RENEWABLE AND NON-RENEWABLE ENERGY-BASED OUTPUT GROWTH ALONG BALANCED-GROWTH PATH

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Abstract

Exhaustible energy source takes important place in the share of world energy consumption in the detriment of renewable energy. This paper investigates output growth based on renewable and non-renewable energy along balanced growth path. As a result, we find that output based on renewable energy grows faster than that based on non-renewable one, and per capita output associated with non-renewable energy would follow decreasing path if the technology is sufficient to sufficient to offset the drag in non-renewable enrgy scaricity in the future, whereas that associated with renewable energy would be increasing or constant. Other results will be discussed along the analysis.

1. INTRODUCTION

There has been a growing concern about long-run positive consumption and environment resulted from exhaustible resource depletion and scarcity since natural resource is an essential element for human-kind. It has gained prominence in different circles of think-tank triggered by the researchers' awareness. The leaders in the world have decided to tackle this issue related to over-extraction of natural resource and green gas emission.

The first oil price shock in early 1970s has awakened the interest of economists to the research on the relationship between finite resource price and economic growth. Precisely, they empirically investigated the relationship between energy price and GDP. In literature, there exists common view on the negative oil price effect on macro-economic parameter in oil-importing-country by using different econometric methods (Hamilton, 2005, 2009; Wei and Guo, 2016; Gadea et al., 2016). Oil price increase is generally prompted by the increasing demand from emerging country, global production trend and speculation (Kilian and Murphy, 2009). High economic growth in emerging country like China, Brazil, India has led to the high energy consumption and hence, unstable price. The issue is not only in demand side but also in the fact that oil supply response to the demand shock is weak and as a result, reflects in price increase. Speculation in production also accounts for oil price increase. The extent impact of such shock depends on the cause of oil price shock (Peersman and Robays, 2012). Recent studies have demonstrated that before affecting the output, oil price effect passes through a different channel such as interest rate and investment which are an essential parameter for the production function (Arshad et al.). These researchers used empirical methodology for estimating a model (GMM, Granger Causality, etc.) to investigate their relationship and the result of their work has been given prominence in the various scholars and policy maker in order to find an issue to secure energy needs in many countries. In the country endowing natural resource, the economy is safe from oil price shock but surprisingly their economy does not show such advantage.

The mainstream in economic theory seems ignore the presence of exhaustible resource among the inputs used in the production process. There is a need to think twice of what steady state means, where exhaustible resource plays an important role (Stiglitz, 1974). In neoclassical growth model, long-run per capita growth is driven by technology and so long as the growth rate technological progress is positive, it would be positive (Solow, 1956). New growth theory put into question this mainstream on the ground of resource depletion which can limit economic growth (Stiglitz, 1974; Dasgupta and Heal, 1974; Hartwick, 1978; Solow, 1976). In case the resource is an essential, they all share the common view that exhaustible resource bounds output to zero when its stock falls to zero. The economic growth would be stagnant and then decline when the resource stock approaches to zero (Schilling and Chireng, 2010). All economy based on typical resource faces the resource scarcity at the end, and positive growth would not be possible. Sustainable production relies on the presence of substitutable input for finite resource. The more heavily production depends on renewable factor, the more it is sustainable.

These previous studies have not sufficiently study and covered all models that may happen in an economy based on renewable and non-renewable energy. The present research investigates and compares the long-run growth of an economy based on renewable and non-renewable energy. The analysis is divided into two cases supposedly occurred in one country; in each case, there is production function such that in the first one, it is based on non-renewable energy while on renewable one in the second case. A model is built for each case and subject to the constraint associated with capital stock and resource stock. At the end of analysis, we have shown that renewable resource based-outputs grow faster than non-renewable energy-based one along the balanced growth path; and other findings related to renewable and non-renewable energy-based outputs growth will be discussed.

2. LITERATURE REVIEW

Hoteling (1931) studied the first analysis in the optimal use of exhaustible resource and published a rule, namely Hoteling's rule. In fact, his work was in response, on one hand, to the wasteful extraction of the exhaustible resource which had led to a very high deletion rate and threatened to leave the future generation with insufficient resources and, on the other hand, to the static equilibrium of the economic theory that is irrelevant with exhaustible natural resource. He concluded that natural resource prices must grow at a rate equal to the interest rate which is, however, in the real world, do not follow such pattern.

