Abatement capital accumulation and the shadow price of carbon

Climate change is calling for an immediate public action in order to provide incentives for carbon abatement. Prior to the design the instruments to implement, the computation of the shadow price of carbon is a measure of the total effort to make whether through taxes, pollution permits or other regulations. Abatement is mainly implemented through abatement capital accumulation. It implies that abatement takes times and this should be accounted for in the computation of the shadow price of carbon. In addition, due to uncertainties surrounding damages from climate change (see Pindyck, 2017), the cost-benefit approach to carbon pricing tend to be replaced a with cost-efficiency approach, however shifting the uncertainty problem to the concentration target. Using a theoretical model calibrated for France, we study the consequences for the shadow price of carbon to account for abatement capital accumulation and target uncertainty.

The theoretical reference used is a stylized model of extraction and use of fossil energies to determine the shadow price of carbon in the case where society imposes a constraint of carbon concentration in the atmosphere (such as Chakravorty, Magne and Moreaux, 2006, or Chakravorty, Moreaux and Tidball, 2008).[1] This amounts to a carbon budget approach, which corresponds to the classic problem of optimal management of an exhaustible resource, the reserves initially available here corresponding to the carbon budget.[2] The problem of optimal use of an exhaustible resource has been solved by Hotelling (1931) which highlights the existence of an inter-temporal arbitrage: the decision-maker must be indifferent, in the sense that the social utility is the same, between withdrawing an additional unit from the resource stock (ie emitting an additional unit of CO2) today or doing it tomorrow. For this to happen, the price associated with this asset must grow over time in line with the discount rate. The price of the resource thus incorporates a scarcity rent, reflecting the exhaustible nature of the resource and thus exceeding the marginal cost of extraction even in the absence of monopolistic competition. Once this carbon budget has been exhausted, the net emissions must remain zero so that the budget continues to be respected.

In addition, valuations of carbon values have made an extensive use of large-scale empirical models. Three broad categories of models are used for carbon valuations: integrated valuation models (IAMs) that are generally part of a cost-benefit approach, techno-economic models, and macroeconomic models. The simulations of these empirical models, whatever their nature (technical-economic or macroeconomic models), reveal (i) the importance of investments in the implementation of the transition to a low-carbon economy and (ii) the existence of costs. marginally depressed as we approach carbon neutrality. If the abatement is mainly achieved through new investments, achieving zero emissions on a given horizon or meeting a carbon

[1] Schubert (2008) also proposes a theoretical model in which the damage due to carbon accumulation is directly taken into account, which corresponds to a cost / benefit approach.

[2] We neglect the natural absorption of CO2, which is a good approximation since the natural absorption rate is very low compared to the discount rate.

budget can be reduced to installing a sufficient abatement capacity. The problem then becomes that of the optimal accumulation of a costly abatement capital. On the one hand, we integrate adjustment costs (see Lucas, 1967, or Gould, 1968 for adjustment costs in the theory of investment) that encourage the spread of the abatement effort over time: it is expensive to make the investment quickly and this encourages the investor not to make all the efforts at the date on which the emissions must become zero. This is a simple way to incorporate recent considerations of marginal abatement costs (Vogt Schilb et al, 2018), that early exercise of emission reduction options may be justified by the fact that investments cannot be deployed immediately and should therefore be anticipated. It is also consistent with studies based on IAMs models that stress the limited ability of economies to switch overnight to low-carbon technologies too modest, subsequent efforts will need to be much stronger (Iyer et al., 2014; Riahi et al., 2015; IPCC, 2014).

We also take into account the observation that the more the economy is advanced in the abatement, the more costly the new investments to be made to reduce emissions, which, on the contrary, encourages to delay investments. Finally, we distinguish two types of abatement technology. The first is similar to a clean-up technology: emissions result from the GDP (increasing in time) to which a constant emission factor is attached and the investment in abatement capital reduces these emissions. For the second, the investment in abatement makes it possible to achieve decoupling, that is, to reduce the emission factor or the energy intensity of the production.

We successively solve two models that are distinguished by the abatement technology chosen, considering that in reality as in the simulation models of this report, the abatement technology is mixed, in the sense that it includes both depollution and reduction of energy intensity. The results obtained by the two models thus provide corridors within which optimal paths must be found in the case of a mixed technology. Dynamic model resolution provides optimal paths for carbon value, marginal value of abatement capacity, investment and emissions. Models are then calibrated using the results of the large-scale empirical models that the Quinet (2019) have run to determine a path for the value of carbon in France. We obtain results for France regarding the shape of the optimal emissions path, the optimal date for carbon neutrality and an illustration of the path of the shadow value of carbon and the non-monotonic behavior of the marginal value of the abatement capital. Finally, the model is extended to account for uncertainty on the concentration target, that reflect the difficulty to estimate the marginal damages generated by greenhouse gases. The model then requires numerical resolution and allows appraising the effect of uncertainty on the shadow price of carbon, the emission path or the abatement capital accumulation.

In the rest of the paper, we present the assumptions of the model in the first section, followed by the resolutions under the alternative assumptions of abatement and reduction in carbon intensity in section 2. The model is then calibrated in the third section in order to compute for France the optimal paths for emissions, investment, carbon value and the value of capital abatement. Section 4 presents the model under uncertainty. Section 5 concludes and proposes extensions.

1. Optimal investment in abatement capital

We first considered that the GHG emissions flow measured in tons of CO2 equivalent (tCO2e) in t is a function of the GDP in t, Y_t, and the installed abatement capacity, A_t, which corresponds to the investments by economic agents (households, firms or institutional actors) in order to reduce their emissions (for example, the purchase of an electric vehicle or heat pump, the adoption of carbon-free insulation of buildings):

$$E_t = f(Y_t, A_t), \qquad (1)$$

The emission function f can take at least two different forms, depending on the type of abatement capacity considered (the reality is probably between the two):

- First, we can consider A_t as a technology that affects the polluting intensity E / Y, which will therefore make it possible to decouple GHG emissions from GDP when it is deployed. If the abatement capacities were only of this type, GDP growth would not require to increase the abatement capacities to offset the new GHG emissions induced by the growth of production. To take the example of the investments required for a change in working methods: if teleworking were to be adopted massively, the costs of this adaptation would be borne once (investments in new production organizations, in large infrastructures, in new forms of urbanization for example) and should not (or a little) be increased with GDP growth. Technology of this type would thus contribute to the decoupling of GHG production and GHG emissions. In order to describe the operation of such a technology, the emission function can be written as follows:

$$E_t = Y_t(\bar{A} - A_t), \qquad (2)$$

  Emissions are here a function of GDP and a coupling coefficient of emissions to output, $(\bar{A} - A_t)$), which decreases as the stock of abatement capital installed at t per unit of GDP, A_t , is close to target$\bar{A}$, also in tCO2e per unit of GDP. Thus, the more one has invested in the abatement capital, the lower the greenhouse gas (GHG) emissions for a given level of GDP. The investment thus makes it possible to decouple the emission level from the GDP.

  Nevertheless, it is unrealistic to consider all abatement technologies are of such a nature. Indeed, if one wants to reach zero emissions, one can think that the increase of the production will require to increase in parallel the capacities of abatement. There are therefore also means of reducing emissions such as "pollution abatement technology", which means considering a second functional form for the emission function.

- Production growth is very likely to lead agents to continually increase their investment in abatement technologies: in this case, the abatement rather looks like depollution. For example, if all the thermal vehicles were to be replaced by electric or hydrogen vehicles in year t and production increased between year t and year t + 1, it would probably be necessary to invest again in electric or hydrogen vehicles in order to maintain the decoupling between GDP and emissions achieved by the investment in t. Indeed, the increase in production would certainly require more goods transport means: it would therefore be necessary to invest again in low-carbon vehicles to avoid the return of thermal vehicles that would increase again the coupling coefficient between emissions and GDP. If all the technologies are of this type, the GHG emission flow measured in tons of CO2 equivalent (tCO2e) in t is then equal to:

$$E_t = Y_t\bar{A} - A_t, \qquad (3)$$

In this specification of the emissions function, an increase in GDP leads to an increase in the emission reduction target to be reached, since $\bar{A}$ corresponds to the pollution coefficient. Note also that A_t still represents the abatement capacity but is no longer expressed per unit of GDP. [3]

Finally, we note that in our model growth is not endogenous. GDP can therefore be rewritten:

$$Y_t = Y_0 e^{gt},$$

where the growth rate, g, is exogenous. However, it is likely that investment in abatement capacities leads economic agents to reduce their investments in other types of capital which could have a negative effect on economic growth, that is not taken into account in the model.[4]

In order to reduce GHG emissions, it is then necessary to invest in abatement capacities. A_t accumulates according to the following dynamics:

$$\dot{A_t} = a_t - \delta A_t, \qquad (4)$$

where a_t is the investment in abatement in t and δ is the depreciation rate of the abatement capital. The cost of this gross abatement of emissions is defined by:

$$c(a, A) = \frac{\alpha}{2} a_t^2 + \beta A_t, \qquad (5)$$

avec $\beta,\alpha>0$.

The convexity with respect to a_t accounts for the adjustment costs which incite to spread the abatement effort over time. This is a simple way to incorporate recent considerations of marginal abatement costs (Vogt Schilb et al, 2018), that early exercise of emission reduction options may be justified by the fact that investments cannot be not be deployed immediately[5].

The presence of A_t in the cost function (5) makes efforts increasingly expensive as the abatement goal is approached, illustrating the fact that agents make first the less expensive investments. The chosen specification takes into account, via a fixed cost dependent on A_t (but not a_t) the fact that approaching the objective makes the abatement more expensive, while smoothing an effect which in reality is probably discontinuous (adoption of electric vehicles or renewable energy sources). However, a disadvantage of this specification is that it does not associate a zero cost with a zero investment, so it is necessary to ensure that the investment a_t is strictly positive to ensure the relevance of this function.
The agent's program consists in minimizing the sum of the discounted costs under the constraint of a stock $\bar{S}$ (the carbon budget). The date T at which this stock is exhausted is then endogenous.

---

[3] It may be noted that At is not expressed in the same units according to the abatement technology chosen, but, without being significant, we will use the same notation in the rest of the presentation of the model in order to simplify the exposure.

[4] Making g determined by the model and not taken as a given would test this intuition and could be an extension of the model proposed here.

[5] For example, a policy of thermal retrofitting of a large number of buildings over a very short period of time would probably face a lack of manpower that could lead to a significant increase in the costs.

2. Optimal accumulation of abatement capital

We consider a stock of GHG, S_t, which increases with the emissions of each period and thus accumulates according to the following dynamics:[6]

$$\dot{S}_t = E_t$$

under the constraint: $S < \bar{S}$.
The accumulation of At and the abatement investment cost are still given equations (4) et (5) respectively. The social planner program is then:

$$\min_{a_{t,}} \int_0^{+\infty} e^{-\rho t} c(a_t, A_t)\, dt$$
$$\dot{A}_t = a_t - \delta A_t,$$
$$\dot{S}_t = E_t$$
$$S_t \leq \bar{S},\ \ {}_0e^{gt}\bar{A} \geq A_t$$
$$A_0, S_0, \bar{A}\ \ et\ \bar{S}\ donnés$$

where ρ is the discount rate. We note λ_t the shadow price of the abatement capital stock, which is the co-state variable associated with the accumulation constraint on At, and μ_t is the shadow price of carbon. We also define $\omega \geq 0$, the multiplier associated to the constraint on the GHG stock, and $\nu \geq 0$, the multiplier associated to the constraint on the abatement capital stock.

2.1. Abatement technology

In case og an abatement technology (voir équation (3)), the dynamic Lagrangian may be written:

$$L_t = -\frac{\alpha}{2} a_t^2 - \beta A_t + \lambda_t(a_t - \delta A_t) - \mu_t(Y_0 e^{gt}\bar{A} - A_t) + \omega(\bar{S} - S) + \nu(Y_0 e^{gt}\bar{A} - A_t)$$

Necessary optimality conditions when the target is not already reached ($\omega = \nu = 0$) are:

$$\frac{\partial c(a, A)}{\partial a} = \alpha a = \lambda,$$
$$\frac{\dot{\lambda}}{\lambda} = \rho + \delta + \frac{\beta}{\lambda} - \frac{\mu}{\lambda}$$
$$\frac{\dot{\mu}}{\mu} = \rho$$

The first equation shows that the marginal cost of the abatement investment is equal to its implicit price.
The second equation provides the rate of growth of the implicit price of the abatement capacity. It corresponds to the user cost, to which we deduct μ / λ. In fact, the user cost of capital includes:
- The rate of preference for the present ρ
- The depreciation rate δ of the abatement capital stock since it is necessary to renew the abatement capacity.

[6] Emissions are net of the sinks and we consider that proportional assimilation to GHG stock is low enough to be ignored.
.

