1}}{\mu_t} && (43)\\
\sum\nolimits_{i=1}^{\infty} \gamma^i \frac{\mu_{t+i}}{\mu_t} \theta_i Y_{t+i} &= F_{E_t} && (44)
\end{aligned}
$$

From equation (44), we know thus that the optimal carbon tax in period $t > 0$ that decentralizes the optimal allocation under distortionary taxation is implicitly defined as follows:

$$\tau_t^E = \sum\nolimits_{i=1}^{\infty} \gamma^i \frac{\mu_{t+i}}{\mu_t} \theta_i Y_{t+i} \tag{45}$$

Notice that, however, from equations (39)-(43), once the government has to rely on distortionary taxation, since the marginal cost of public funds is not longer one, this creates a wedge between the marginal rate of substitution for consumption and the marginal rate of transformation i.e., the return on physical capital investment. In order to describe the implications of these distortions on the setting of optimal climate and fiscal policy, along the lines of Chari et al. (2007), I define wedges in terms of the marginal cost of public funds for consumption and labor as follows:

Definition 6. *The consumption-savings wedge, Ξ_t^C, and the labor supply wedge, Ξ_t^L, reflect inter-temporal welfare costs from using distortionary taxes, and can be represented by the ratio between the marginal cost of public funds in period $t+1$ and t:*

$$\Xi_t^C = \frac{\Lambda^{C_{2,t+1}}}{\Lambda^{C_{1,t}}} \tag{46}$$

$$\Xi_t^L = \frac{\Lambda^{L_{2,t+1}}}{\Lambda^{L_{1,t}}} \tag{47}$$

From the perspective of the generation born at period t, optimal choices for consumption and labor supply in both periods depend on how these decisions are affected by the climate and fiscal policy in place. Therefore, in order to decentralize the second-best allocations, the government makes use of these wedges to derive prices and taxes that support such an allocation.

Lemma 2. *The optimal carbon tax in an economy with distortionary fiscal policy equals the market costs of carbon:*

$$\tau_t^E = MCC_t \tag{48}$$

but not attains its Pigouvian level

$$\tau_t^E = \sum\nolimits_{i=1}^{\infty} \beta^i \prod_{j=1}^{i} \frac{U_{C_{2,t+j}}}{U_{C_{1,t+j-1}}} \Xi_{t+j-1}^C \theta_i Y_{t+i} \neq \tau_t^{PIGOU} \tag{49}$$

as long as $\Xi_t^C \neq 1$.

Proof. To get the first result, equation (48), replace the first-order condition for capital from the social planner's problem, (43), into (44). The second result, equation (49), follows from conditions (39-40) and (44). ∎

Lemma 2 provides the basic characterization for the setting of optimal carbon prices in a second-best world. In general, the optimal carbon tax in an economy with distortionary fiscal policy equals the market costs of carbon, but not always attains its Pigouvian level unless I provide certain conditions for the consumption-savings wedge to be one. For instance, the requirements for the marginal cost of funds to be constant over time. In addition, under this fiscal structure, optimal income taxes can be derived as:

Lemma 3. *The optimal capital and labor income taxes in an economy with distortionary fiscal policy are given by:*

$$\tau_{t+1}^K = 1 - \Xi_t^C \tag{50}$$

$$\frac{1 - \tau_{2,t+1}^L}{1 - \tau_{1,t}^L} = \frac{\Xi_t^C}{\Xi_t^L} \tag{51}$$

Proof. Using the primal approach, see Lemma 1, by combining the FOC's for the social planner (39)-(40) and for the households (4), it yields the optimal capital income tax (50). In addition, from (39-42) and (5-6), we can get the path for labor income taxes, (51). ∎

Taking together Lemma 2 and Lemma 3, it turns out that if $\Xi_t^C = 1$, then from (50) the optimal capital income tax is zero, and using (49) the optimal carbon tax attains its Pigouvian level, without imposing any restrictions on the path of labor income taxes. The following proposition points out the conditions, in terms of wedges, under which optimal carbon prices would differ from the Pigouvian level and the setting of other taxes in the economy.

Proposition 2. *In a second-best fiscal policy:*

1. *The optimal carbon tax always equals the market costs of carbon, (48); however, it is below (above) its Pigouvian level, (49), if the consumption-savings wedge, (46), is below (above) one.*

2. *The optimal capital tax, (50), is positive (negative) if the consumption-savings wedge, (46), is below (above) one.*

3. *The labor income taxes, (51), decrease over time, $\tau_t^L > \tau_{t+1}^L$, if the consumption-savings wedge, (46), is greater than the labor supply wedge, (47).*

Proof. The results follow directly from Lemma 2 and Lemma 3. ■

From proposition 2, a noteworthy implication is that the government always finds optimal to discount future marginal damages using the market interest rate and tax carbon emissions below (or above) its Pigouvian rate relatively to the consumption-savings wedge, Ξ_t^C. Since the government would like to avoid intertemporal distortions, using the same discount rate to evaluate future marginal damages from current carbon emissions, and investment in capital, is optimal. This link between climate and capital investments is also presented in Barrage (2018). However, once the capital income tax rate is different from zero, it is optimal to adjust the carbon price relatively to the Pigouvian level in order to take into account that distortion in consumption-savings trade-offs.