Following the oil price shock in 1970s, many researchers have paid attention to the economic growth and exhaustible resource price, and investigated the relationship between economic growth and energy price. Energy is a critical support of industrialization throughout many centuries and sufficient energy sources are vital to sustain economic and social development. The research on the link between oil price shock and economic growth has attracted the interest of economist. It is well-known that this interest dates back to the first oil price shock in early 1970s which were described as the period of severe oil price fluctuation. There is a common opinion on this relationship in which oil price shock may cause inflation and decline in output. Hamilton (2009), and Kilian and Murphy (2009) emphasized the increasing demand from emerging countries, global production insufficiency and the speculation are the principal causes of oil price increase. Intuitively, high price has negative effect on oil-importing country and positive on exporting country. A number of empirical analyses has studied the nexus between energy price and economic growth and has shown the negative energy price effect on importing country's economic growth (Peersman and Robays, 2012; Idrisov et al.; 2015; Gadea et al.; 201). Idrisov et al. (2015) studied the global oil price effect on Russian economy and performed econometric test and argued that oil price rise cannot affect long-term economic growth butt only short-term positive effect. This result is in line with the Solow model which states, on a steady-state path, output-capital ratio is constant i.e. output and capital grow at same constant rate equal to the sum of technology change and population growth rate. Wei and Guo (2016) have investigated the relationship between chines macro-economy and oil prices. The method used in the analysis is the frequency domain causality test developed by Breitung and Candelon (2006) which allows for statistically testing the causality at different frequencies. The results have shown that energy price negatively influence output in energy-importing country.

Other researchers have carried out an analysis in the context of economic growth with resource depletion in order to find an optimal path of resource use. In neoclassical growth model, long-run growth of per capita output is led by technological progress. So long as technological growth rate is positive, per capita output would also increase (Solow, 1956). All long-run growth stems from factors outside the model. However, there is another concept called endogenous growth that provided further analysis on growth engine. Romer (1990) has argued that private firms seeking for own profit induce more research and this, in turn, accelerates output growth. Contrary to Solow, he added production of knowledge through research into the model as the third factor that drives growth process in long-run. They share the opinion on the positive long-run growth with technological progress. New growth theory criticized the mainstream in growth model on the ground that resource depletion limits long-run growth. There exists a growing concern on the current context of increasing resource needs on one hand and depletion of exhaustible resource expected in the future on the other.

New growth theory has addressed this issue and studied the long-run growth with the presence of exhaustible resource. Some authors emphasized that the limited available stock of non-renewable natural resource involves that output is bounded since that stock falls to zero over infinite horizon whereas others proved that positive long-run growth is feasible with some technological properties. Example of them includes, among others, Meadows (1972), Stiglitz (1974), Solow (1976), Hartwick (1978), Dasgupta-Heal (1974) and Simmon (2005). Then, exhaustible resource should be optimally extracted in order to prevent from being early used up all resources. Schilling and Chireng (2010) argued the extensive depletion rate of non-renewable resource yields a significant impact on sustainable growth. The future generation would be deprived of enough resource to achieve their future goal. Then, the economic growth would become stagnant and after decline since many essential resources would be exhausted.

Robson (1980) has investigated output growth path of an economy with exhaustible resource and costly innovation. His model extended the work of Solow (1978) and included innovation, and assumed those following assumptions: constant extraction rate and constant fraction of output used into innovation fund. He finds the asymptotic growth is altered by the fact that innovation is endogenous, and there is symmetric relationship between costly innovation and natural resource depletion. That is to say, if the rent from resource extraction is invested into innovation, the effect of resource depletion and innovation can counteract one another. More efficient innovation can result in a slower rate of optimal resource depletion. Hamilton and Hartwick (2005) have set up growth model in which natural resource rent is invested into capital to analyse consumption and investment path. They have demonstrated rules of relationship existing between the motions of consumption and net investment at specific path i.e. at a point of locally constant consumption, the level of net investment is negative and the percentage change in net investment is equal to dominant market rate of interest. Hartwick (1977) concluded that if economic profit from exhaustible resource extraction is invested into capital accumulation, sustainable development may be possible but this conclusion has been criticised in the sense that it is an efficiency condition but not that of sustainability. Takekuma (1983) has investigated on optimal programs of capital accumulation with exhaustible resources and considered a model in multi-sector economy with several exhaustible resources, which differs it from the previous works carried out by Solow, Stiglitz and Hartwick. He proved that optimal program of resource depletion exists if capital accumulation can prevent the exhaustibility of resource depletion.

3. OUTPUT GROWTH BASED ON RENEWABLE AND NON-RENEWABLE ENERGY

In this research, we aim at finding and comparing the renewable and non-renewable energy-based output growth along balanced-growth path. We divided the analysis into two different cases, which differ with regard to the types of energy used in the production.

3.1. General model

Let assume a Cobb-Douglass production function (Y_t) with four inputs such as capital stock (K_t), labor (L_t), technology (A_t), and energy use (E_t), which is either renewable or non-renewable energy source. The function *F* is continuously differentiable, concave and increasing.

$$Y = F\big(K(t), R(t)\big) = A(t)K^{\alpha}(t)E^{\theta}(t)L^{\beta}(t) \tag{1}$$

where the parameters $\alpha, \beta\ and\ \theta$ represent the share of capital stock, labor and the energy, respectively. B is environmental issue associated with non-renewable energy. F(Xi) satisfies the following properties: $\frac{dF}{dX_i} \geq 0; \frac{d^2F}{d^2X_i} \leq 0; \lim_{X_i \to \infty} F(.) = 0; \lim_{X_i \to 0} F(.) = \infty$