- An element that reflects the change in the price of the investment, since the company has to invest in increasingly expensive technologies as emissions are reduced and residual emissions become more and more difficult to eliminate. This effect comes from the presence of the parameter β in the cost function of equation (5).

The ratio μ/λ is excluded from this usage cost. It can be interpreted as the relative social value of the abatement and GHG stocks (rent associated with the valuation of avoided emissions) and which indicates the contribution of the abatement stock to the reduction the stock of GHGs

We obtain the following optimal paths (see appendix 1):

$$\mu_t = \mu_0 e^{\rho t}$$

$$\lambda_t = \bar{x} e^{(\rho+\delta)t} + \frac{\mu_0}{\delta} e^{\rho t} - \frac{\beta}{\rho + \delta}$$

$$S_t = \frac{Y_0}{g}(e^{gt} - 1)\bar{A} + \frac{\bar{z}}{\delta}\left(e^{-\delta t} - 1\right) - \frac{\bar{x}}{\alpha(\rho + 2\delta)(\rho+\delta)}\left(e^{(\rho+\delta)t} - 1\right) - \frac{\mu_0}{\delta\alpha(\delta+\rho)\rho}(e^{\rho t} - 1) + \frac{\beta t}{\alpha\delta(\rho+\delta)} + S_0$$

where μ_0 , $\bar{z}$ and $\bar{x}$ are constants that can be identified thanks to the conditions on A_0, A_T, and the continuity of λ_t at T.

The complete resolution of this model is carried out in Appendix 1. In particular, the constant $\bar{x}$ is negative, which opens the way to a non-monotonic dynamic of λ_t and therefore of the investment a_t (and it will be the case in the numerical illustrations, see next section). In addition, we observe that the size of the carbon budget (*ie.* the value of $\bar{S}$) has no effect on the growth rate of μ_t and λ_t, but on the starting point of these values.

After T, dynamics of the two co-state variables become (voir appendix):

$$\lambda_t = \alpha Y_T e^{g(t-T)} \bar{A}(g + \delta)$$

$$\mu_t - \nu_t = \beta + (\rho + \delta - g)\alpha Y_T e^{g(t-T)} \bar{A}(g + \delta)$$

where ν_t is the Lagrange multiplier associated with the constraint $A_t \leq Y_0 e^{gt} \bar{A}$ (after T, it is necessary to have $A_t = Y_0 e^{gt} \bar{A}$, therefore $\nu_t \geq 0$ for emissions to be zero. This means that once the carbon budget has been exhausted and the carbon neutrality achieved thanks to the abatement capital installed, the shadow price of the abatement capital must grow at the same rate as the production to offset the new emissions induced by the economic growth. This result comes from the assumption made in this section that the technologies adopted are of the "abatement" type and thus do not allow the decoupling of production and emissions.

2.2. Technology affecting carbon intensity

In the case of a technology that affects the carbon intensity, that is, that allows a decoupling between the GDP and the emissions, the dynamic Lagrangian associated with the problem can be written:

$$L_t = -\frac{\alpha}{2}a_t^2 - \beta A_t + \lambda_t(a_t - \delta A_t) - \mu_t Y_0 e^{gt}(\bar{A} - A_t) + \omega(\bar{S} - S) + \nu Y_0 e^{gt}(\bar{A} - A_t)$$

The necessary optimality conditions before the target is reached ($\omega = \nu = 0$) are:

$$\frac{\partial c(a, A)}{\partial a} = \alpha a = \lambda,$$
$$\frac{\dot{\lambda}}{\lambda} = \rho + \delta + \frac{\beta}{\lambda} - Y_0 e^{gt}\frac{\mu}{\lambda}$$
$$\frac{\dot{\mu}}{\mu} = \rho$$

As in the case of a clean-up technology, we find (see the second equation) that the rate of growth of the shadow price of the abatement capacity corresponds to the user cost, minus μ / λ. The only significant change is in the way μ / λ (the contribution of the abatement stock to the reduction of the GHG stock) is accounted for, as it appears with a weight $Y_0 e^{gt}$ which increases over time.

Again, μ_t follows a Hotelling rule and is close to the generally accepted design of the carbon value, while λ_t is now growing at a rate that depends on the growth rate of the economy, which was not the case previously. The complete resolution of this model is carried out in Appendix 2. We find again that the dynamics for λ_t and investment may be non-monotonic.

After the saturation of the carbon budget and the carbon neutrality at the endogenous date T, the dynamics of the two co-state variables become (see appendix 2):

$$\lambda_t = \alpha\delta\bar{A} = \bar{\lambda}$$

Investment $a_t = \delta\bar{A}$ is constant as well and

$$\mu_t - \nu_t = e^{-g(t-T)}[\beta + (\rho + \delta)\alpha\delta\bar{A}]/Y_T$$

which decreases in time. Thus, in the case where the technology is such that the carbon intensity of the GDP is gradually reduced, it is no longer necessary to increase the shadow price of the abatement capital once a sufficient amount of investment has been made to achieve carbon neutrality. Indeed, at time T, emissions have been fully decoupled from GDP and the production of an additional euro of wealth no longer generates any emissions.

3. Numerical illustration

In this section, the theoretical models with abatement technology on the one hand and with a technology that reduces carbon intensity on the other hand are calibrated using the simulation models results of the Quinet (2019) report. The sensitivity to the values chosen for the parameters is then tested by varying the latter sufficiently to cover the likely ranges. This makes it possible to compare the sensitivity of the different variables of interest (optimum date for carbon neutrality, investment paths, emissions, carbon value) to the various parameters.

The discount rate is chosen equal to 4.5%, as it is the value used to evaluate long term public policies in France (see Bureau and Gollier, 2009). The depreciation rate, set at 4.35%, is obtained by weighting the depreciation rates of the various sectors proposed in Vogt-Schilb et al. (2018). The GDP growth rate is calibrated at 1.6%, consistent with the results of the simulation models used in Quinet (2019). The GDP of year 0 is the one observed in 2015 by INSEE, ie € 2173.69 billion. The 2015 abatement stock is assumed to be zero and $\bar{A}$ is computed as the ratio between the 2015 emissions (458MtCO2) and the GDP of the same year.
The abatement cost function is calibrated from the TIMES model results in the Quinet (2019) report in the case of a 95 MtCO2 carbon sink. In order to be consistent with the logic of the TIMES model, we determine in Appendix 3, according to the parameters of the cost function, the expression of the carbon price path μ_t which allows the producer to choose the path imposed in TIMES. This carbon price path must therefore correspond to that obtained by the TIMES model and the parameters α and β are identified by equalizing our theoretical μ with the carbon value obtained by TIMES in 2030 and in 2040 for a spontaneous growth rate of zero emissions.[7] Note also that α and β differ depending on whether an abatement technology or a technology that reduces the carbon intensity is considered. Finally, in order to allow a comparison with the results obtained when a linear emission trajectory is imposed, the carbon budget from 2015 is chosen equal to that obtained in the case of a linear reduction of emissions from 2015 onwards. to arrive at zero in 2050.[8] The table below summarizes the values of the various parameters.

ρ	δ	g	Y_0	$\bar{A}$	Abatement tech.		Red. Carbon intensity		$\bar{S}$
					α	β	α	β	
4,5%	4,35%	1,6%	2173,69 Mds€	$2{,}08.10^{-4}$ $tCO_2e/€$	$1{,}05.10^{-4}$	180,67	$5{,}1.10^{20}$	$3{,}98.10^{14}$	$8.10^{9} tCO_2e$

3.2. Model with abatement

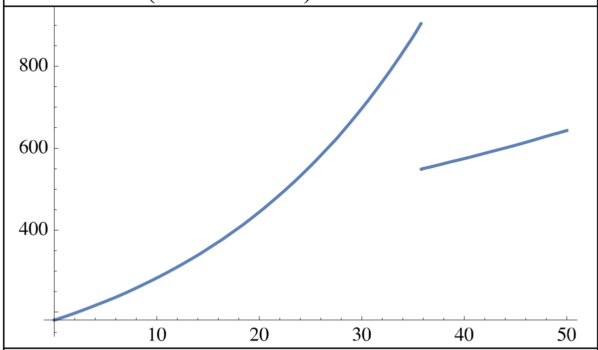
With this calibration for the parameters, in a framework of optimization of the consumption of the carbon budget, emissions become null after 35,8 years, which brings the optimal date for the carbon neutrality to almost 2051, therefore very close to 2050 which is the one fixed by the French political objective. In addition, the temporal profiles of the shadow price of carbon,

7 This is an assumption made in TIMES.

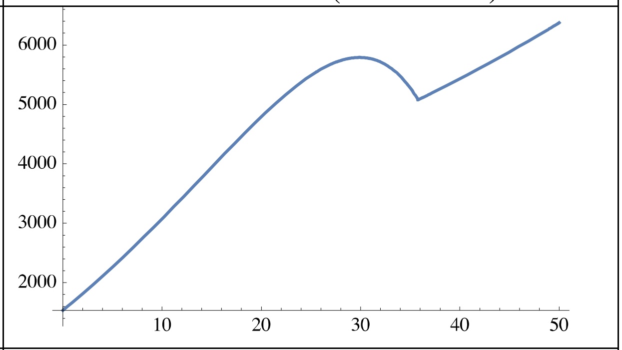
8 It should be noted that this carbon budget is not very far from a carbon budget of 7092MtCO2 calculated as follows: France represents about 0.9% of global emissions, which leads to a carbon budget of 9000MtCO2 in 2011, subtracted emissions between 2011 and 2015 to obtain the carbon budget in 2015.

the shadow price of abatement capacity, abatement capacity and emissions are presented in the graphs below. We note in particular that the emissions path exhibit a shape first slightly concave and then convex, that is not too far from a linear relationship. In addition, the shadow price of the abatement capacity is non-monotonic (which implies that investments are non=monotonic as well since $a_t = \lambda_t / \alpha$). The rationale is the following. Like any investment in capacity, it has a tendency to decrease during the accumulation. However, the abatement capacity has the additional characteristics that it depends positively on the carbon price, which increases over time. It is observed that this last effect prevails in the first periods, while the first prevails after 2045. Finally, the non-continuity of the shadow price of carbon net of the Lagrange multiplier ν (t), needs to be noticed,. This is because the carbon stock variable is constrained to a particular level at date T when the regime changes. On the other hand, μ (t) - ν (t) continues to grow after 2051, but at a slower pace.

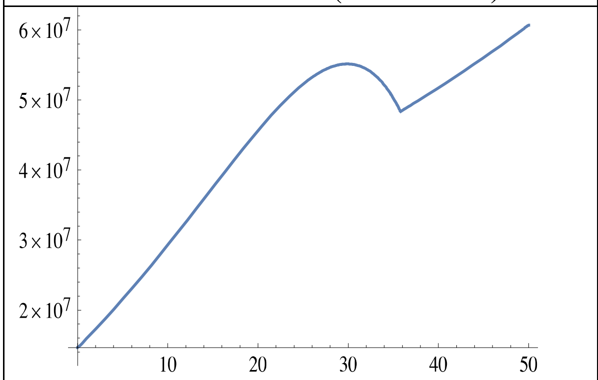
Path of μ(t)-ν(t) in €/tCO_2e between 2015 and 2065 (t=0 in 2015)



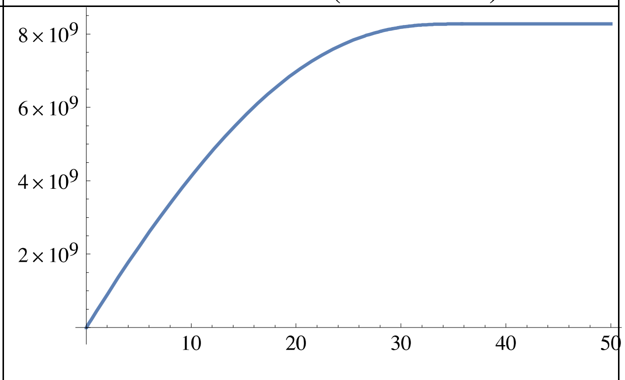
Path of λ(t) in €/tCO_2e between 2015 and 2065 (t=0 in 2015)



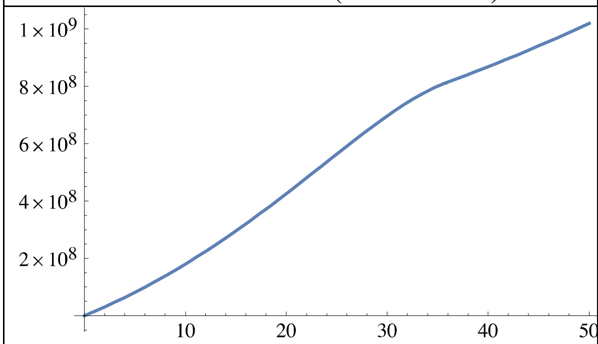
Path of a(t) in tCO_2e between 2015 and 2065 (t=0 in 2015)