The results presented in numerals 2 and 3 are not novel, though. For instance, Conesa et al. (2009) and Erosa and Gervais (2002) find out that under certain assumptions on preferences, the capital income tax rate is different from zero, which in terms of this paper, would imply a consumption-savings wedge distinct of one.

It is important to note that so far I have assumed that the government has access to a full set of income taxes i.e., capital and age-dependent taxes and no constraints on households' preferences. That is, the second-best problem is not restricted. In particular, I show below that if we extend or restrict the set of available tax instruments to the government, different capital income tax policies could be optimal, conditional to the assumptions on separability in preferences over consumption and leisure. In this sense, to put more structure on the model, in the next section I also proceed to use separable and non-separable preferences as in Conesa et al. (2009) to draw some implications in terms of tax instruments and preferences modeling for the setting of carbon policies and other taxes in general.

4. Age-dependent labor income taxes

Previous literature has pointed out that the existence of individual-specific taxation could generate welfare gains in the implementation of fiscal policies in the presence of consumption externalities (Jacobs and de Mooij, 2015; Kaplow, 2012). As mentioned in the introduction, recent research has also indicated that age-dependent labor income taxation in economies with heterogeneous agents, e.g, in terms of abilities, could reduce the costs associated to distortionary fiscal policy (DaCosta and Santos, 2018; Bastani et al., 2013; Gervais, 2012; Weinzierl, 2011; Blomquist and Micheletto, 2008). In order to understand the role of age-dependent labor income taxes in the setting of optimal climate and fiscal policy when there are production externalities, as special cases, I first consider a policy with age-dependent taxation under two different assumptions about separability and non-separability in preferences over consumption and leisure. Then, I provide additional general results for how the set of optimal tax rates changes when I assume a constrained government who cannot enact differential labor income taxes.[13]

4.1. Age-dependent taxes

In this subsection, I describe under which conditions the optimal carbon price can attain its Pigouvian level using two different preference specifications. It has been shown that the assumption about complementarity between consumption and leisure has important implications for the setting of optimal income taxes, since those interactions constrain how the government can reduce the distortions in the economy when individualized lump-sum taxation is not possible (see e.g., Conesa et al. (2009), Erosa and Gervais (2002)).

4.1.1. Separable preferences

One of the main implications of using separable preferences over consumption and leisure is that there are not complementary effects, $U_{C_{1,t},L_{1,t}} = U_{C_{2,t+1},L_{2,t+1}} = 0$. For instance, suppose that the households' preferences can be represented by the following utility function as in Conesa et al. (2009):

$$U(C_t, L_t) = \frac{C_t^{1-\sigma_1} - 1}{1-\sigma_1} + \chi \frac{(1-L_t)^{1-\sigma_2}}{1-\sigma_2} \tag{52}$$

[13] A general discussion of the implications of age-dependent taxation can be found in Woodland (2016).

where σ_1 and σ_2 denote consumption and labor supply elasticities, respectively; and χ measures the distaste for work with respect to consumption. Under this assumption, as a special case for Proposition 2, it follows:

Proposition 3. *If the government has access to age-dependent labor income taxes, and the households have preferences over consumption and labor which can be represented by an utility function defined as in (52), then in a second-best world:*

1. *The optimal carbon tax equals the market costs of carbon, (48), and attains its Pigouvian level, (49).*
2. *The optimal capital tax, (50), is zero.*
3. *If $L_{1,t} > L_{2,t+1}$, then $\tau^L_{1,t} > \tau^L_{2,t+1}$.*

Proof. In appendix A. ∎

By using a utility function which is separable in consumption and labor, the tax rate on capital income is zero and the carbon tax fully internalizes climate damages from carbon emissions that affect output. This result is equivalent to the one in Barrage (2018), in an infinitely-lived agent model, when climate change only affects production.[14] The intuition for these findings is straightforward. Non-complementarity between consumption and labor reduces the costs of implementing the second-best fiscal policy given that in this case the consumption-savings wedge, Ξ^C_t, is constant over time. Besides, since the government has access to a full set of age-dependent labor income taxes, it is optimal to avoid the distortions in inter-temporal consumption-savings decisions. Thus, considering that a zero optimal capital income tax rate does not affect the relative price between consumption at period t and consumption at period $t+1$, the marginal rate of substitution equals the marginal rate of transformation, and as a consequence the optimal carbon tax is set at its first-best (The Pigouvian level).