Following Solow model (1956), technology grows at exogenous rate (ε) such as:

$$A(t) = e^{\varepsilon t}A_0 \tag{2}$$

where A_0 denotes the initial level of technology. The labor grows at exogenous rate equal to the population growth, and there is no distinction between population and labor:

$$\frac{\dot{L}}{L} = n \tag{3}$$

The central planner maximizes the instantaneous utility function of the representative household with a constant rate of time preference (ρ>0).

$$U = \int_0^{\infty} \frac{C^{1-\omega} - 1}{1-\omega} e^{-\rho t}; 0 < \omega < 1 \tag{4}$$

where ω is the constant value of elasticity of marginal utility or risk aversion; U(c_t) : monotonically increasing, strictly concave, and continuously differentiable. Taking logarithmically time derivative of equation (1), one obtains:

$$g_Y = \alpha g_k + \theta g_E + \varepsilon + \beta n \quad (5)$$

where g_k and g_E denote respectively the growth of capital stock and energy use.

3.2. Non-renewable energy-based output

Non-renewable resource is natural resource that cannot be regenerated by natural means and is then depleted over time. The total energy source that can be extracted should abide by the following conditions:

$$\int_0^{\infty} E(t)\,\partial t \leq S_0 \quad (6)$$

where S_0 represent the natural resource stock at initial period. This stock decreases by each resource extracted at each period, and then its change follows the equation below:

$$\dot{S} = -E(t) \quad (7)$$

Following the previous works in the literature (among others, Dasgupta and Heal, 1976), the extraction cost and uncertainty are ignored. Of output, a part is invested in capital stock K(t) and the rest is consumed C(t). The change of capital stock follows the general pattern such as:

$$\dot{K} = Y - C - \delta \mathrm{K} \quad (8)$$

where δ represents the depreciation rate of capital stock at each period.

3.2.1. Optimal Condition

The problem associated with the model is in equation (9). The aim is to find the optimal path by maximizing utility function subject to the change in the resource stock and capital stock.

Max (9)

$$\int_0^\infty \frac{C^{1-\omega}}{1-\omega} e^{-\rho t} \partial t;\ 0 < \omega$$

Subject to

$$\dot{K} = Y_t - C_t - \delta \mathrm{K}_t$$

$$\dot{S}_t = -E_t$$

$R_t \geq 0$, $C_t \geq 0$ and $K_t \geq 0$

K_0 and S_0 are given

The current value of Hamiltonian function associated with the problem in equation (9) is:

$$H = \frac{c^{1-\omega}}{1-\omega} + \mu_{(t)}(Y_t - C_t - \delta \mathrm{K}_t) - \lambda_{(t)} E_t \tag{10}$$

where μ and λ are shadow price associated with capital stock and exhaustible resource stock. Hamiltonian function consists of objective function, control and state variables. The objective function, utility function, is the one that is maximized. The state variable is the capital stock, and the control ones are the consumption and the investment allocated in renewable energy source.

Computing the First Order Conditions with respect to the consumption and resource gives the following equations at optimal path:

$$c^{-\omega} = \mu \tag{11}$$

$$\mu F_E = \lambda \tag{12}$$

The conditions associated with the shadow price μ(t) and λ(t) are:

$$\dot{\mu} - \rho\mu = -\mu(F_k - \delta) + \rho\mu \tag{13}$$

$$\dot{\lambda} = \rho\lambda \tag{14}$$

With $F_K = \frac{dF(K,E)}{dK}$ and $F_E = \frac{dF(K,E)}{dE}$. Let $\dot{X} = \frac{dX}{dt}$ and by taking time-derivative of equation (11) logarithmically and using equation (13), one obtains:

$$\omega \frac{\dot{c}}{c} = F_k - (\rho + \delta) \tag{15}$$

By using equation (12), (13) and (14),

$$F_k - \rho = g_y - g_E \tag{16}$$

Since $C = cL$ and taking logarithmic derivative of it yields:

$$\frac{\dot{C}}{C} = \frac{\dot{c}}{c} + n \tag{17}$$

Let g_S denote the depletion growth rate and be presented by the following equation:

$$g_S = \frac{\dot{S}}{S} = -\frac{E}{S} = -m \tag{18}$$

3.2.2. Balanced Growth Path (BGP)

Let $h \equiv \frac{C}{Y}$ $q \equiv \frac{Y}{K}$ $and\ m \equiv -\frac{\dot{S}}{S} = \frac{E}{S}$. Along balanced-growth path, the dynamic model follows a trajectory such that all variables grow at a constant rate. Following Groth and Schou (2007), an equilibrium path for a set of $(\mathrm{Y}, \mathrm{K}, \mathrm{C}, \mathrm{E})_0^\infty$ is named balanced growth equilibrium (BGE).