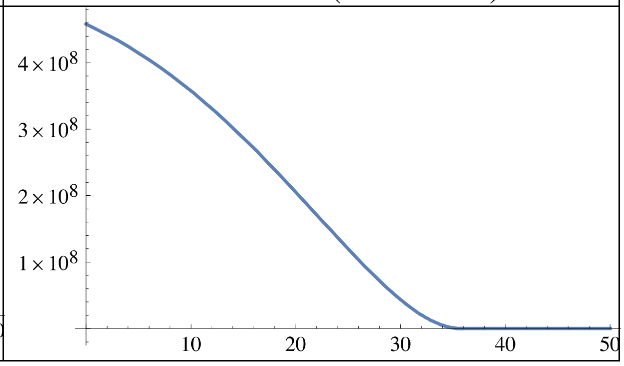
Path of S(t) in tCO_2e between 2015 and 2065 (t=0 in 2015)



Path of A(t) in tCO_2e between 2015 and 2065 (t=0 in 2015)

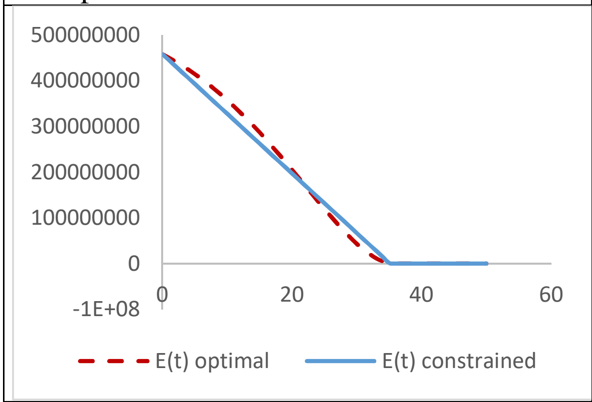


Path of E(t) in tCO_2e between 2015 and 2065 (t=0 in 2015)

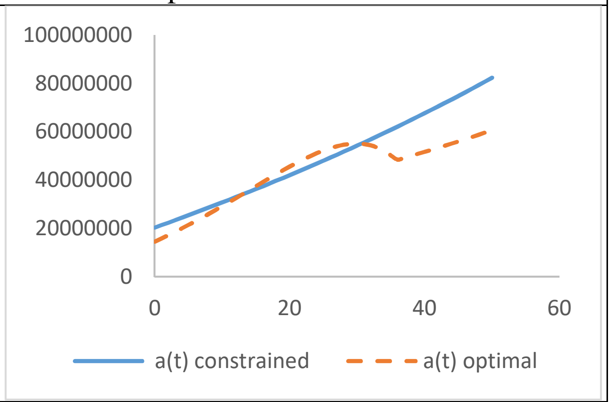


The graphs below compare the investment and the value of μ (t) -ν (t) (remember that up to T, ν(t) = 0) depending on whether the trajectory is optimal or imposed and linear, for the same carbon budget. In particular, we note the close proximity of the emissions paths

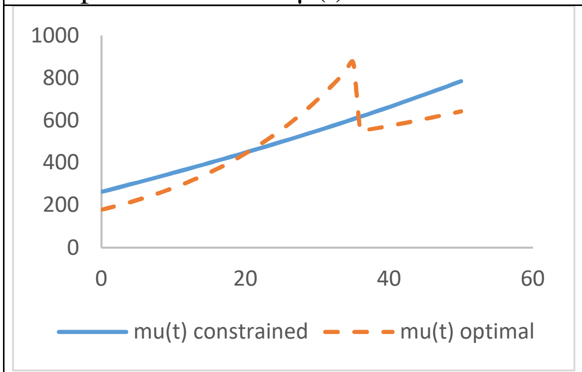
Comparison between emissions



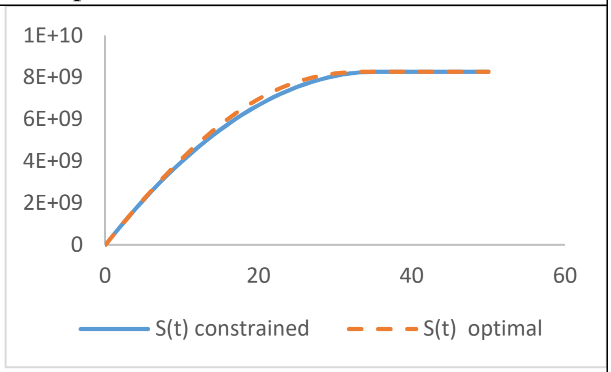
Comparison between investments



Comparison between μ(t)



Comparison between S(t)



We then conduct a comparative dynamics exercise to test the sensitivity of the results to the values chosen for calibration. The figures in Appendix 4 show that:

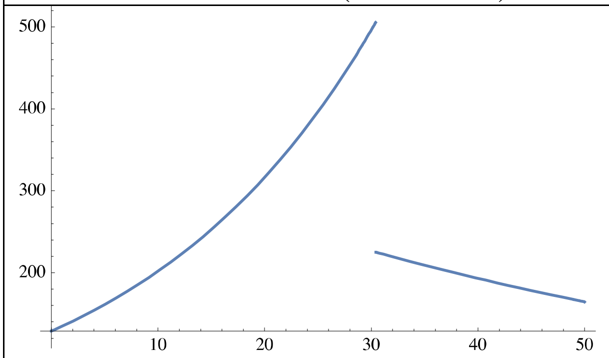
- The optimal date of carbon neutrality is rather insensitive to parameters, with the exception of the discount rate (for ρ between 2.5% and 6%, it varies between 2048 and 2057, which is still relatively close to 2050) and the carbon budget constraint (if the latter varies by more or less 10%, the optimal date goes from 2048 to 2053).
- The emission path is unchanged for the ranges chosen for g and the depreciation rate. It is more sensitive to changes in the carbon budget (at each date, it is higher for a higher carbon budget) or the discount rate (trajectories intersect, with higher initial emissions when the discount rate is higher).
- The investment (as well as λ (t)) reacts significantly to the rate of growth of the economy (we recall that we consider here an abatement technology), as well as the rate of depreciation. A growth rate equal to 2% rather than 1.6% leads to investments 22.5% higher in 2050, while if the depreciation is 6.7% (as in the transport sector, according to Vogt Schilb et al. , 2018) rather than 4.3%, investment is 40% higher in 2050. The effects are less impressive when the carbon budget or the discount rate is changed.
- As the growth rate of the carbon value is equal to the discount rate, the latter parameter directly influences the trajectory of μ (t). It can also be noted that, in level, μ (t) is more affected by the modifications of the depreciation rate than changes in g.

3.2. Model with a technology reducing the carbon intensity

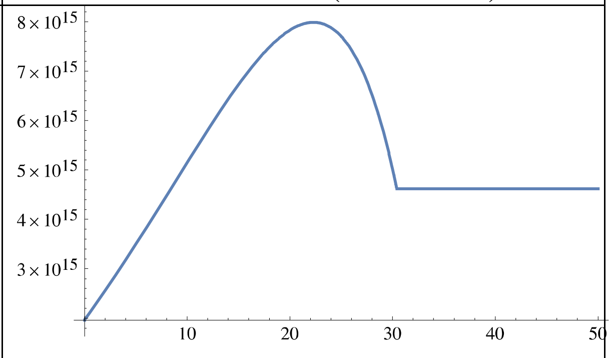
With the proposed calibration for the parameters, emissions become zero after 30.3 years, *ie.* an optimal carbon neutrality date around 2045, which again is relatively close to the horizon set by the French political objective. The paths of the shadow price of carbon, the shadow price of the abatement capacity, the abatement capacity and the emissions are presented in the graphs below. We observe the same characteristics as the ones observed in the case of an abatement technology, in particular:

- An emissions path almost linear despite a shape with an inflexion point.
- The non-monotonicity of the shadow price of the abatement capacity and the investment. They both exhibit a bell curve; however, the maximum is reached earlier than in the case of an abatement technology (2035 vs. 2045).

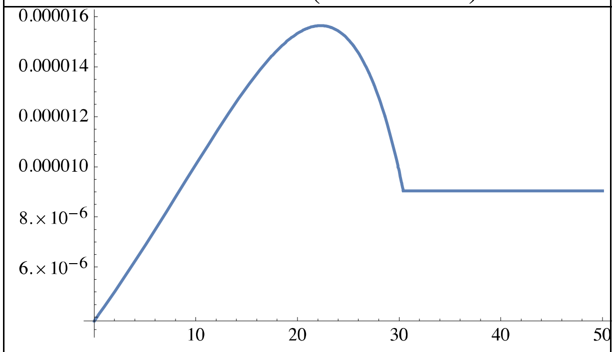
Path of μ(t)-ν(t) in €/tCO_2e
between 2015 and 2065 (t=0 in 2015)



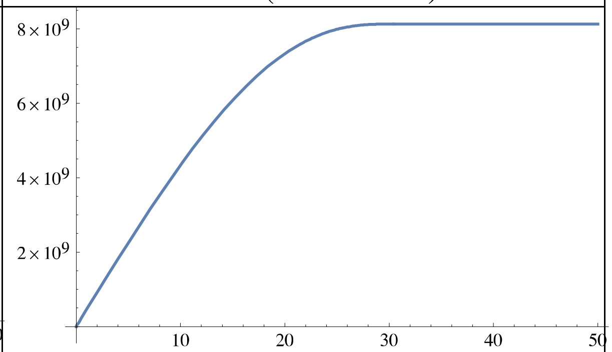
Path of λ(t) in €^{2}/tCO_2e
between 2015 and 2065 (t=0 in 2015)



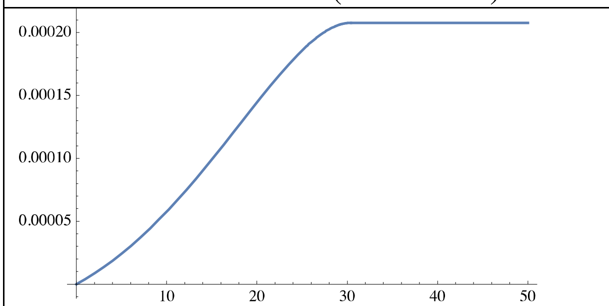
Path of a(t) in tCO_2e/€
between 2015 in 2065 (t=0 in 2015)



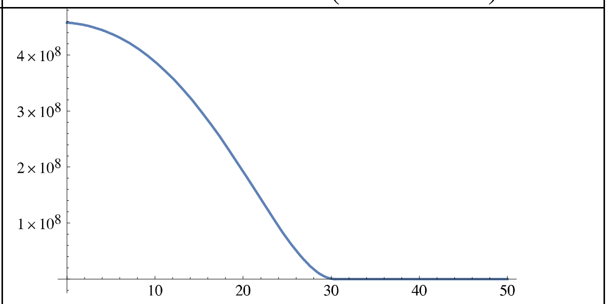
Path of S(t) en tCO_2e
entre 2015 et 2065 (t=0 en 2015)



Path of A(t) in tCO_2e/€
between 2015 and 2065 (t=0 in 2015)

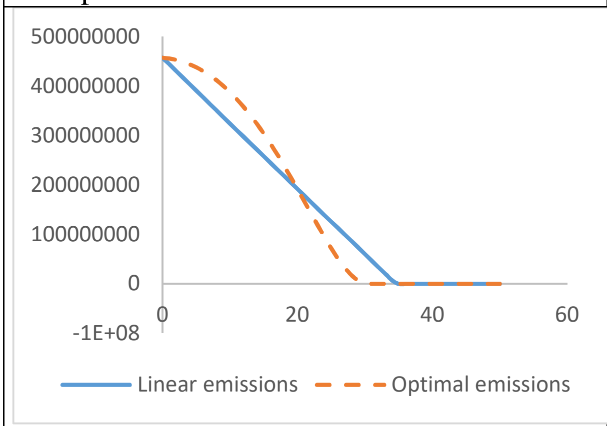


Path of E(t) in tCO_2e
between 2015 and 2065 (t=0 in 2015)

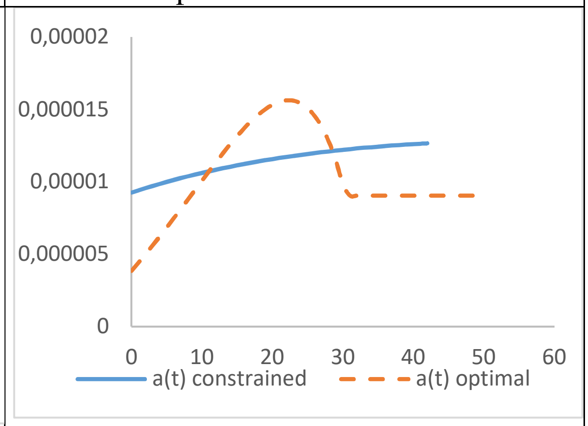


The paths obtained with the optimal emissions path are compared again with those obtained with a linear emission reduction (see graphs below). Although the optimal emissions still exhibit an inflexion point, the proximity of the emission trajectories is less obvious than in the case of an abatement technology. One can also notice the negative slope of the value of the carbon in the case where the emissions path is imposed. It should be remembered, however, that in the real world, there exists a mix of abatement technologies and technologies that reduce carbon intensity. This suggests intermediate results between those obtained in the models. with each type of technology