4.1.2. Non-separable preferences

Here, I assume that preferences are represented by the following Cobb-Douglas utility function which is not separable in consumption and labor as in Conesa et al. (2009):

[14] Barrage (2018) also shows that this result holds using non-separable preferences. In an OLG framework, however, it is not valid since young and old households have different consumption and labor supply profiles, and that complementarity creates a motive for the government to set both labor and capital income taxes at the same time to smooth optimal paths for consumption and leisure (see e.g., Erosa and Gervais (2001) and Erosa and Gervais (2002)).

$$U(C_t, L_t) = \frac{\left(C_t^{\xi}(1-L_t)^{1-\xi}\right)^{1-\sigma}}{1-\sigma} \tag{53}$$

where ξ measures the degree of substitutability between consumption and leisure and $1/\sigma$ denotes the intertemporal elasticity of substitution. Notice that in this case, the previous non-complementarity vanishes, $U_{C_{1,t},L_{1,t}}, U_{C_{2,t+1},L_{2,t+1}} \neq 0$, and therefore, labor income taxes affect both labor supply and consumption decisions. Under this specification, as a special case for Proposition 2, I get the following:

Proposition 4. *If the government has access to age-dependent labor income taxes, and the households have preferences over consumption and labor which can be represented by the usual Cobb-Douglas utility function (53), then in a second-best world:*

1. *The optimal carbon tax equals the market costs of carbon, (48), however, it is below (above) its Pigouvian level, (49), as long as the labor supply is decreasing (increasing) over the life-cycle*

2. *The optimal capital tax, (50), is positive (negative) as long as the labor supply is decreasing (increasing) over the life-cycle.*

3. $\tau^L_{1,t} > \tau^L_{2,t+1}$*, as long as* $L_{1,t} > L_{2,t+1}$ *and the intertemporal elasticity of substitution,* $1/\sigma$*, is above one.*

Proof. In appendix A. ■

The non-separability of consumption and leisure creates new interactions between the optimal allocation of consumption and labor supply over time and, therefore, it also affects the allocation of leisure. In this case, since labor income taxes distort both consumption and labor optimal paths, the government finds optimal to set a non-zero capital income tax to offset the changes in demand for leisure and consumption. In particular, the allocation of labor over the life-cycle will determine the sign of the capital income tax.

It is important to note that in contrast to Barrage (2018), since in infinitely-lived agent models it is optimal to set a zero capital income tax, here I can derive the implications for the optimal price on carbon emissions when non-zero capital income taxes are optimal as well. It turns out that due to non-separability in preferences and heterogeneity in the age of households, it is possible to get positive (or negative) tax rates on capital returns as an optimal fiscal policy. Thus, I find that again, as in Proposition 2, the optimal carbon tax equals the market costs of carbon, although it differs from the Pigouvian level. The reason is straightforward. Even with a non-zero capital income tax, the marginal productivity of capital does not change, and it only affects the households' decisions with respect to consumption and leisure.

4.2. Age-independent taxes

In the previous apart, I consider special cases in terms of preferences modeling that complement and change the results pointed out in Proposition 2. Here, I assume that the government has no access to age-dependent labor income taxes along the lines of Conesa et al. (2009), that is, $\tau_{1,t}^L = \tau_{2,t}^L = \tau_t^L$. In this case, notice that the government losses one degree of freedom in the set of policy instruments it can use to finance its stream of expenditures and inherited debt. Thus, bearing in mind that, an additional constraint has to be added to the government's problem described using the primal approach (see Lemma 1).

$$\phi_2 U_{L_{1,t}} U_{C_{2,t}} - \phi_1 U_{L_{2,t}} U_{C_{1,t}} = 0 \tag{54}$$

As shown in Erosa and Gervais (2002) and Conesa et al. (2009), this restriction generates a robust role for capital income taxes as they can help the government to tax individuals at different rates without condition on age. To see why it is the case, notice that from Lemma 3, in steady-state it follows:

$$\frac{1-\tau_2^L}{1-\tau_1^L} = \frac{1-\tau^K}{\Xi^L} \tag{55}$$

Given that under age-independent labor income taxes, $\tau_1^L = \tau_2^L = \tau^L$, using the previous equation I can obtain the following:

$$\tau^K = 1 - \Xi^L \tag{56}$$

Since labor supply in an OLG framework is in general not flat, given the heterogeneity with respect to age and productivities, the labor wedge, Ξ_t^L, is not equal to one even in steady-state, and it then depends on the allocation of labor over the life-cycle. Thus, capital taxes can be used to generate (to mimic) the same wedge as in the situation with age-dependent taxation (see e.g., Conesa et al. (2009), Gervais (2012)). In order to see how the main results change when I introduce this constraint, I can define modified versions of the marginal cost of public funds, the consumption-savings and labor supply wedges. To do so, let $\gamma^t\psi_t$ denote the Lagrange multiplier associated to the labor-income taxation constraint (54).

Definition 7. *For $X = \{C, L\}$, let $\Upsilon^{X_{i,t}}$ correspond to the welfare costs involved with the no availability of age-dependent labor income taxation, constraint (54).*

$$\Upsilon^{X_{1,t}} = \frac{\phi_2 U_{L_{1,t}X_{1,t}} U_{C_{2,t}} - \phi_1 U_{L_{2,t}} U_{C_{1,t}X_{1,t}}}{U_{X_{1,t}}} \tag{57}$$

$$\Upsilon^{X_{2,t+1}} = \frac{\phi_2 U_{L_{1,t+1}} U_{C_{2,t+1}X_{2,t+1}} - \phi_1 U_{L_{2,t+1}X_{2,t+1}} U_{C_{1,t+1}}}{U_{X_{2,t+1}}} \tag{58}$$