Proposition 1: For any balanced growth equilibrium, the following characteristics hold:

i. $g_E = g_S = -m^*$ (m is a positive constant)
ii. $g_C = g_Y = g_k = g^*$
iii. $m^*\ and\ g^*$ satisfy the following conditions:

$$(1-\alpha)g^* + \theta m^* = \beta n + \varepsilon \tag{19}$$

$$(1-\omega)g^* + m^* = \rho - \omega n \tag{20}$$

Proof of proposition 1:

- Equation (i)

Dividing equation (7) by S

$$\frac{\dot{S}}{S} = -\frac{E}{S} = -m \tag{6.a}$$

Since m is constant, $S_t = S_0\exp(-mt)$ and $E = mS = mS_0\exp(-mt)$; by taking logarithmic time-derivative of E_{nr}, we have $\frac{\dot{E}}{E} = -m$, equation (i)

- Equation (ii)

It is a feature of balanced growth equilibrium.

- Equation (19)

By using equation (5), (i) and (ii),

$$g^* = \alpha g^* - \theta m^* + \beta n + \varepsilon$$

By rearranging it, one obtains equation (19)

$$(1-\alpha)g^* + \theta m^* = \beta n + \varepsilon$$

- Equation (20)

By using equation (15), (17) and (26),

$$g^* = \frac{1}{\omega}(g^* + m^* - \rho) + n$$

By rearranging it, one obtains equation (20).

$$(1-\omega)g^* + m^* = \rho - \omega n$$

We have two equations (19) and (20) with two unknown variables and then, it is possible to find a solution.

Proposition 2: Along BGE, output growth and depletion rate are equal to g* and m*, respectively. Given the determinant of matrix associated with the equations (19) and (20) is different from nil, the output growth and depletion rate are the following ones along the BGP:

$$g^* = \frac{\beta n + \varepsilon - \theta(\rho - \omega n)}{1 - \alpha - \theta(1-\omega)} \quad (21)$$

At constant return to scale, this denominator of g* is likely to be positive. The resource extraction (m*) along BGE would be equal to:

$$m^* = \frac{-(1-\omega)(\beta n + \varepsilon) + (1-\alpha)(\rho - \omega n)}{1 - \alpha - \theta(1-\omega)} \; ; m > 0 \quad (22)$$

Proof of g* and m*:

The matrix associated with the system (19) and (20):

$$A = \begin{pmatrix} 1-\alpha & \theta \\ 1-\omega & 1 \end{pmatrix}$$

In order to solve the equations (19) and (20), we should find the inverse of A and apply the following formula:

$$\begin{pmatrix} g^* \\ m^* \end{pmatrix} = A^{-1} \begin{pmatrix} \beta n + \varepsilon \\ \rho - \omega n \end{pmatrix}$$

We know that $A^{-1} = \frac{1}{detA} \begin{pmatrix} 1 & -\theta \\ \omega - 1 & 1-\alpha \end{pmatrix}$ an

d $detA = \omega\theta + 1 - \alpha - \theta$. Then,

$$\begin{pmatrix} g^* \\ m^* \end{pmatrix} = \frac{1}{detA} \begin{pmatrix} 1 & -\theta \\ \omega - 1 & 1-\alpha \end{pmatrix} \begin{pmatrix} \beta n + \varepsilon \\ \rho - \omega n \end{pmatrix}$$

We have solved the output growth and depletion rate along the balanced growth path. Since g* and m* are already known, it is now easy to find the solution for output-capital (q^*) and consumption-capital (v^*) ratio along the BGP:

By using equation (i), (ii) and (15), one obtains:

$$q^* = \frac{\omega(\beta n + \varepsilon)}{\alpha(1-\alpha-\theta)} + \frac{\rho + \delta - \omega n}{\alpha} \tag{23}$$

By dividing equation (7) by K and using equation (15), we have:

$$v^* = \frac{(\omega - \alpha)(\beta n + \varepsilon) + (1-\alpha)(\rho + \delta - \omega n)(1-\alpha-\theta)}{\alpha(1-\alpha-\theta)} \tag{24}$$

Proposition 3: At constant return to scale, whether per capita consumption growth is positive or not depends on technological progress, discount rate and the share of non-renewable energy source.

Proof of proposition 3:

We know that g* is the growth rate of output, capital stock and consumption along the balanced growth equilibrium (g* is equation (21)). The total consumption equals C= h.L where h is per capita consumption, and L is population number; by taking time-derivative of C logarithmically, we have $\frac{\dot{C}}{C} = \frac{\dot{h}}{h} + n$ where $\frac{\dot{L}}{L} = n$. Subtracting n to the left side,

$$\frac{\dot{h}}{h} = \frac{\dot{c}}{c} - n = g^* - n$$

Computing this difference gives:

$$g^* - n = \frac{\beta n + \varepsilon - \theta(\rho - \omega n)}{1 - \alpha - \theta(1 - \omega)} - n$$

and then

$$\frac{\dot{h}}{h} = \frac{\varepsilon - \theta\rho}{\theta\omega} \qquad (25)$$

Proposition 4: Assume a production function with constant return to scale. Along the balanced-growth equilibrium, the per capita output growth is negative if there is no technological progress.