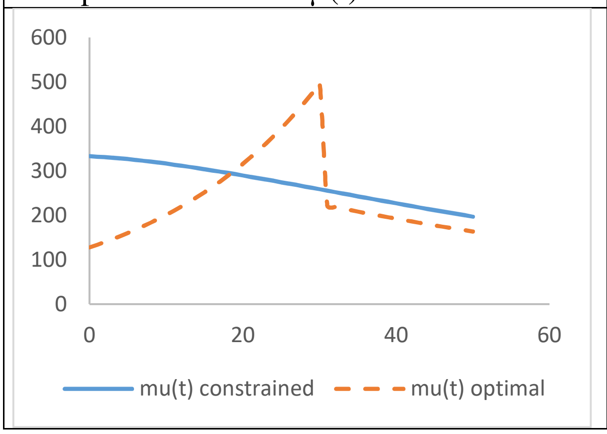
Comparison between emissions



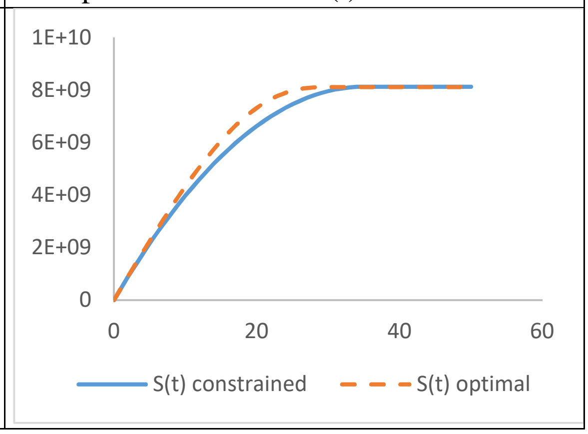
Comparison between investments



Comparisons between μ(t)



Comparisons between S(t)



A comparative dynamics exercise similar to the one carried out for the abatement technology is conducted to test the sensitivity of the results to the values chosen for the calibration. In particular, the same value ranges for the parameters are considered. The figures in Appendix 5 show again that the optimum date for carbon neutrality varies little, only between 2045 and 2048 (for the ranges envisaged for the carbon budget or the discount rate) and that the emissions path is slightly sensitive to the carbon budget and the discount rate. Lastly, the path of the shadow price of carbon is not very sensitive to the rate of growth of the economy, while the discount rate modifies it significantly.

4. Optimal investment in abatement capital under target uncertainty

There is a large literature on the large uncertainty surrounding the marginal damages generated by greenhouse gases. In fact, such an uncertainty is a rationale to rely on a cost/efficiency approach rather than a cost/benefit one: once a target has been defined at the political level (like the 2°C at COP 21), the uncertainty on the damages generated by GHGs does no longer affect the analysis. However, uncertainty on marginal damages has only been shifted to the target, and we are not immune to a sudden discovery that we should immediately stop (net) emissions. In such a case, a large adjustment cost would have to be faced to make the required investment.
The carbon budget is uncertain and there is a non-zero probability that it is reached at every moment. The model is solved in the case of a technology reducing the carbon intensity: $E_t = {}_0e^{gt}(\bar{A} - A_t)$. Therefore, there is a point in time, τ at which there is a jump in the state A_t in order to immediately reach $\bar{A}$ so that the net emissions are zero.

The value of the program once the carbon budget has been reached, at the date τ is therefore:

$$-\int_T^{\infty} e^{-\rho(t-T)}\left(\mathrm{c}(\delta\bar{\mathrm{A}},\bar{\mathrm{A}})\right) dt = -\frac{1}{\rho}\left(\frac{\alpha}{2}\delta^2\bar{\mathrm{A}}^2 + \beta\bar{\mathrm{A}}\right) = \bar{W}$$

While the value of the program before τ is :

$$\max_{a_{t,t<\tau},} \int_0^{+\infty} e^{-\rho t}[(-\mathrm{c}(\mathrm{a}_t, A_t) + \mathrm{h}(\bar{W} - W(A_t, S_t) - \mathrm{c}(\bar{\mathrm{A}} - A_t, A_t)]\, dt$$

$$\dot{\mathrm{A}}_\mathrm{t} = \mathrm{a}_\mathrm{t} - \delta\mathrm{A}_\mathrm{t},$$

$$a_t = \delta\bar{A}\ for\ t \geq \tau$$

$$\dot{\mathrm{S}}_\mathrm{t} = {}_0e^{gt}(\bar{A} - A_t)$$

$$A_0, S_0\ donnés$$

where h is the hazard rate *ie*. The probability that the carbon budget is reached at time t knowing that it is not been reached yet. For simplicity we assume a constant hazard rate, but a more realistic setting would feature a hazard rate that depends on the stock S_t.

The Hamiltonian corresponding to the problem can be written :

$$H = -\left(\frac{\alpha}{2}a^2 + \beta A\right) + h[\bar{W} - W(A_t, S_t) - \mathrm{c}(\bar{\mathrm{A}} - A_t, A_t)] + \lambda(a - \delta A) - \mu Y_0 e^{gt}(\bar{A} - A_t) + \nu Y_0 e^{gt}(\bar{A} - A)$$

Necessary optimality conditions are:

$$\frac{\partial c(a,A)}{\partial a} = \alpha a = \lambda, \qquad (1)$$

$$\dot{\lambda} = (\rho + \delta + h)\lambda + \beta - (\mu - \nu)Y_0 e^{gt} + \mathrm{h}\frac{\partial \mathrm{c}(\bar{\mathrm{A}} - A_t, A_t)}{\partial \mathrm{A}} \qquad (2)$$

$$\dot{\mu} - (\rho + h)\mu = 0 \qquad (3)$$

The problem has to be further solved using numerical techniques, but the first order conditions already convey some insights. In particular, the growth rate of the carbon value includes the hazard rate while the relationship between the abatement investment and the shadow price of abatement capital remains the same.

5. Conclusion and extensions

We have proposed two optimal investment models: one based on an abatement technology while the other featured decoupling. The reality falls between these two specifications for the technology. In the models studied, investment is spread over time, in particular because of the presence of adjustment costs, but we also take into account the fact that the more advanced the economy in the emissions reduction, the more expensive the new investments to achieve carbon neutrality. The resolution of these models highlights the existence of a shadow price of carbon and a shadow price of the capital of abatement, which illustrates the duplicity of a carbon value described as a value of the effort to provide to achieve a climate goal. The shadow price of capital corresponds to the cost of the last technological system making it possible to reach the target while the shadow price of carbon is the price signal which leads economic agents to respect the emissions path constraint.

One of the main lessons of this theoretical model is that the realistically calibrated optimization model leads to an emission trajectory very close to the linear trajectory. A second result concerns the optimal date of carbon neutrality between 2045 and 2053, regardless of the type of abatement technology and parameter values (including the discount rate or the size of the carbon budget) considered. We also obtain a non-monotonic path for the marginal value of the abatement capacity, while the carbon value follows a Hotelling rule. However, even if the rate of growth of the price of carbon only depends on the discount rate and the hazard rate when the target is uncertain, its initial value is affected by the characteristics of the investment technology.

Some extensions to this model can be developed to take into account technical progress, and to endogenise the GDP dynamics. Learning-by-doing would indeed work in the opposite direction of the effect according to which abatement is increasingly expensive as one approaches the objective. In addition, investors' liquidity constraints could be taken into account. Finally, considering a growth model would make it possible to take into account the tradeoff between productive investment and investment in abatement; in this case a cost-benefit approach would be suitable.

<u>Références</u>

Boucekkine R., Pommeret A. and Prieur F. (2013), « « Optimal regime switching and threshold effects: theory and application to a resource extraction problem under irreversibility » *Journal of Economic Dynamics and Control*, 37(12): 2979-2997

Bureau D. and Gollier C. (2009), Evaluation des projets publics et développement durable, CEDD Références économiques pour le développement durable n°8.

Chakravorty U., Moreaux M. and Tidball M. (2008), “Ordering the extraction of polluting non renewable resources”, American Economic Review, vol. 98, n° 3,juin, p. 1128-1144.

Chakravorty U., Magné B. and Moreaux M. (2006), “A Hotelling model with a ceiling on the stock of pollution”, Journal of Economic Dynamics and Control, 30, 2875-2904.

Gould, J.P., (1968). Adjustment costs in the theory of investment of the firm. Rev. Econ. Stud. 35 (1), 47.

Hotelling H., (1931), “The Economics of Exhaustible Resources”, Journal of Political Economy, 39, 137-175.

IPCC, (2014). Summary for Policymakers. In: Climate Change 2014, Synthesis Report. Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, cambridge university press Edition. Cambridge, United Kingdom and New York, NY, USA.

Iyer, G., Hultman, N., Eom, J., McJeon, H., Patel, P., Clarke, L., (2014). Diffusion of low-carbon technologies and the feasibility of long-term climate targets. Technol. Forecast. Social. Change.

Lucas Jr.,R.E.,(1967).Adjustment costs and the theory of supply. J.Political Econ.75(4),321–334.

Pindyck R. (2017) The use and misuse of models for climate policy, Review of Environmental Economics and Policy, vol 11(1), p.100=114.

Quinet A. et al. (2009), “La valeur tutélaire du carbone”, la Documentation Française.

Quinet A. (2019) La valeur de l’action climatique, une boussole pour l’investissement et l’action, France Stratégie.

Riahi, K., Kriegler, E. , Johnson, N. , Bertram, C. , denElzen, M. , Eom, J. , Schaeffer, M. , Edmonds, J. , Isaac, M. , Krey, V. , Longden, T. , Luderer, G. , Méjean, A. ,McCollum, D.L. , Mima, S., Turton, H. , van Vuuren , D.P. , Wada, K. , Bosetti, V. , Capros , P. , Criqui , P. , Hamdi-Cherif, M. , Kainuma, M. , Edenhofer, O. , (2015) .Locked into Copenhagen pledges Implications of short-term emission targets for the cost and feasibility of long-term climate goals. Technological Forecasting and Social Change 90,PartA, 8–23.

Schubert (2009), « La valeur du carbone : niveau initial et profil temporel optimaux », la Documentation Française.

Vogt-Schilb, A, G Meunier, and S Hallegatte (2018), “When Starting with the Most Expensive Option Makes Sense: Optimal Timing, Cost and Sectoral Allocation of Abatement Investment”, *Journal of Environmental Economics and Management,* 88 (March), 210–33.

<u>Appendix 1 : resolution of the model</u>

1. Résolution avec technologie de dépollution

The model is solved in case of an abatement technology (see equation (3)). The dynamic Lagrangien associated to the problem is (we omit time indices):

$$L = -\frac{\alpha}{2}\,a^2 - \beta A + \lambda(a - \delta A) - \mu(Y_0 e^{gt}\bar{A} - A_t) + \omega(\bar{S} - S) + \nu(Y_0 e^{gt}\bar{A} - A_t)$$

where $\omega \geq 0$ is the multiplier associated to the stock of CO_2 and $\nu \geq 0$ is the multiplier associated to the constraint on the abatement capital stock.

Necessary optimality condition are:

$$\frac{\partial c(a,A)}{\partial a} = \alpha a = \lambda, \qquad \text{(a.1)}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho + \delta + \frac{\beta}{\lambda} - \frac{\mu - \nu}{\lambda} \quad \text{(a.2)}$$

$$\frac{\dot{\mu}}{\mu} = \rho + \omega/\mu \quad \text{(a.3)}$$

- We first study the phase t>T when the cap is reached: (a.3) provides the expressions for ω.

$$\dot{S} = 0 \Rightarrow A_t = Y_T e^{g(t-T)} \bar{A} \Rightarrow a_t = Y_T e^{g(t-T)} \bar{A}(g+\delta) \Rightarrow \lambda_t = \alpha Y_T e^{g(t-T)} \bar{A}(g+\delta)$$

And we deduce from (a.2) :

$$\mu_t - \nu_t = \beta + (\rho + \delta - g)\alpha Y_T e^{g(t-T)} \bar{A}(g+\delta)$$

- We study the phase t<T, when the cap is not yet reached ($\omega = \nu = 0$). We deduce $\mu_t = \mu_0 e^{\rho t}$.