In a third-best world, bearing in mind the additional constraint in the set of policy instruments, I can rewrite the marginal cost of public funds as follows:

Definition 8. *For $X = \{C, L\}$ and $i = \{1, 2\}$, let $\underline{\Lambda}^{X_{i,t}}$ be the modified marginal cost of public funds (MMCF), that is:*

$$\underline{\Lambda}^{X_{1,t}} = 1 + \lambda_t \left(1 + \Theta^{X_{1,t}}\right) + \psi_t \Upsilon^{x_{1,t}} \tag{59}$$

$$\underline{\Lambda}^{X_{2,t+1}} = 1 + \lambda_t \left(1 + \Theta^{X_{2,t+1}}\right) + \psi_{t+1} \Upsilon^{X_{2,t+1}} \tag{60}$$

Notice that the non-availability of individualized labor income taxes implies a marginal cost of public funds in the implementation of a distortionary fiscal policy that depends now on two terms, and not in only one as described in Proposition 2: (i) the implementability condition, and (ii) the age-independent labor income tax constraint. Solving the constrained social planner's problem, the optimality conditions become:[15]

$$U_{C_{1,t}} \cdot \underline{\Lambda}^{C_{1,t}} = \mu_t \tag{61}$$

$$\frac{\beta}{\gamma} U_{C_{2,t+1}} \cdot \underline{\Lambda}^{C_{2,t+1}} = \mu_{t+1} \tag{62}$$

$$U_{L_{1,t}} \cdot \underline{\Lambda}^{L_{1,t}} = -\mu_t \phi_1 F_{L_t} \tag{63}$$

$$\frac{\beta}{\gamma} U_{L_{2,t+1}} \cdot \underline{\Lambda}^{L_{2,t+1}} = -\mu_{t+1} \phi_2 F_{L_{t+1}} \tag{64}$$

$$\frac{1}{F_{K_{t+1}}} = \gamma \frac{\mu_{t+1}}{\mu_t} \tag{65}$$

$$\sum_{i=1}^{\infty} \gamma^i \frac{\mu_{t+i}}{\mu_t} \theta_i Y_{t+i} = F_{E_t} \tag{66}$$

[15] Notice that I multiply the constraint, (54), by $\frac{\beta}{\gamma}$ for the ease of calculations.

Definition 9. *Under age-independent labor income taxes, the modified consumption-savings wedge, $\underline{\Xi}_t^C$, and the modified labor supply wedge, $\underline{\Xi}_t^L$, reflect inter-temporal welfare costs from using distortionary taxes, and can be represented by the ratio between modified marginal cost of public funds in period $t+1$ and t:*

$$\underline{\Xi}_t^C = \frac{\underline{\Lambda}^{C_{2,t+1}}}{\underline{\Lambda}^{C_{1,t}}} \tag{67}$$

$$\underline{\Xi}_t^L = \frac{\underline{\Lambda}^{L_{2,t+1}}}{\underline{\Lambda}^{L_{1,t}}} \tag{68}$$

As in the previous apart, I can derive optimal tax rates using the consumption-savings and labor supply wedges:

Proposition 5. *In a third-best fiscal policy:*

1. *The optimal carbon tax always equals the market costs of carbon, (48), however, it is below (above) its Pigouvian level, (49), if the modified consumption-savings wedge,(67), is below (above) one.*

2. *The optimal capital tax, (50), is positive (negative) if the modified consumption-savings wedge, (67), is below (above) one.*

3. *The labor income taxes decrease over time, $\tau_t^L > \tau_{t+1}^L$, if the modified consumption-savings wedge, (67), is greater than the modified labor wedge, (68).*

Proof. The results follow directly from Lemma 2 and Lemma 3, but now taking into account the modified version for both the consumption-savings and labor wedges. ■

Proposition 5 implies that the main results derived in Proposition 2 still hold, however, now they are more general in the sense that allow us to determine how the reduction in the set of policy instruments alters the second-best optimal fiscal policy. In the context of optimal environmental policies, it is also easy to check that again the optimal carbon price equals the market costs of carbon, since the production side of the economy is not affected, that is, the marginal productivity of capital does not change under an age-independent taxation fiscal structure.

5. Exogenous capital income taxes

In the previous section I derived the implications for optimal climate and fiscal policy of facing constraints with respect to the use of age-related labor income taxation. Here, in the spirit of Barrage (2018), I describe why those prescriptions are no longer valid when the government is constrained to implement optimal capital income taxes due to an exogenous constraint. Following the same procedure as before, an additional restriction has to be added to the government's problem illustrated in Lemma 1:

$$\frac{U_{C_{1,t}}}{U_{C_{2,t+1}}} - \beta(1-\overline{\tau}^K)F_{K_{t+1}} = 0 \tag{69}$$

Let $\gamma^t\varphi_t$ denote the Lagrange multiplier associated to the capital income tax rate constraint. Under this policy, I get the following expression for the additional welfare costs:

Definition 10. *For $X = \{C, L\}$ and $i = \{1, 2\}$, let $\Pi^{X_{i,t}}$ correspond to the welfare costs involved with the non-availability of a flexible capital income tax rate, constraint (69).*

$$\begin{aligned} \Pi^{X_{1,t}} &= \frac{1}{U_{C_{2,t+1}}}\frac{U_{C_{1,t}X_{1,t}}}{U_{X_{1,t}}} && (70) \\ \Pi^{X_{2,t+1}} &= \frac{U_{C_{1,t}}}{\beta}\frac{U_{C_{2,t+1}X_{2,t+1}}}{U_{X_{2,t+1}}} && (71) \end{aligned}$$

Moreover, in order to write the results in terms of the marginal cost of public funds, I provide an additional definition as follows:

Definition 11. *For $X = \{C, L\}$ and $i = \{1, 2\}$, let $\overline{\Lambda}^{X_{i,t}}$ be the adjusted marginal cost of public funds (AMCF), that is:*

$$\begin{aligned} \overline{\Lambda}^{X_{1,t}} &= 1 + \lambda_t\left(1 + \Theta^{X_{1,t}}\right) + \varphi_t\Pi^{X_{1,t}} && (72) \\ \overline{\Lambda}^{X_{2,t+1}} &= 1 + \lambda_t\left(1 + \Theta^{X_{2,t+1}}\right) - \varphi_t\Pi^{X_{2,t+1}} && (73) \end{aligned}$$

The optimality conditions are now given by:

$$\begin{aligned} U_{C_{1,t}} \cdot \overline{\Lambda}^{C_{1,t}} &= \mu_t && (74) \\ \frac{\beta}{\gamma}U_{C_{2,t+1}} \cdot \overline{\Lambda}^{C_{2,t+1}} &= \mu_{t+1} && (75) \\ U_{L_{1,t}} \cdot \overline{\Lambda}^{L_{1,t}} &= -\mu_t\phi_1 F_{L_t} && (76) \\ \frac{\beta}{\gamma}U_{L_{2,t+1}} \cdot \overline{\Lambda}^{L_{2,t+1}} &= -\mu_{t+1}\phi_2 F_{L_{t+1}} && (77) \\ \frac{1}{F_{K_{t+1}}} + \frac{\varphi_t}{\mu_t}\beta(1-\overline{\tau}^K)\frac{F_{K_{t+1}K_{t+1}}}{F_{K_{t+1}}} &= \gamma\frac{\mu_{t+1}}{\mu_t} && (78) \\ \sum_{i=1}^{\infty}\gamma^i\frac{\mu_{t+i}}{\mu_t}\theta_i Y_{t+i} - \alpha\beta(1-\overline{\tau}^K)\sum_{i=1}^{\infty}\theta_i\frac{\gamma^{i-1}\varphi_{t+i-1}}{\mu_t}\frac{Y_{t+i}}{K_{t+i}} &= F_{E_t} && (79) \end{aligned}$$

Notice that the introduction of a constrained capital income tax rate, since now the government has lost again a policy instrument, creates an inefficiency in the accumulation of capital, see equation (78) second term of the left-hand side, and one more direct interaction between future damages and capital returns, see equation (79) second term of the left-hand side, and therefore, the government has to take this into account when it wants to calculate the present value of future marginal damages from current marginal carbon emissions. Thus,

Proposition 6. *Under no flexibility in the setting of optimal capital income taxes, it follows:*

1. *The optimal carbon tax is given by:*

$$\begin{aligned}\tau_t^E = \sum_{i=1}^{\infty}\prod_{j=1}^{i}\left\{\frac{1}{F_{K_{t+j}}}+\frac{\varphi_{t+j-1}}{\mu_{t+j-1}}(1-\overline{\tau}^K)\frac{F_{K_{t+j}K_{t+j}}}{F_{K_{t+j}}}\right\}\theta_i Y_{t+i} \\ -\alpha\beta(1-\overline{\tau}^K)\sum_{i=1}^{\infty}\theta_i\frac{\gamma^{i-1}\varphi_{t+i-1}}{\mu_t}\frac{Y_{t+i}}{K_{t+i}}\end{aligned} \tag{80}$$

and it differs from both the market costs of carbon, (48), and its Pigouvian level, (49).

Proof. Substituting condition (78) into (79), and according to our definitions, we get the result. ■

This result is equivalent to the one in Barrage (2018) where a constrained set of policy instruments generate efficiency losses in the second-best fiscal policy. That is, exogenous tax rates on capital returns impedes the government to use the market interest rate to discount future marginal damages from carbon emissions, since the constraint on that policy instrument leads to a level of aggregate capital that is not the one a social planner would like to see in the economy. As a result, the optimal carbon tax does not equals the market costs of carbon, (48), a finding that as shown above, is robust to assumptions on preferences modeling and age-dependent taxation in a second-best world.