Proof of proposition 4:

From equation (21) and referring to the assumption that there is no technological change, we have

$$g^* = \frac{\beta n - \theta(\rho - \omega n)}{1 - \alpha - \theta(1 - \omega)}$$

Computing the difference between g* and n gives the following result:

$$g^* - n = -\left[\frac{\theta(\rho - \omega n) + n\theta\omega}{1 - \alpha - \theta(1 - \omega)}\right]$$

Since $\rho - \omega n$ is positive according to the equation (20), the g*- n is negative.

3.2.3. Transitional dynamics

Recalling that $q \equiv \frac{Y}{K}, x \equiv \frac{C}{K}\, and\ m \equiv \frac{R}{S}$, and Let q*, x* and m* denote the steady state value associated with the system of q, x and m, respectively. It has saddle-path stability if it has a unique solution converging to the steady-state for a time period towards infinity. Taking the time-derivative logarithmically for the equations (q, x and m) gives the following system of equation.

$$\dot{q} = \left[-(1-\alpha)(q - x - \delta) + \theta m + \beta n + \varepsilon\right] + q \qquad (26)$$

$$\dot{x} = \left(\frac{\alpha}{\omega} - 1\right)q + x + \delta$$

$$\dot{m} = g_r - m$$

Proposition 5: The system of equations q, x and m would have saddle-path stability if $\alpha > \frac{1-2\theta}{1-\theta}$.

The dynamics of q, x and m are described by the system of equations (21). We form the Jacobian matrix from the system of equations (26):

$$J = \begin{pmatrix} \frac{\partial \dot{q}}{\partial q} & \frac{\partial \dot{q}}{\partial x} & \frac{\partial \dot{q}}{\partial m} \\ \frac{\partial \dot{x}}{\partial q} & \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial u} \\ \frac{\partial \dot{m}}{\partial q} & \frac{\partial \dot{m}}{\partial x} & \frac{\partial \dot{m}}{\partial m} \end{pmatrix} = \begin{pmatrix} -(1-\alpha)q & (1-\alpha)q & \theta q \\ \left(\frac{\alpha}{\omega} - 1\right)x & x & 0 \\ 0 & -m/_{1-\theta} & -m \end{pmatrix}$$

Now, we need to find the determinant of J in order to decide on the stability of the system. It is equal to $\det J = \alpha qxm[1 - 2\theta - \alpha(1-\theta)]$. The determinant of J would be negative if only if $\alpha > \frac{1-2\theta}{1-\theta}$. In that case, according to the boundary value condition, the system has saddle-path stability. However, in constant return to scale (CRS), the system is saddle-path stable without any condition.

3.3. Renewable energy-based output

Renewable energy source is defined as a resource that can be replenished by natural means or that has an infinity supply. The renewable resource used as input in this analysis is the latter case, i.e. a resource having infinity supply such as solar, wind energy etc. In this research, we assume that the firm invests in renewable energy, and the rate of renewable energy follows the equation below:

$$\dot{E}_r = f(I) - \delta E(t) \tag{27}$$

where $f(I) = I^{\gamma}(t)$; f(I) is the energy production function; I and δ represent the investment allocated in renewable energy power at each period and the loss of energy from the depreciation of power plant, respectively; γ is the investment-elasticity of energy. Referring to the fact that the capacity of renewable energy production generally increases with the amount of investment allocated in it, that is the larger the investment, the higher is the increase of production, f(I) is an increasing function (or at least a constant return) which means that γ is greater than (or at least equal to) one ($\gamma \geq 1$).

Since a part of income is invested in renewable energy, the change of capital stock follows:

$$\dot{K} = Y_t - C_t - I - \delta \mathrm{K}_t \tag{28}$$

3.3.1. Optimal Path

The problem associated with the model is in equation (29). The aim is to find the optimal path by maximizing utility function subject to the change in the resource stock and capital stock.

Max (29)

$$\int_0^{\infty} \frac{C^{1-\omega}}{1-\omega} e^{-\rho t} \partial t;\ 0 < \omega$$

Subject to

$$\dot{K} = Y_t - C_t - I - \delta \mathrm{K}_t$$

$$\dot{E}_r = f(I) - \delta E(t)$$

$R_t \geq 0$, $C_t \geq 0$ and $K_t \geq 0$

K_0 and R_0 are given

The current value Hamiltonian function associated with the problem (29) is:

$$H = \frac{C^{1-\omega}(t) - 1}{1-\omega} + \mu(t)[Y(t) - C(t) - I(t) - \delta \mathrm{K}_t] + \lambda(t)[I^{Y}(t) - \delta E(t)] \tag{30}$$

where λ and μ are shadow price associated with physical capital and renewable energy resource. In the following step, we apply the first order condition by taking the derivative with respect to the consumption and the renewable resource-allocated investment.