We set $x_t = \lambda_t e^{-(\rho+\delta)t}$ which implies $\dot{x} = \dot{\lambda}_t e^{-(\rho+\delta)t} - (\rho+\delta)\lambda_t e^{-(\rho+\delta)t}$ and therefore $\dot{x} e^{(\rho+\delta)t} = \dot{\lambda}_t - (\rho+\delta)\lambda_t = \beta - \mu_t$, thus:

$$x_t = \bar{x} + e^{-\delta t}\frac{\mu_0}{\delta} - e^{-(\rho+\delta)t}\frac{\beta}{(\rho+\delta)}$$

with $\bar{x}$ a constant to be determined.

We deduce:

$$\lambda_t = \bar{x} e^{(\rho+\delta)t} + \frac{\mu_0}{\delta} e^{\rho t} - \frac{\beta}{\rho+\delta} \qquad \text{et } a_t = \frac{\bar{x}}{\alpha} e^{(\rho+\delta)t} + \frac{\mu_0}{\delta\alpha} e^{\rho t} - \frac{\beta}{\alpha(\rho+\delta)}$$

Moreover, $\dot{A} + \delta A = a$

We set $z_t = A_t e^{\delta t}$ which implies $\dot{z} = \dot{A}_t e^{\delta t} + \delta A_t e^{\delta t}$ and therefore $\dot{z} e^{-\delta t} = \dot{A}_t + \delta A_t = a_t$, thus:

$$\dot{z}_t = \frac{\bar{x}}{\alpha} e^{(\rho+2\delta)t} + \frac{\mu_0}{\delta\alpha} e^{\delta t} e^{\rho t} - \frac{\beta e^{\delta t}}{\alpha(\rho+\delta)}$$

We take the integral to obtain:

$$z_t = \bar{z} + \frac{\bar{x}}{\alpha(\rho+2\delta)} e^{(\rho+2\delta)t} + \frac{\mu_0}{\delta\alpha(\delta+\rho)} e^{(\delta+\rho)t} - \frac{\beta e^{\delta t}}{\alpha\delta(\rho+\delta)}$$

with $\bar{z}$ a constant to be determined.

Therefore:

$$A_t = z_t e^{-\delta t} = \bar{z} e^{-\delta t} + \frac{\bar{x}}{\alpha(\rho+2\delta)} e^{(\rho+\delta)t} + \frac{\mu_0}{\delta\alpha(\delta+\rho)} e^{\rho t} - \frac{\beta}{\alpha\delta(\rho+\delta)}$$

Since $\dot{S} = Y_0 e^{gt} \bar{A} - A_t$, we obtain using integration :

$$S_t = \frac{Y_0}{g}(e^{gt}-1)\bar{A} + \frac{\bar{z}}{\delta}\left(e^{-\delta t}-1\right) - \frac{\bar{x}}{\alpha(\rho+2\delta)(\rho+\delta)}\left(e^{(\rho+\delta)t}-1\right) - \frac{\mu_0}{\delta\alpha(\delta+\rho)\rho}(e^{\rho t}-1) + \frac{\beta t}{\alpha\delta(\rho+\delta)} + S_0$$

A_0 , $\bar{A}$ and $\bar{S}$ that are given, as well as the continuity of λ_t at T allows identifying the unknowns T, $\bar{x}$, $\bar{z}$ et μ_0.

2. Resolution when the technology reduces carbon intensity

–

Necessary optimality conditions are :

$$\frac{\partial c(a,A)}{\partial a} = \alpha a = \lambda, \quad \text{(a.4)}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho + \delta + \frac{\beta}{\lambda} - Y_0 e^{gt}\frac{\mu-\nu}{\lambda} \quad \text{(a.5)}$$

$$\frac{\dot{\mu}}{\mu} = \rho + \omega/\mu \quad \text{(a.6)}$$

- We first study the phase t>T when the cap is reached: (a.6) provides the expression for ω.

$$\dot{S} = 0 \Rightarrow A_t = \bar{A} \Rightarrow a_t = \delta\bar{A} \Rightarrow \lambda_t = \alpha\delta\bar{A}$$

And we deduce using (a.5) :

$$\mu_t - \nu_t = e^{-g(t-T)}[\beta + (\rho+\delta)\alpha\delta\bar{A}]/Y_T$$

- We study the phase t<T, when the cap is no reached yet ($\omega = \nu = 0$). We deduce $\mu_t = \mu_0 e^{\rho t}$.

We set $x_t = \lambda_t e^{-(\rho+\delta)t}$ which implies $\dot{x} = \dot{\lambda}_t e^{-(\rho+\delta)t} - (\rho+\delta)\lambda_t e^{-(\rho+\delta)t}$ and therefore $\dot{x}e^{(\rho+\delta)t} = \dot{\lambda}_t - (\rho+\delta)\lambda_t = \beta - \mu_t Y_0 e^{gt}$, thus:

$$x_t = \bar{x} - e^{(g-\delta)t}\frac{\mu_0 Y_0}{g-\delta} - e^{-(\rho+\delta)t}\frac{\beta}{(\rho+\delta)}$$

With $\bar{x}$, a constant to be determined.

We deduce :

$$\lambda_t = \bar{x}e^{(\rho+\delta)t} + \frac{\mu_0 Y_0}{\delta-g}e^{(g+\rho)t} - \frac{\beta}{\rho+\delta} \quad \text{et } a_t = \frac{\bar{x}}{\alpha}e^{(\rho+\delta)t} + \frac{\mu_0 Y_0}{(\delta-g)\alpha}e^{(g+\rho)t} - \frac{\beta}{\alpha(\rho+\delta)}$$

Moreover, $\dot{A} + \delta A = a$

We set $z_t = A_t e^{\delta t}$ which implies $\dot{z} = \dot{A}_t e^{\delta t} + \delta A_t e^{\delta t}$ and therefore $\dot{z}e^{-\delta t} = \dot{A}_t + \delta A_t = a_t$, thus :

$$\dot{z}_t = \frac{\bar{x}}{\alpha}e^{(\rho+2\delta)t} + \frac{\mu_0 Y_0}{(\delta-g)\alpha}e^{(g+\rho+\delta)t} - \frac{\beta e^{\delta t}}{\alpha(\rho+\delta)}$$

We compute the integral to obtain:

$$z_t = \bar{z} + \frac{\bar{x}}{\alpha(\rho+2\delta)} e^{(\rho+2\delta)t} + \frac{\mu_0 Y_0}{(\delta-g)\alpha(\delta+\rho+g)} e^{(\delta+\rho+g)t} - \frac{\beta e^{\delta t}}{\alpha\delta(\rho+\delta)}$$

with $\bar{z}$ a constant to be determined.

Therefore:

$$A_t = z_t e^{-\delta t} = \bar{z}e^{-\delta t} + \frac{\bar{x}}{\alpha(\rho+2\delta)} e^{(\rho+\delta)t} + \frac{\mu_0 Y_0}{(\delta-g)\alpha(\delta+\rho+g)} e^{(\rho+g)t} - \frac{\beta}{\alpha\delta(\rho+\delta)}$$

Since $\dot{S} = Y_0 e^{gt}(\bar{A} - A_t)$, we obtain using integration:

$$S_t = \frac{Y_0}{g}(e^{gt} - 1)\bar{A} + \frac{\bar{z}Y_0}{\delta - g}\left(e^{(g-\delta)t} - 1\right) - \frac{\bar{x}Y_0}{\alpha(\rho + 2\delta)(\rho + \delta + g)}\left(e^{(\rho+\delta+g)t} - 1\right) - \frac{\mu_0 {Y_0}^2}{(\delta - g)\alpha(\delta + \rho + g)(\rho + 2g)}\left(e^{(\rho+2g)t} - 1\right) + \frac{\beta Y_0(e^{gt} - 1)}{\alpha\delta g(\rho + \delta)} + S_0$$

A_0 , $\bar{A}$ and $\bar{S}$ that are given as well as the continuity of λ_t at T allow identifying the unknowns T, $\bar{x}$, $\bar{z}$ and μ_0.

3. Model with a given linear emissions path

3.1. Abatement technology

Program of a decentralized producer:

$$Max \int_0^\infty -\left(\frac{\alpha}{2}a_t^2 + \beta A_t\right) - \mu_t E_t \, dt$$

$$s.c. \quad \dot{A}_t = a_t - \delta A_t \quad et \quad Y_0 e^{gt}\bar{A} - A_t$$

The Hamiltonian can be written:

$$H = -\left(\frac{\alpha}{2}a^2 + \beta A\right) - \mu_t E_t + \lambda(a_t - \delta A_t)$$

$$H = -\left(\frac{\alpha}{2}a^2 + \beta A\right) - \mu_t(Y_0 e^{gt}\bar{A} - A_t) + \lambda(a_t - \delta A_t)$$

First order conditions :

$$\alpha a_t = \lambda_t \Leftrightarrow \lambda_t = \alpha a_t \Leftrightarrow \dot{\lambda}_t = \alpha \dot{a}_t \qquad (1)$$

$$-\frac{\partial H}{\partial A_t} = \dot{\lambda}_t - \rho\lambda_t = \delta\lambda_t + \beta - \mu_t \qquad (2)$$

(1) and (2) lead to:

$$\mu_t = -\alpha\dot{a}_t + \alpha a_t(\rho + \delta) + \beta \ (3)$$

Assuming that we impose a linear emissions path (the path is therefore not optimized): $E_t = \bar{E} - \epsilon * t$ with $\epsilon > 0$ and $\bar{E}$ are the initial emissions and ϵ is the yearly emissions reduction.

Moreover, we consider an abatement technology, therfore $A_t = Y_0 e^{gt}\bar{A} - E_t$ et :

$$a_t = \dot{A}_t + \delta A_t = (g + \delta)Y_0 e^{gt}\bar{A} - \frac{\partial E_t}{\partial t} - \delta E_t = (g + \delta)Y_0 e^{gt}\bar{A} + \epsilon - \delta\bar{E} + \delta\epsilon t$$

And thus $\lambda_t = \alpha\left((g + \delta)Y_0 e^{gt}\bar{A} + \epsilon - \delta\bar{E} + \delta\epsilon t\right)$.

Using (3) :

$$\mu_t = \alpha Y_0 e^{gt}\bar{A}(g + \delta)(-g + \rho + \delta) + \alpha(\rho + \delta)\delta\epsilon t - \alpha\delta\epsilon + \alpha(\rho + \delta)(\epsilon - \delta\bar{E}) + \beta$$

The TIMES model (with a 95MtCO_2 sink) provides a path for μ_t . Using two points of the trajectory (μ_{2030}=322€/tCO^2 and μ_{2045}=375€/tCO_2), and setting $g = 0$ as in TIMES, we identify α and β.

3.2. Technology reducing carbon intensity:

$$Max \int_0^{\infty} -\left(\frac{\alpha}{2}a_t^2 + \beta A_t\right) - \mu_t E_t \, dt$$

$$s.c. \quad \dot{A}_t = a_t - \delta A_t \quad et \quad E_t = Y_0 e^{gt}(\bar{A} - A_t)$$

The Hamiltonian is:

$$H = -\left(\frac{\alpha}{2}a^2 + \beta A\right) - \mu_t E_t + \lambda(a_t - \delta A_t)$$

$$H = -\left(\frac{\alpha}{2}a^2 + \beta A\right) - \mu_t Y_0 e^{gt}(\bar{A} - A_t) + \lambda(a_t - \delta A_t)$$

First order conditions :

$$\alpha a_t = \lambda_t \Leftrightarrow \lambda_t = \alpha a_t \Leftrightarrow \dot{\lambda}_t = \alpha \dot{a}_t \qquad (1)$$

$$-\frac{\partial H}{\partial A_t} = \dot{\lambda}_t - \rho\lambda_t = \delta\lambda_t + \beta - Y_0 e^{gt}\mu_t \qquad (2)$$

(1) and (2) lead to :

$$\mu_t Y_0 e^{gt} = -\alpha\dot{a}_t + \alpha a_t(\rho + \delta) + \beta \; (3)$$

Assume we impose a linear path for emissions reduction (so the path is not optimized): $E_t = \bar{E} - \epsilon * t$ with $\epsilon > 0$ where $\bar{E}$ is the initial level of emissions and ϵ the yearly emissions reduction.

Moreover, we consider a technology reducing carbon intensity, therefore $A_t = \bar{A} - e^{-gt}E_t/Y_0$, and :

$a_t = \dot{A}_t + \delta A_t = \frac{(\bar{E}-\epsilon t)(g-\delta)e^{-gt}}{Y_0} + \frac{\epsilon e^{-gt}}{Y_0} + \delta\bar{A}$

thus $\lambda_t = \alpha\left(\frac{(\bar{E}-\epsilon t)(g-\delta)e^{-gt}}{Y_0} + \frac{\epsilon e^{-gt}}{Y_0} + \delta\bar{A}\right)$.