6. Exogenous carbon taxes

So far, it has been assumed that the social planner has access to both fiscal and climate policies. However, one can think of situations in which there is an external regulator who fixes carbon taxes without caring about the welfare of the successive generations, and in that case, the government can only implement capital and labor income taxes. Here, I define the implication of this new restriction on policy instruments. As before, I need to introduce an additional constraint in the social planner's problem:

$$\tau^E = F_{E_t} \tag{81}$$

Let $\gamma^t \delta_t$ denote the Lagrange multiplier associated to the carbon tax rate constraint. Solving this problem, the optimality conditions are given by:

$$\begin{aligned}
U_{C_{1,t}} \cdot \Lambda^{C_{1,t}} &= \mu_t && (82)\\
\frac{\beta}{\gamma} U_{C_{2,t+1}} \cdot \Lambda^{C_{2,t+1}} &= \mu_{t+1} && (83)\\
U_{L_{1,t}} \cdot \Lambda^{L_{1,t}} &= -\mu_t \phi_1 F_{L_t} + \delta_t \phi_1 F_{E_t L_t} && (84)\\
\frac{\beta}{\gamma} U_{L_{2,t+1}} \cdot \Lambda^{L_{2,t+1}} &= -\mu_{t+1} \phi_2 F_{L_{t+1}} + \delta_{t+1} \phi_2 F_{E_{t+1} L_{t+1}} && (85)\\
\frac{1}{F_{K_{t+1}}} + \gamma \frac{\delta_{t+1}}{\mu_t} \frac{F_{E_{t+1} K_{t+1}}}{F_{K_{t+1}}} &= \gamma \frac{\mu_{t+1}}{\mu_t} && (86)\\
\sum_{i=1}^{\infty} \gamma^i \frac{\mu_{t+1}}{\mu_t} \theta_i Y_{t+i} - \sum_{i=1}^{\infty} \gamma^i \frac{\delta_{t+i}}{\mu_t} \theta_i F_{E_{t+i} E_t} &= F_{E_t} + \frac{\delta_t}{\mu_t} F_{E_t E_t} && (87)
\end{aligned}$$

From the conditions for labor (84-85) and capital (86), one can see that as a result of the constraint on carbon taxes, optimal levels of labor and capital should be adjusted to take into account the interaction between the marginal product of energy and changes in inputs, so that the constraint (81) binds. Under this fiscal scheme, optimal income taxes can be derived as:

Lemma 4. *The optimal capital and labor income taxes in an economy with exogenous carbon taxes are given by:*

$$\tau_{t+1}^K = 1 - \frac{\mu_t \Xi_t^C}{\mu_t + \gamma \delta_{t+1} F_{E_{t+1} K_{t+1}}} \tag{88}$$

$$\frac{1 - \tau_{2,t+1}^L}{1 - \tau_{1,t}^L} = \frac{\Xi_t^C}{\Xi_t^L} \cdot \left[\frac{1 - \frac{\delta_{t+1} F_{E_{t+1} L_{t+1}}}{\mu_{t+1} F_{L_{t+1}}}}{1 - \frac{\delta_t F_{E_t L_t}}{\mu_t F_{L_t}}} \right] \tag{89}$$

Proof. Using the primal approach, see Lemma 1, by combining the FOC's for the social planner (82)-(83) and for the households (4), it yields the optimal capital income tax (88). In addition, from (82-85) and (5-6), we can get the path for labor income taxes, (89). ■

Lemma 4 refines the results of Lemma 3 by including the effects of exogenous carbon prices on the setting of income taxes. At the same time, it also presents a novel result in the literature of optimal taxes on capital:

Corollary 1. *An exogenous carbon price, τ^E, provides a firm rationale for positive capital income taxes even if preferences are separable over consumption and leisure, in contrast to Proposition 3, that is,*

$$\tau_{t+1}^K = 1 - \frac{\mu_t}{\mu_t + \gamma \delta_{t+1} F_{E_{t+1}K_{t+1}}} > 0 \tag{90}$$

From Proposition 3 we know that separability in preferences over consumption and leisure implies a constant marginal cost of public funds, and therefore, a consumption-savings wedge, $\Xi_t^C = 1$. Here, the presence of non-optimal carbon prices leads to the social planner to make adjustments on available fiscal instruments, particularly, implementing a non-zero optimal capital income tax.

7. Discussion

This paper has studied the optimal climate and fiscal policy in an OLG economy. I show that in general the optimal carbon tax in an economy with distortionary fiscal policy equals the market costs of carbon, but not always attains its Pigouvian level. That is, future marginal damages to current carbon emissions are discounted using the market rate of interest. This result resembles the opportunity cost of capital investment, supporting the Nordhaus (2008) recommendation that claims that climate change mitigation investments should earn the same net return than other investments in the economy.

Moreover, I addressed the implications of separability in the utility function and different tax instruments available to the government on the setting of carbon taxes. In particular, I show that with a full set of tax instruments and separable preferences over consumption and labor supply, the optimal carbon tax attains its Pigouvian level. The intuition behind this outcome relies on the fact that separability implies a constant general equilibrium elasticity in consumption, result that avoids the distortions due to the introduction of proportional labor and capital income taxes. Thus, it turns out that since a zero capital income tax is optimal, the way households value current and future consumption is not affected, and therefore, the carbon tax equals both its Pigouvian level and the market costs of carbon.