First order conditions:

(i) Maximizing (30) with respect to C gives the following condition:

$$C^{-\omega} - \mu = 0 \tag{31}$$

Taking time-derivative of equation (31) logarithmically yields

$$-\omega\frac{\dot{C}}{C}=\frac{\dot{\mu}}{\mu} \tag{32}$$

(ii) Likewise maximizing (30) with respect to I

$$\mu(t)=\gamma\lambda(t)I^{\gamma-1}(t) \tag{33}$$

Taking time-derivative of (33) logarithmically,

$$\frac{\dot{\mu}}{\mu}=\frac{\dot{\lambda}}{\lambda}+(\gamma-1)\frac{\dot{I}}{I} \tag{34}$$

The condition associated with λ is $\dot{\lambda}=-H_E+\rho\lambda$. Then,

$$\dot{\lambda}=-\mu F_E+(\rho+\delta)\lambda \tag{35}$$

By using equation (33) and (35), one obtains:

$$\frac{\dot{\lambda}}{\lambda}=-\gamma F_E I^{\gamma-1}(t)+(\rho+\delta) \tag{36}$$

The condition associated with μ is $\dot{\mu}=-H_k+\rho\mu$. Then,

$$\dot{\mu}(t)=-\mu(t)F_K+(\rho+\delta)\mu(t) \tag{37}$$

Dividing this equation by $\mu(t)$

$$\frac{\dot{\mu}}{\mu}=-F_K+\delta+\rho \tag{38}$$

By substituting $\frac{\dot{\mu}}{\mu}$ in the equation (32) for (38), one obtains:

$$\omega g_c = \alpha \frac{Y}{K} - (\delta + \rho) \tag{39}$$

By using equation (34) and (38),

$$(1-\gamma) g_I = -\alpha \frac{Y}{K} + \gamma \frac{Y}{I} \frac{I^{\gamma}}{E} \tag{40}$$

Equation (40) states the relationship between marginal productivities associated with the renewable energy and physical capital stock along optimal path. For a special case in which $\gamma = 1$, the marginal productivity of capital stock would be equal to that of renewable energy $F_K = F_E$ at optimal path.

3.3.2. Balanced-growth path

Let $h \equiv \frac{Y}{I}$ $q \equiv \frac{Y}{K}$ $z \equiv \frac{q}{h}$ $and\ u \equiv \frac{\dot{E}}{E} = \frac{I^{\gamma}}{E} - \delta$. Along balanced-growth path, the dynamic model follows a trajectory such that all variables grow at a constant rate. Following Groth and Schou (2007), an equilibrium path for a set of $(\mathrm{Y}, \mathrm{K}, \mathrm{I}, \mathrm{R})_0^{\infty}$ is named balanced growth equilibrium.

Proposition 6: For any balanced growth equilibrium, the following characteristics hold.

i. $g_y = g_I = g_c = g_k = g^*$
ii. $u = \frac{\dot{E}}{E} = \gamma g^*$
iii. $(1-\alpha-\theta\gamma) g = \beta n + \varepsilon$
iv. $-\omega g + \alpha q = \delta + \rho$
v. $(1-\gamma) g - \alpha q + \theta\gamma h u = 0$

Proof of proposition 6:

- Equation (i)

The equation (i) stems from the equilibrium characteristics.

- Equation (ii)

Along the balanced growth, it is noted that input grows at constant rate. Then, the change of rate of renewable energy use equals nil,

$$\frac{\partial}{\partial t}\log\left(\frac{\dot{E}}{E}\right) = \frac{\gamma g_I - u}{1 - \frac{\delta}{u}} = 0$$

, which is the equation (ii).

- Equation (iii)

From (i), (ii) and (1), one obtains (iii).

- Equation (iv)

Using equations (39) and (i), we have the equation (iv).

- Equation (v)

For the last equation, it is obtained from the equation (40).

Proposition 7: Along the balanced growth equilibrium, output grows at positive constant rate equal to g*.

Proof of proposition 7:

By using the equations (i) and (ii), one obtains the equation (41).

$$g^* = \frac{\beta n + \varepsilon}{1 - \alpha - \gamma\theta} \tag{41}$$

Proposition 8: Assume a production function with constant return to scale and a positive denominator of g*. Along the balanced-growth equilibrium, the per capita output growth is positive (or at least nil) if there is no technological progress.