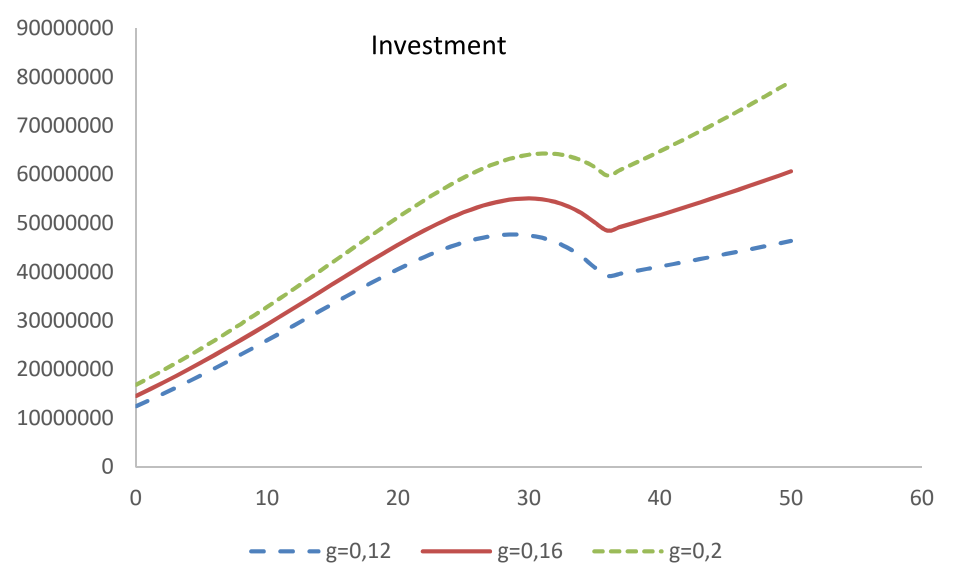
Therefore using (3) :

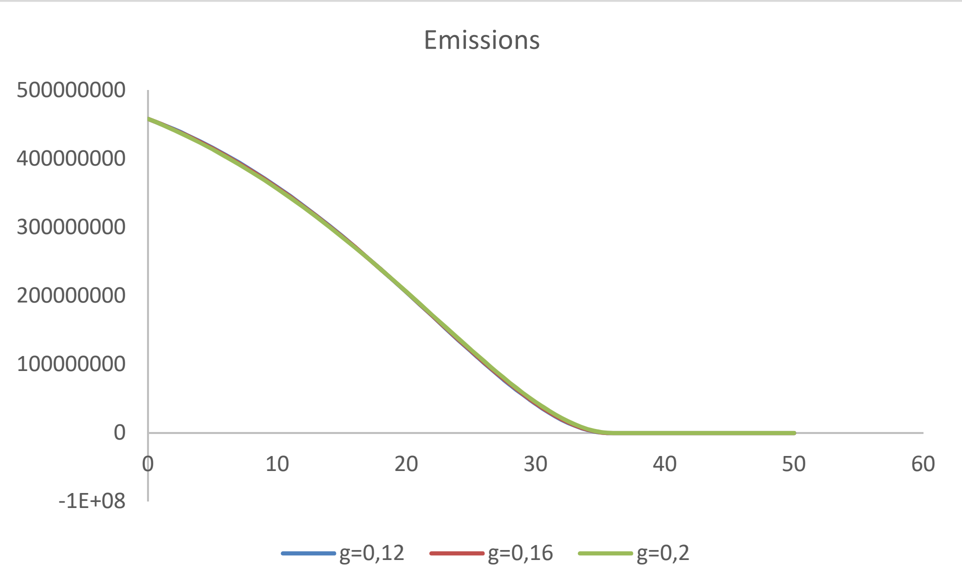
$$\mu_t = \beta + \frac{\alpha e^{-gt}}{Y_0}.[(g-\delta)(g+\rho+\delta)\bar{E} + \epsilon(\rho+2g) - (g-\delta)\epsilon t(g+\rho+\delta)] + (\rho+\delta)\delta\bar{A}$$

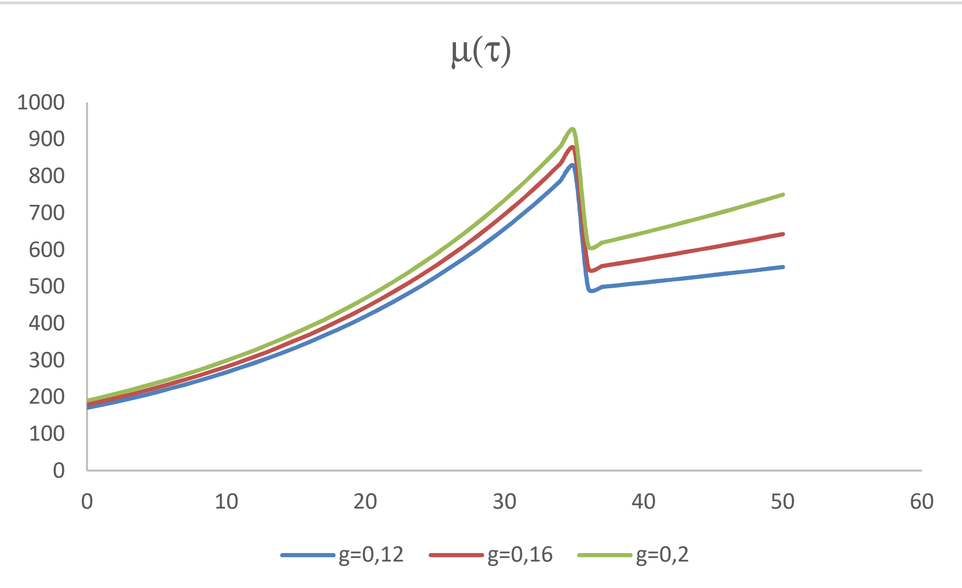
TIMES model (with a 95MtCO_2 sink) provides a path for μ_t. Using two points of the trajectory, (μ_{2030}=322€/tCO^2 and μ_{2040}=375€/tCO_2), and setting $g = 0$ as in TIMES, we can identify α and β.

4. Sensitivity to parameters in the case of an abatement technology

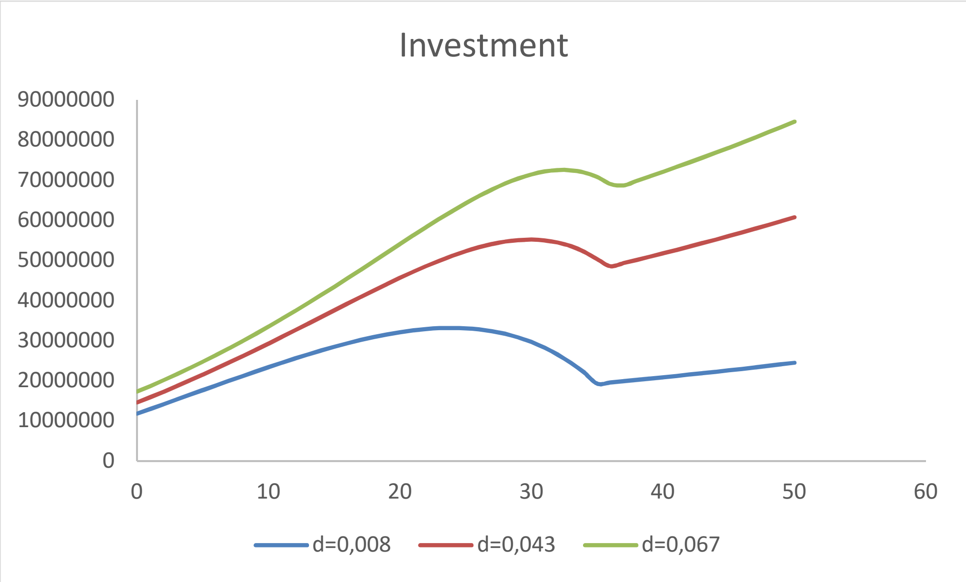
- **Sensitivity to the GDP growth rate**

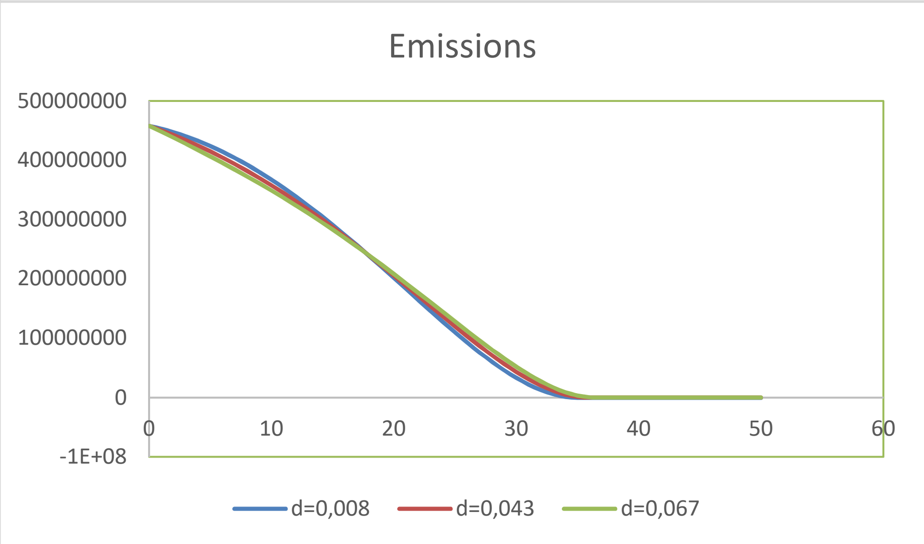


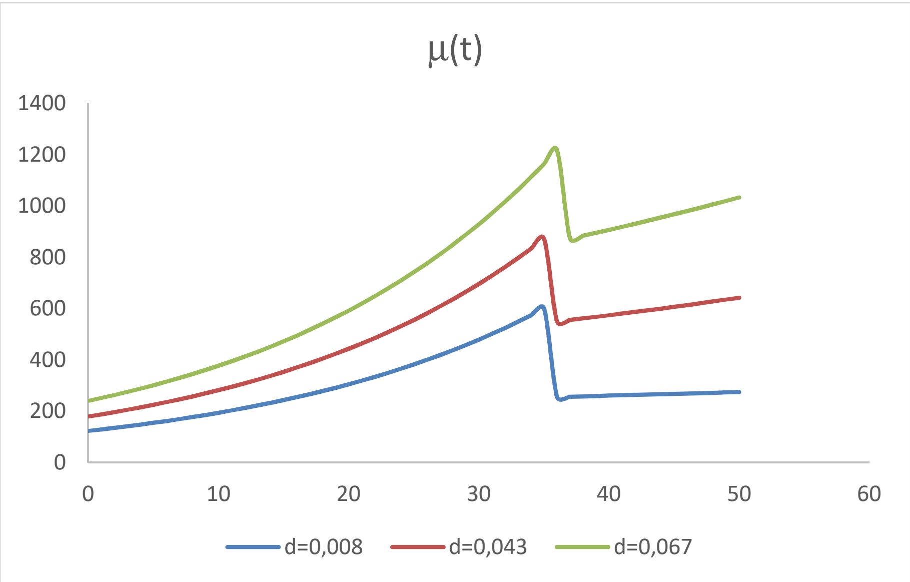




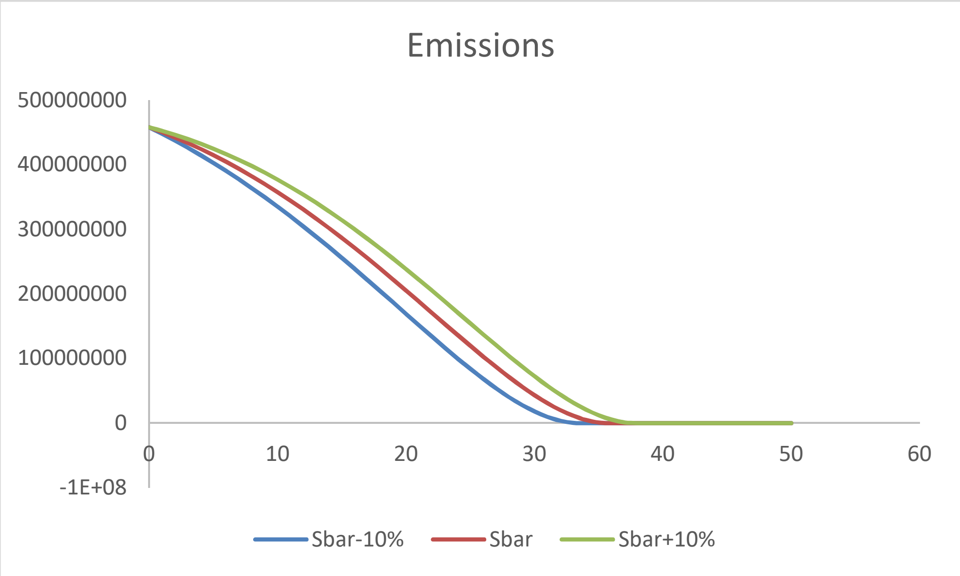
- **Sensitivity to the depreciation rate**