However, once the government is constrained in the set of tax instruments, for instance, no age-dependent labor income taxation, there is space for an endogenous non-zero tax rate on capital income and, as a result, the optimal carbon tax does not attain its Pigouvian level; that is, either a positive or a negative tax rate on capital income creates a wedge between the marginal rate of substitution for consumption and the marginal rate of transformation. Likewise, when preferences are not separable over consumption and labor, even if I allow for the existence of age-dependent labor income taxes, the optimal carbon price differs from the Pigouvian tax. In general, I find out that the conditions which optimally generate a capital income tax rate different from zero, would also imply a distortion in the setting optimal environmental policies.

Finally, I also provide a novel result in the literature of optimal capital taxation. The existence of constraints on environmental regulations such as an exogenous carbon price leads to the social planner to adjust its fiscal policy scheme by changing capital and labor income tax rates in order to offset the inefficiency coming from a carbon tax that does not fully internalize climate damages from current carbon emissions.

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A. Proofs

Lemma 1

Proof. The procedure follows closely the one in Iqbal and Turnovsky (2008) and Garriga (2001). I begin by showing that a competitive equilibrium must satisfy equations (30)-(33). Notice that equations (30) and (31) are equivalent to the first two constraints that come from the market clearing conditions in definition 1. To derive (32), I proceed to use the intertemporal budget constraint for the households which is given by:

$$\begin{aligned} C_{1,t} + \frac{C_{2,t+1}}{(1-\tau^K_{t+1})r_{t+1}} = (1-\tau^L_{1,t})\phi_1 w_t L_{1,t} + \frac{(1-\tau^L_{2,t+1})\phi_2 w_{t+1} L_{2,t+1}}{(1-\tau^K_{t+1})r_{t+1}} \\ + B^D_{t+1}\left[\frac{R_{t+1}}{(1-\tau^K_{t+1})r_{t+1}} - 1\right] \end{aligned} \tag{A.1}$$

Then, using the optimality conditions (4)-(7) from the household's problem, it follows that:

$$C_{1,t} + \frac{\beta U_{C_{2,t+1}}}{U_{C_{1,t}}} C_{2,t+1} = -\frac{U_{l_{1,t}}}{U_{C_{1,t}}} L_{1,t} - \frac{U_{L_{2,t+1}}}{U_{C_{2,t+1}}} L_{2,t+1} \frac{\beta U_{C_{2,t+1}}}{U_{C_{1,t}}} \tag{A.2}$$

which yields (32). The same procedure can be applied to derive (33) by considering that the budget constraint for the initial old is:

$$C_{2,0} = (1-\tau^L_{2,0}) w_0 \phi_2 L_{2,0} + \left[(1-\tau^K_0) r_0\right] K_0 + R_0 B_0 \tag{A.3}$$

Using the first-order conditions from the household's problem, (4-6), we arrive to conditions (54-69). To prove the last part of the proposition, the prices can be derived using the first-order conditions (11)-(13) from the firm's problem, in addition to a carbon tax, to make them consistent with a competitive equilibrium, that is:

$$\tau^E_t = F_{E_t} - p^E \tag{A.4}$$

$$w_t = F_{L_t} \tag{A.5}$$

$$r_t = F_{K_t} \tag{A.6}$$

Likewise, the set of policies for labor and capital income can be constructed by replacing allocations and equilibrium prices into the first order conditions from the household's problem such that tax rates satisfy those conditions, Therefore, using equations

(4)-(6), the following conditions characterize labor and capital income taxes:

$$\tau_{t+1}^{K} = 1-\frac{U_{C_{1,t}}}{\beta r_{t+1}U_{C_{2,t+1}}} \tag{A.7}$$

$$\frac{1-\tau_{2,t+1}^{L}}{1-\tau_{1,t}^{L}} = \frac{\phi_1 w_t U_{L_{2,t+1}} U_{C_{1,t}}}{\phi_2 w_{t+1} U_{L_{1,t}} U_{C_{2,t+1}}} \tag{A.8}$$

Finally, the return on debt holdings can be defined using the no arbitrage condition (7). Notice that the household's budget constraint also holds under those allocations and prices. To see that, replace the first order conditions from the household's problem into equations (32) and (33) to get the intertemporal budget constraint for the households. Since the feasibility constraint and the intertemporal budget constraint for the households are satisfied, by Walras' Law, the government budget constraint holds as well and government debt B_{t+1} is set accordingly.

■

Proposition 3

Proof. Consumption-leisure separability implies $U_{CL} = U_{LC} = 0$, thus:

$$\begin{aligned} U_{C_{i,t}} &= C_{i,t}^{-\sigma_1} \\ U_{C_{i,t}C_{i,t}} &= -\sigma_1 C_{i,t}^{-\sigma_1-1} \\ U_{L_{i,t}} &= -\chi(1-L_{i,t})^{-\sigma_2} \\ U_{L_{i,t}L_{i,t}} &= -\chi\sigma_2(1-L_{i,t})^{-\sigma_2-1} \end{aligned}$$