Proof of proposition 8:

From equation (41), we have g*. By doing the difference between g* and n, one obtains:

$$g^* - n = \frac{n(\alpha + \beta + \gamma\theta - 1)}{1 - \alpha - \gamma\theta} \tag{41.a}$$

Since $\gamma \geq 1$, one can transform it in such that $\gamma = 1 + a\ (41.b)$. The only condition on "$\gamma\theta$" for this proposition is $0 < \gamma\theta < \beta$ since we assume that the denominator of g* is positive. Replacing γ in equation (41.a) with (41.b) gives:

$$g^* - n = \frac{na}{\beta - \theta a} > 0 \tag{41.c}$$

The positive (or zero) long-run per capita output growth comes from the renewable energy production function which is an increasing (or at least constant) return to scale. By using equations (ii), (iv) and (v), one has the solutions for the ratio of output-capital (q) and output-investment in renewable energy (h) along the balanced growth equilibrium.

$$q^* = \frac{\omega(\beta n + \varepsilon)}{\alpha(1 - \alpha - \gamma\theta)} + \frac{(\delta + \rho)}{\alpha} \tag{42}$$

And

$$h^* = \frac{(\gamma + \omega - 1)}{\theta\gamma^2} + \frac{(\delta + \rho)(1 - \alpha - \gamma\theta)}{\theta\gamma^2(\beta n + \varepsilon)} \tag{43}$$

3.3.3. Transitional dynamics

Recalling that $h \equiv \frac{Y}{I}\ q \equiv \frac{Y}{K}\ x \equiv \frac{C}{K} and\ u \equiv \frac{I^{\gamma}}{E}$, and let q*, x* and m* denote the steady state value associated with the system of q, x and m, respectively. It has saddle-path stability if it has a unique solution converging to the steady-state for a time period towards infinity. By taking the time-derivative logarithmically of equations (q, x, h and u) and knowing that from equation (40), $g_I = \alpha\frac{Y}{K} - \gamma\frac{Y}{I}\frac{I^{\gamma}}{E}$, one obtains the following system of equation:

$$\dot{q} = \left[-(1-\alpha)(q - x - \frac{q}{h} - \delta) + \eta u + \beta n + \varphi \right] q \tag{44}$$

$$\dot{x} = \left[q\left(\frac{\alpha}{\omega} - 1\right) + x + \frac{q}{h} - \delta\left(1 - \frac{1}{\omega}\right) - \frac{\rho}{w} \right] x$$

$$\dot{h}=\left[\alpha\left(\frac{-\gamma}{1-\gamma}q-x-\frac{q}{h}-\delta\right)+\eta u+\beta n+\varphi+\frac{\gamma^2}{1-\gamma}hu\right]$$

$$\dot{u}=\left[-\left(\frac{\gamma^3}{1-\gamma}\right)\eta hu+\frac{\alpha\gamma}{1-\gamma}q-u\right]u$$

Proposition 9: The system q, x, h and u has saddle-path stability.

The dynamics of system q, x, h and u are described by the system (44). We form the Jacobian matrix along the steady state:

$$J=\begin{pmatrix}\frac{\partial\dot{q}}{\partial q} & \frac{\partial\dot{q}}{\partial h} & \frac{\partial\dot{q}}{\partial x} & \frac{\partial\dot{q}}{\partial u}\\ \frac{\partial\dot{h}}{\partial q} & \frac{\partial\dot{h}}{\partial h} & \frac{\partial\dot{h}}{\partial x} & \frac{\partial\dot{h}}{\partial u}\\ \frac{\partial\dot{x}}{\partial q} & \frac{\partial\dot{x}}{\partial h} & \frac{\partial\dot{x}}{\partial x} & \frac{\partial\dot{x}}{\partial u}\\ \frac{\partial\dot{u}}{\partial q} & \frac{\partial\dot{u}}{\partial h} & \frac{\partial\dot{u}}{\partial x} & \frac{\partial\dot{u}}{\partial u}\end{pmatrix}=\begin{pmatrix}-(1-\alpha)\left(1-\frac{1}{h}\right)q & -(1-\alpha)\frac{q^2}{h^2} & (1-\alpha)q & nq\\ \alpha\left(-\frac{\gamma}{1-\gamma}-\frac{1}{h}\right)h & \frac{\alpha q}{h}+\frac{\gamma^2}{1-\gamma}hu & -\alpha h & \left(\frac{\gamma^2}{1-\gamma}h+\eta\right)h\\ \left(\frac{\alpha}{\omega}-1+\frac{1}{h}\right)x & -\frac{q}{h^2}x & x & 0\\ \frac{\alpha\gamma}{1-\gamma}u & -\frac{\gamma^3}{1-\gamma}\eta u^2 & 0 & -\left(1+\frac{\gamma^3}{1-\gamma}\eta h\right)u\end{pmatrix}$$

$$\det J=-\frac{\alpha(\gamma u)^2}{(1-\gamma)^2}qhx[(1-\alpha)(\gamma-1)+\gamma\eta(1-\gamma\eta)+\eta\gamma\omega(1-\eta)]$$

The determinant of J is negative. Following the condition of boundary value, the system has saddle-path stability.

4. Growth comparison

There are two kinds of output in this analysis, and one is based on renewable energy whereas the other one on non-renewable energy. We have subtracted Hamiltonian function and computed the first order difference for each output, and we obtained long-run output growth for each.

Proposition 10: Assume that the denominator of g_r and g_{nr} are positive, renewable energy-based output grows faster than non-renewable energy-based one along the balanced growth equilibrium (equation (21) and (46)).