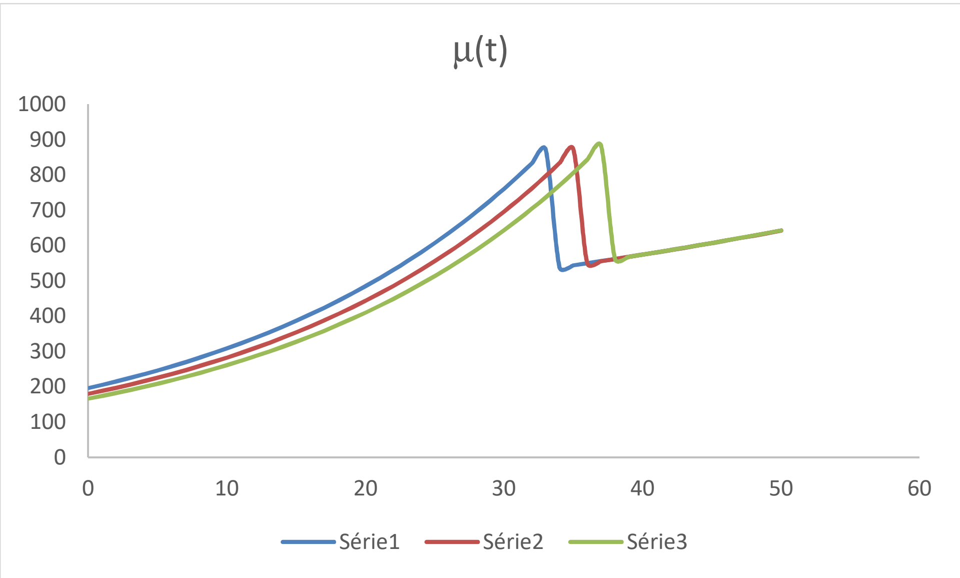




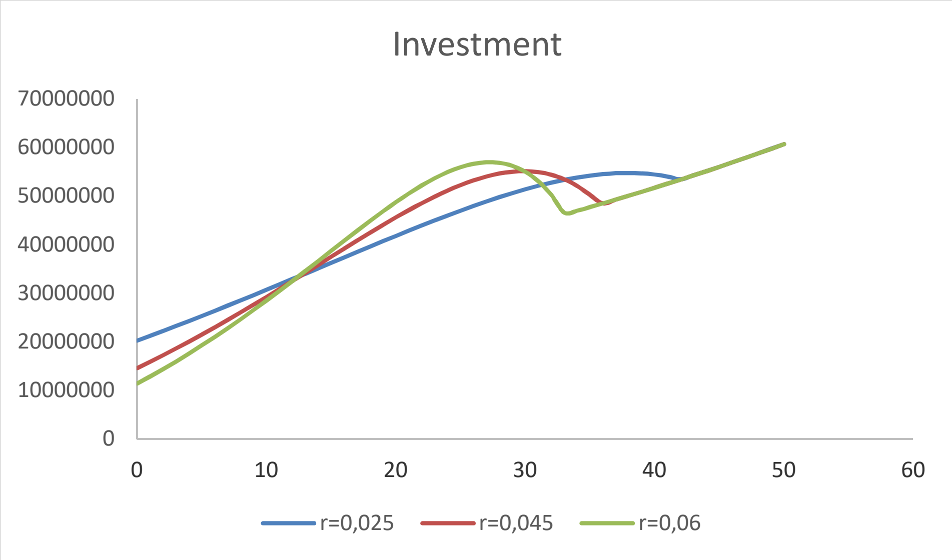


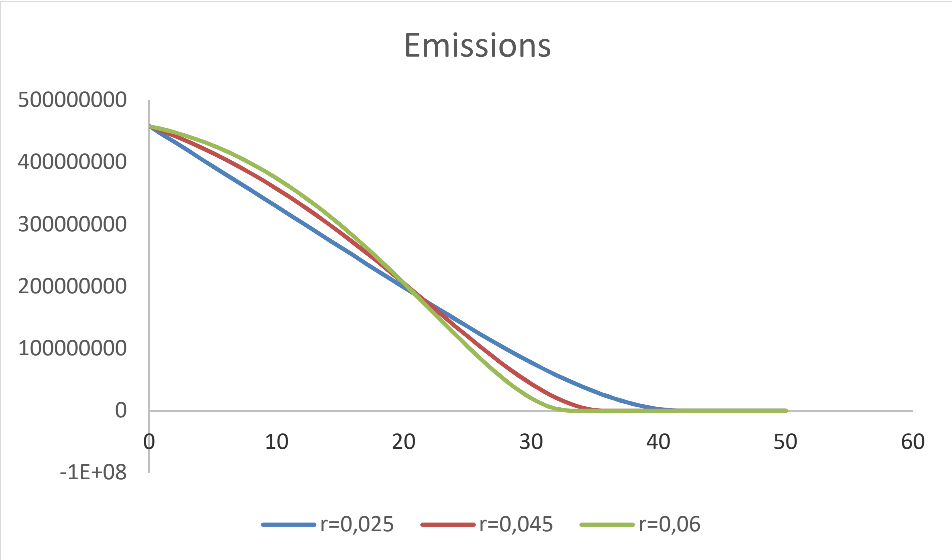
- **Sensitivity to the carbon budget**

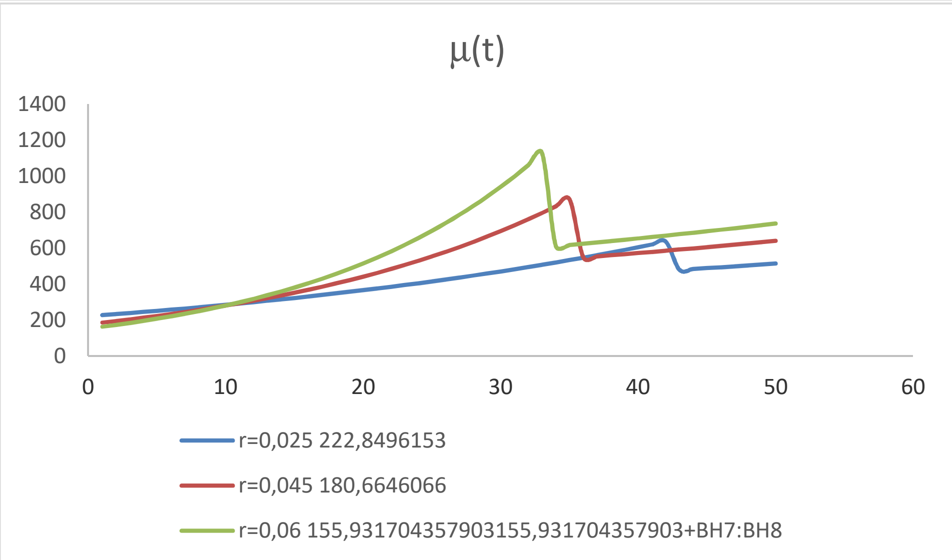




- **Sensitivity to the discount rate**

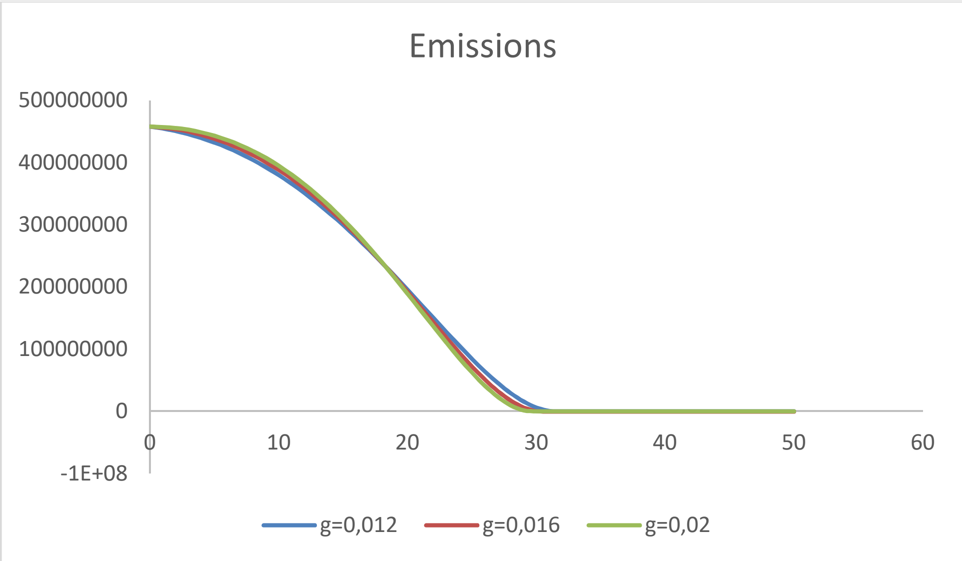


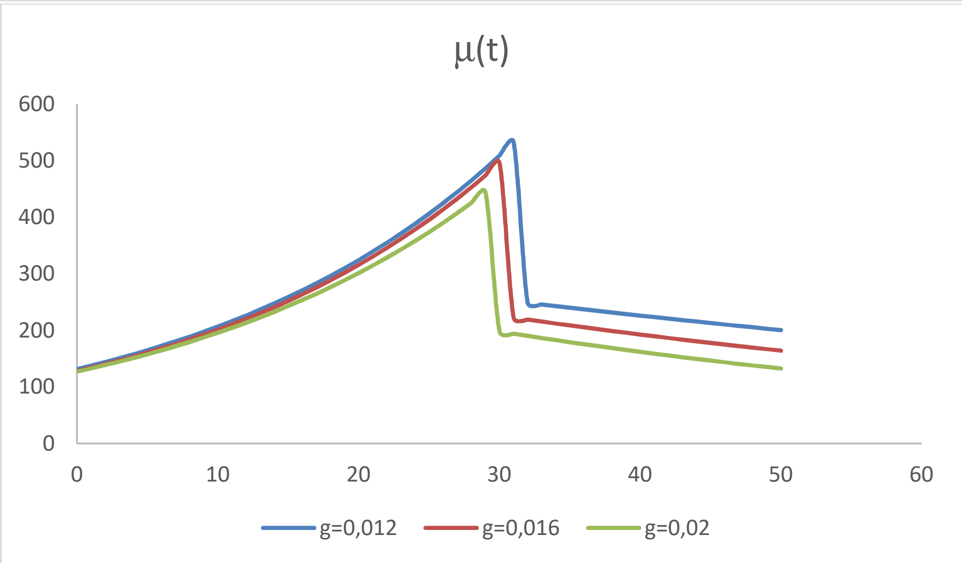




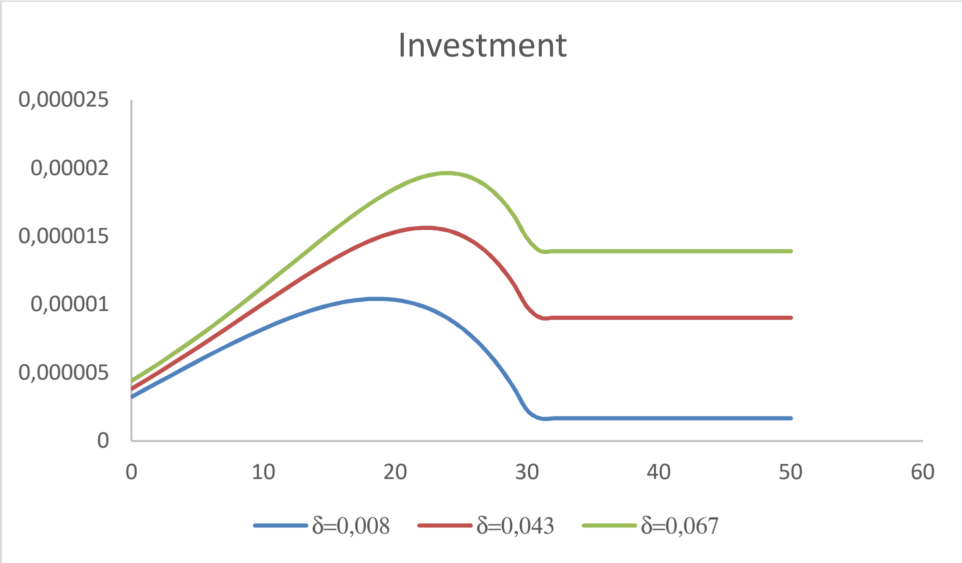
5. Sensitivity to parameters in the case of technology reducing carbon intensity