Then, I can find the expressions for the general equilibrium elasticities as follows:

$$\Theta^{C_{i,t}} = -\sigma_1 \tag{A.9}$$

$$\Theta^{L_{i,t}} = \frac{\sigma_2 L_{i,t}}{1-L_{i,t}} \tag{A.10}$$

Hence, if $\Theta^{C_{1,t}} = \Theta^{C_{2,t+1}} = -\sigma_1$, then $\Lambda^{C_{1,t}} = \Lambda^{C_{2,t+1}}$, and using (46), it yields $\Xi_t^C = 1$. To check that the first two numerals of the proposition hold under this condition, replace $\Xi_t^C = 1$ in equations (50) and (49), to get the zero optimal capital income tax and the optimal carbon tax at the Pigouvian level, respectively. Finally, to derive optimal labor income taxes, we use (51) to obtain:

$$\frac{1-\tau^L_{2,t+1}}{1-\tau^L_{1,t}} = \frac{1}{\Xi^L_t} \tag{A.11}$$

From the previous equation, $\tau^L_{2,t+1} < \tau^L_{1,t}$, as long as $\Xi^L_t < 1$. This latter condition requires that $\Lambda^{L_{1,t}} > \Lambda^{L_{2,t+1}}$, which is only possible if $L_{2,t+1} < L_{1,t}$. That is, a declining labor supply requires labor income taxes that decrease with age, then $\Xi^L_t < 1$, and we obtain the third result.

■

Proposition 4

Proof. Without separability, marginal utilities can be derived as:

$$\begin{aligned}
U_{C_{i,t}} &= \frac{\xi(1-\sigma)U_{i,t}}{C_{i,t}} \\
U_{L_{i,t}} &= -\frac{(1-\xi)(1-\sigma)U_{i,t}}{1-L_{i,t}} \\
U_{C_{i,t}C_{i,t}} &= \frac{\left[(1-\sigma)\xi^2-\xi\right](1-\sigma)U_{i,t}}{C^2_{i,t}} \\
U_{C_{i,t}L_{i,t}} = U_{L_{i,t}C_{i,t}} &= -\frac{(1-\xi)\xi(1-\sigma)^2U_{i,t}}{C_{i,t}(1-L_{i,t})} \\
U_{L_{i,t}L_{i,t}} &= -\frac{(1-\xi)(1-\sigma)U_{i,t}}{(1-L_{i,t})^2}\left[(1-\xi)\sigma+\xi\right]
\end{aligned}$$

Moreover, the general equilibrium elasticities imply the following:

$$\Theta^{C_{i,t}} = -1+(1-\sigma)\left[\xi - \frac{(1-\xi)L_{i,t}}{1-L_{i,t}}\right] \tag{A.12}$$

$$\Theta^{L_{i,t}} = \frac{[(1-\sigma)\xi+\sigma]L_{i,t}}{1-L_{i,t}} + \xi(1-\sigma) \tag{A.13}$$

Using the previous results, it follows that:

$$\Xi^C_t = \frac{1+\lambda_t(1-\sigma)\left[\xi - \frac{(1-\xi)L_{2,t+1}}{1-L_{2,t+1}}\right]}{1+\lambda_t(1-\sigma)\left[\xi - \frac{(1-\xi)L_{1,t}}{1-L_{1,t}}\right]} \tag{A.14}$$

In this case, notice that $\Xi^C_t = 1$ if and only if households feature a flat labor supply. So, if the labor supply is decreasing (increasing) over the life-cycle, it is optimal to set a positive (negative) capital income tax, and the optimal carbon tax would be lower (higher) than the Pigouvian level.

Likewise, with respect to labor income taxes, I obtain that:

$$\Xi_t^L = \frac{1+\lambda_t(1-\sigma)\left[\xi - \frac{(1-\xi)L_{2,t+1}}{1-L_{2,t+1}}\right] + \frac{\lambda_t}{1-L_{2,t+1}}}{1+\lambda_t(1-\sigma)\left[\xi - \frac{(1-\xi)L_{1,t}}{1-L_{1,t}}\right] + \frac{\lambda_t}{1-L_{1,t}}} \tag{A.15}$$

We know thus that:

$$\frac{1-\tau_{2,t+1}^L}{1-\tau_{1,t}^L} = \frac{\Xi_t^C}{\Xi_t^L} > 1 \tag{A.16}$$

as long as $L_{2,t+1} < L_{1,t}$ and $1+\lambda_t(1-\sigma) > 0$. To see this, define $m_{i,t}$ and $n_{i,t}$ as:

$$m_{i,t} = 1+\lambda_t(1-\sigma)\left[\xi - \frac{(1-\xi)L_{i,t}}{1-L_{i,t}}\right]$$

$$n_{i,t} = \frac{\lambda_t}{1-L_{i,t}}$$

such that,

$$\frac{\Xi_t^C}{\Xi_t^L} = \frac{\frac{m_{2,t+1}}{m_{1,t}}}{\frac{m_{2,t+1}+n_{2,t+1}}{m_{1,t}+n_{1,t}}} \tag{A.17}$$

After some algebra, and by replacing our auxiliary variables, it yields the condition for $\Xi_t^C/\Xi_t^L > 1$:

$$\{L_{1,t} - L_{2,t+1}\}\{1+\lambda_t(1-\sigma)\} > 0 \tag{A.18}$$

Note that this expression is particularly true whenever that the intertemporal elasticity of substitution, $1/\sigma$, is above one.

■