Proof of proposition 10:

$$v=g_r^*-g_{nr}^*=\frac{\beta n+\varepsilon}{1-\alpha-\gamma\theta}-\frac{\beta n+\varepsilon-\theta(\rho-\omega n)}{1-\alpha-\theta(1-\omega)}$$

And then, $v = \frac{(\beta n+\varepsilon)[(\theta(\gamma-1)+\theta\omega)]+\theta(\rho-\omega n)}{(1-\alpha-\gamma\theta)(1-\alpha-\theta(1-\omega))}$. Since $\gamma \geq 1$ and $\rho - \omega n$ is positive (according to the equation (17)), $v > 0$.

Proposition 11: Without technological growth and at constant return to scale, per capita consumption associated with non-renewable energy-based output grows at negative rate whereas that of renewable one at positive rate.

Proof of proposition 11: Proposition (3) and (6).

Proposition 12: The growth difference between both outputs would depend on the exogenous rate of increase in non-renewable energy use (φ) if we let a positive growth of non-renewable energy use.

Proof of proposition 12:

Normally, along the balanced growth path, the growth rate of non-renewable energy use should be negative so that it can last for infinite period. But in this proposition, we assume it to be positive for the sake of comparison. The energy use is assumed to follow exogenous pattern: $E_t = \varphi E_{t-1}$ and then, $g_E^* = \varphi - 1$ (45) where $\varphi > 1$ since we have assumed $g_E^* > 0$. Using equation (8) and (45),

$$g_{nr}^* = \frac{\beta n + \varepsilon + \theta(\varphi - 1)}{1 - \alpha}$$

With this new value of g_{nr}, we calculate $v = g_r^* - g_{nr}^*$ where g_r^* is the equation (41). After computing this difference,

$$v = \frac{(\beta n + \varepsilon)\theta\gamma - \theta(1 - \alpha - \gamma\theta)(\varphi - 1)}{(1 - \alpha - \gamma\theta)(1 - \alpha)}$$

v is greater than zero if $\varphi < 1 + \frac{(\beta n+\varepsilon)\theta\gamma}{\theta(1-\alpha-\gamma\theta)}$ and negative otherwise. Here, γ and φ are the investment-elasticity of renewable energy and rate of increase in non-renewable resource use.

Proposition 13: Renewable energy-based output-capital ratio is higher than that based on non-renewable energy.

Proof of proposition 13:

Let q_1^* and q_2^* denote the output-capital associated with renewable and non-renewable energy, respectively.

$$q_1^* = \frac{\omega(\beta n + \varepsilon)}{\alpha(1 - \alpha - \theta)} + \frac{\rho + \delta - \omega n}{\alpha}$$

and

$$q_2^* = \frac{\omega(\beta n + \varepsilon)}{\alpha(1 - \alpha - \gamma\theta)} + \frac{\delta + \rho}{\alpha}$$

Obviously, $q_1^* > q_2^*$ since $\gamma \geq 1$ and $\omega n > 0$.

5. CONCLUSION

After setting the current value Hamiltonian function, we derived FOC conditions and subtracted relationship between input and output. The system associated with the output based on non-renewable energy would be saddle-path stable if the share of capital is sufficiently high, whereas the one associated with renewable energy has saddle-path stability without any condition.

Along balanced-growth equilibrium, non-renewable energy-based per capita output growth depends on technological progress, discount rate and the share of non-renewable energy source. It grows at negative rate if there is no technological progress since it is forced to follow a path associated with negative growth of non-renewable energy use. However, the per capita output growth that is associated with renewable energy is positive (or at least nil) along the balanced-growth equilibrium. We have also proved that renewable energy-based output grows faster than non-renewable energy-based one along the balanced growth equilibrium. Among the balanced-growth characteristics, output-capital ratio is constant. It turns out that renewable energy-based output-capital ratio is higher than that based on non-renewable energy. This is accounted for by the contribution of renewable energy use in the production since it grows at almost same rate as capital stock, whereas the contribution of non-renewable energy in output decreases by time-period since it experiences negative growth energy use.

The poor growth performance of non-renewable energy-based output comes from the negative rate of energy use over long run period since it is a finite resource, while renewable energy grows at positive rate. Furthermore, we have supposed a case where the rate of non-renewable energy use is positive along the balanced-growth path. The result has revealed that the growth difference between both outputs depends on the rate of increase in non-renewable energy use, and this rate must be higher enough to make non-renewable energy-based output equal or outpace the one based on renewable energy because non-renewable energy is non-accumulable production factor. Finally, the negative rate of non-renewable energy use along balanced-growth path and being non-accumulable input are the main reasons that lower non-renewable energy-based output growth comparatively to that of renewable energy-based one.

The result from this analysis should inspire policy maker to give prominence in renewable energy source-oriented policy since it is not only a great advantage for addressing the environmental issue but also play an important role in sustaining positive economic growth. It is an important tool to mitigate green gas emission in the world, which is on the edge of imminent climate change.

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