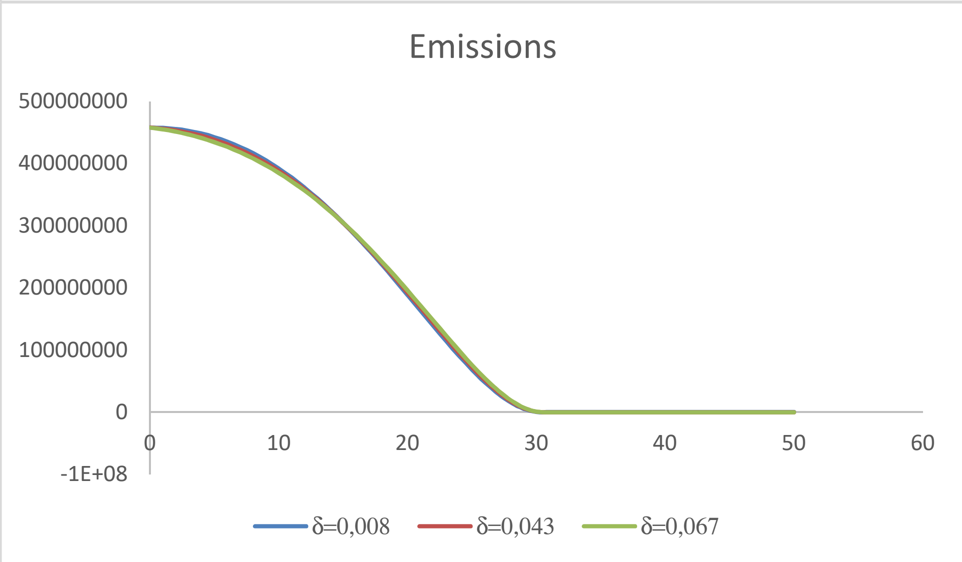
- **Sensitivity to the GDP growth rate**

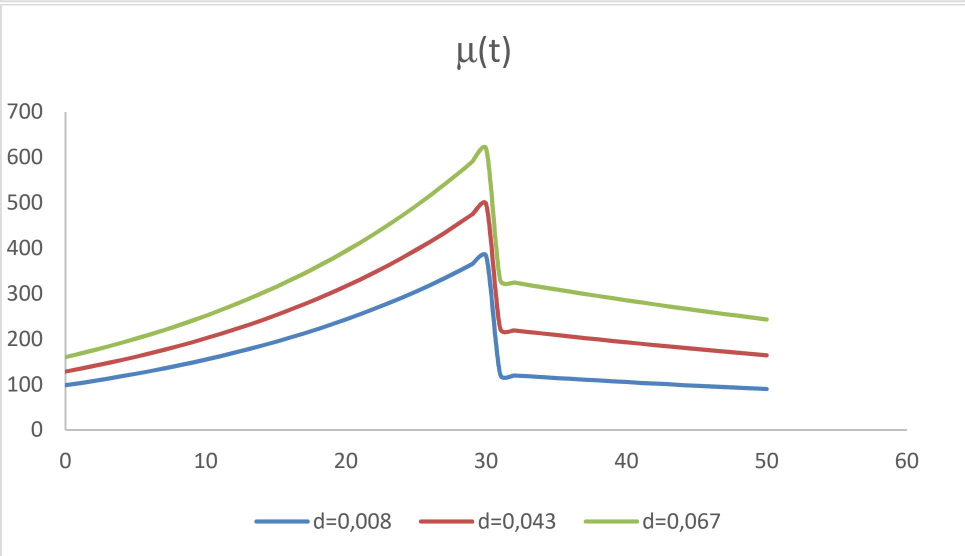




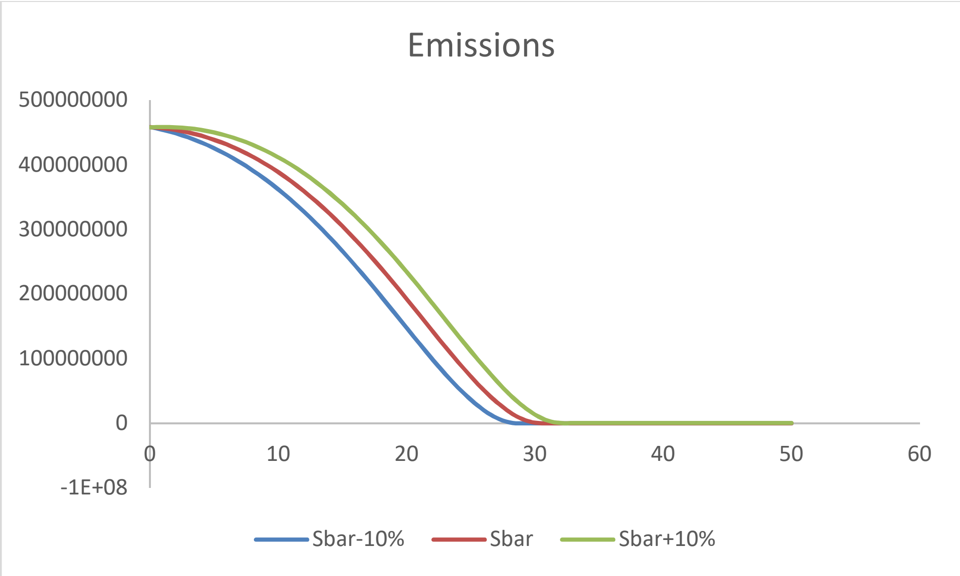
- **Sensitivity to the depreciation rate**

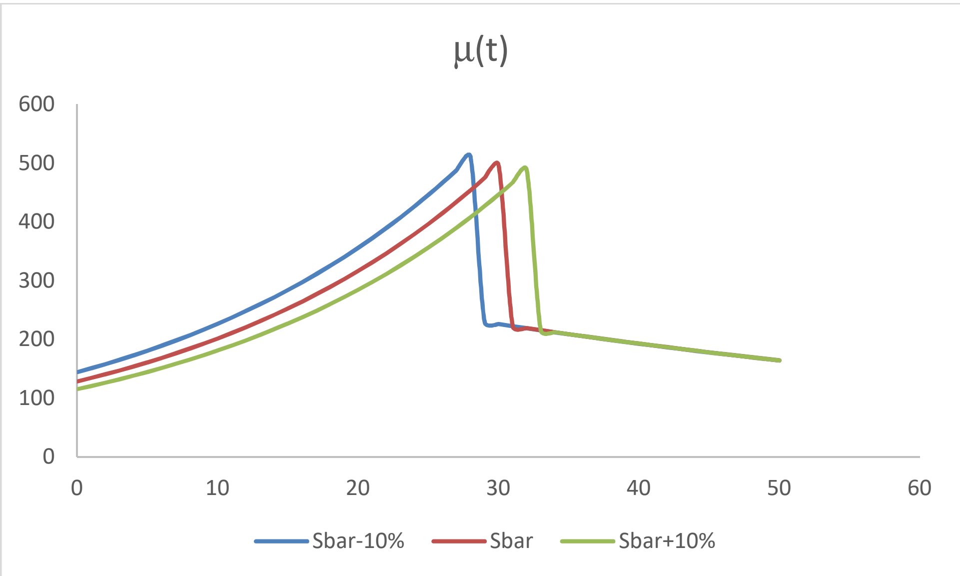




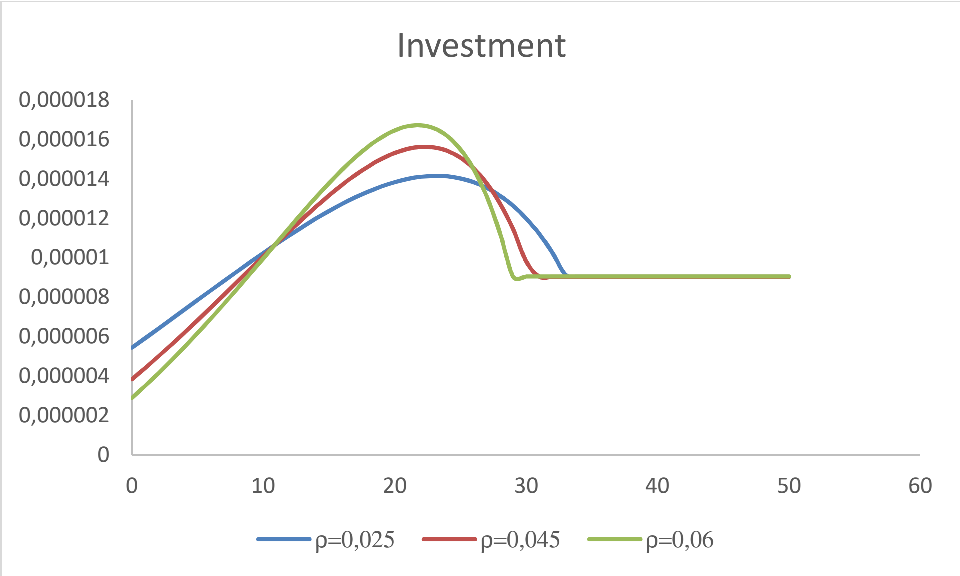


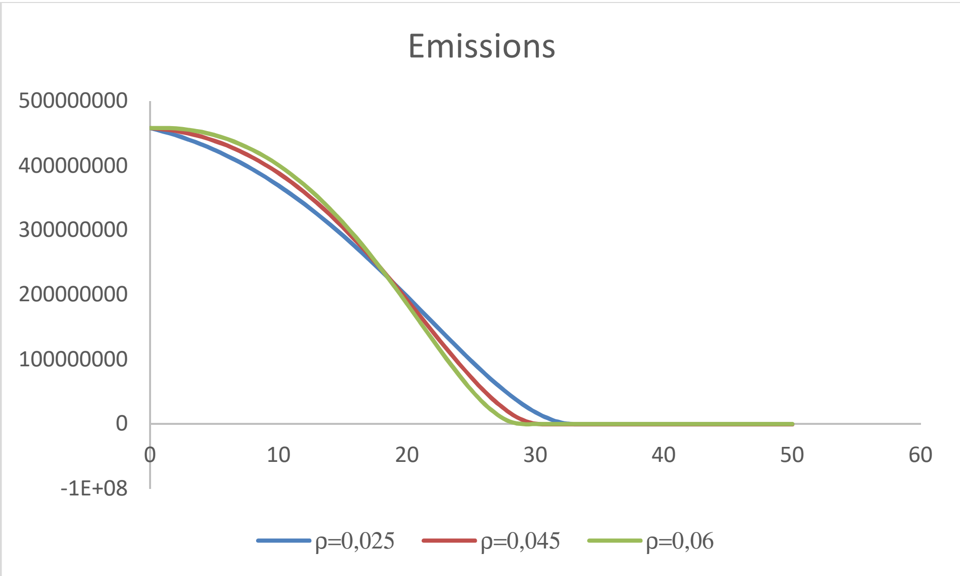
- **Sensitivity to the carbon budget**

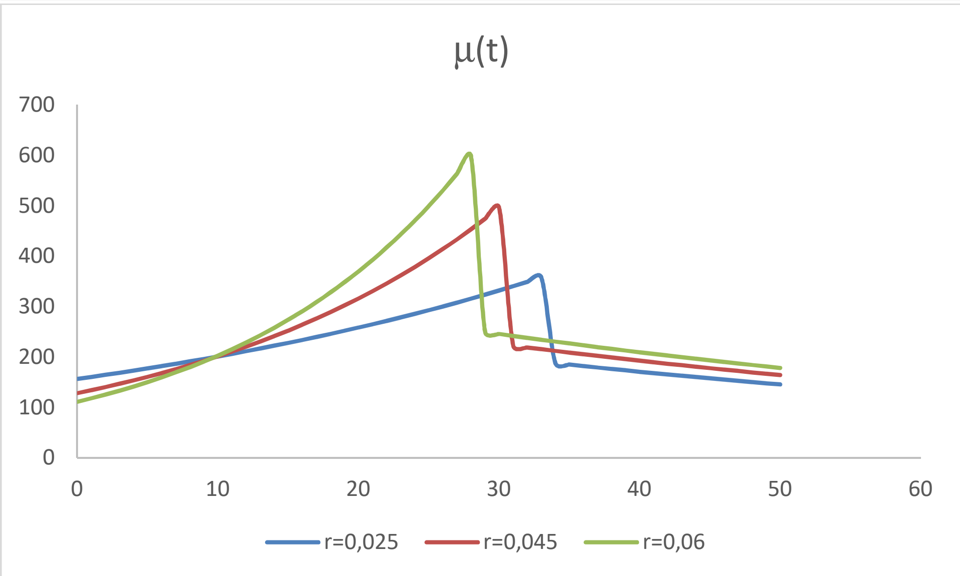




- **Sensitivity to the discount rate**


Investment
0,000018
0,000016
0,000014
0,000012
0,00001
0,000008
0,000006
0,000004
0,000002
0
0
10
20
30
40
50
60
ρ=0,025
ρ=0,045
ρ=0,06


Emissions
500000000
400000000
300000000
200000000
100000000
0
-1E+08
0
10
20
30
40
50
60
ρ=0,025
ρ=0,045
ρ=0,06


μ(t)
700
600
500
400
300
200
100
0
0
10
20
30
40
50
60
r=0,025
r=0,045
r=0,06