Environmental Tax Reform with Heterogeneous Regions and Imperfect Labour Markets

Diane Aubert*

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Abstract

This paper investigates how a rise in the pollution tax rate may affect unemployment, migration and welfare in a general equilibrium model. We build a model of two different regions (Harris-Todaro), with imperfect labour markets (unemployment) and migration. Pollution is due to the consumption of a dirty commodity by households and to the use of a dirty input in the production process. We allow for non-homothetic preferences for taking account the potential regressivity of green taxes (the polluting good is assumed to be a necessary good). We show that frictional unemployment and non-homothetic preferences bring about inter-region wage differential. Thus, an economy almost always exhibits distortions in the absence of the government intervention. Green tax may exacerbate these distortions by generating spillovers, if the labour market is initially more frictional in the region where the subsistence level of the polluting good is the lowest one. Wages subsidies and transfers among regions are explored as solution to remove distortions.

JEL classification: - H23 - J64 - Q52 - Q56 - R13.

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*Department of Management, Technology and Economics, ETH Zurich, Zrichbergerstrasse 18, 8092 Zurich, Switzerland.

1 Introduction

While environmental taxes are considered by economists as one of the most efficient policy instruments to fight climate change, governments seem reluctant to implement green tax shifts, for fear of opposition from public opinion.[1] Indeed, as any kind of indirect taxation, environmental taxes lower consumers' purchasing power and might be detrimental for employment. Besides, they are usually deemed to be strongly regressive, harming the poor more than the rich, since polluting goods are often necessary goods.

A large literature already addressed the efficiency and distributional incidence of green taxes through the analysis of the vertical distributive effects of these policies - i.e. distributive effects between households along the income dimension.[2] However, recent empirical works (Cronin et al., 2017; Douenne et al., 2018; Berry, 2019) point out that the political opposition might come not only from vertical, but mostly from horizontal distributive effects - i.e. between households with similar incomes but different location. On the consumer side, households generally face a minimum consumption of energy that meets their transports and heating needs. But, these needs might strongly differ across regions due to climate differences or gaps between the coverage of public transport network.

To offset the potential negative impacts of green taxes, economists suggest using other fiscal tools beside, as lump-sum transfers. But given the importance of unobserved heterogeneity in the determinants of energy consumption, horizontal distributive effects are much more difficult to tackle than vertical ones (Douenne et al., 2018). Understanding the incidence of environmental taxes among different regions seems then key to facilitate the implementation of ambitious environmental policies.

This paper aims to provide answers to these questions. Recent micro-simulation studies have paid particular attention to the way in which households consume and therefore use their income differently depending on their location. We bring a different perspective and look at the impact of green taxes on the formation of households' revenues between areas. Indeed, on the production side, some regions may present a slightest degree of industrialization and fear to face higher costs due to a global carbon tax. Consequently, households working in industrialized sectors that are often intensive in energy input, feel particularly more vulnerable to environmental taxes and exposed to unemployment. We intend to combine these arguments in a theoretical model. We develop a general equilibrium framework to analyses the incidence of a global green tax, implemented in two regions that differ through: (i.) the polluting consumption basket of households;

[1]Some examples among others: in 2012, Australia abolished its initial carbon tax, in line with the campaign promises of the new Prime Minister; in 2016, the United States withdrew from the Paris Agreement on Climate Change, in line also with the electoral promises of President Donald Trump; in 2018, in France, the government was forced to abandon the increase in taxes on gasoline following the protest of yellow jackets.

[2]See for instance Hassett et al. (2007), West and Williams III (2012); Wier et al. (2005), Berry (2019), Sterner (2012)

(ii.) the intensity of the production sector in energy inputs, (iii.) the level of unemployment. We evaluate the consequences of environmental fiscal policy in terms of unemployment, consumption and migration of workers. To the best of our knowledge, this paper is the first one to combine a sectoral analysis with polluting input; structural unemployment due to frictions (search and matches); and non-homothetic preferences. This allows us to analyze the green taxes incidences on the labour revenue formation (sources side of income) without neglecting the incidence on the consumption part (the uses side of income).

More precisely, in a general equilibrium framework, we build a two-sector model, within labour is assumed mobile (Harris and Todaro (1970)) and there is a pollution externality. As in Bovenberg and De Mooij (1994), pollution is due to two different sources: the use, in the production processes, of a polluting input; and the consumption of polluting commodities by household. We represent this pollution commodity as a necessity and we allow its subsistence level to differ between regions. Both sectors present structural unemployment caused by hiring costs, and we use a static search and matching model to formulate frictions on labour markets with individual worker-firm bargaining. The model is solved analytically as we have specified, in the simplest way, preferences and technologies. The main results are the following: frictional unemployment and non-homothetic preferences bring about an inter-region wage differential. Thus, an economy almost always exhibits distortions in the absence of government intervention. A green tax may exacerbate these distortions by generating spillovers, if the labour market is initially more frictional in the region where the subsistence level of the polluting good is lower. Under some conditions on the minimum of polluting goods consumption, simulations show that wage subsidies to the sector that is the most polluting could be part of the solution even if it contributes to maintain labour inside polluting industries.

Several empirical papers already addressed this issue, underlying the local dimension in the regressivity of green taxes. Sterner (2012) shows that the elasticity of substitution between clean and dirty good seems higher in a rural region compared to urban region in a developed country. More recently, Carraro and Zatti (2014), using a micro-simulation model, show that geographic and social-economic features of households greatly influence redistributive patterns of duties on fuel sources and vehicle taxes. Rural households and large families tend to be more affected within each income quintile. Moreover, richer households are normally those capable of shifting towards more fuel-efficient vehicles. Ciaschini et al. (2012), Williams et al. (2014), and Hassett et al. (2007) confirm these results by using CGE models.

Yet, the distribution of green taxes burden has not been extensively analyzed from the perspective of regional inequalities in a theoretical framework. Some theoretical works in environmental economics investigate the difficulties of setting an optimal green tax in an economic federation with different regions. These papers refer to fiscal externalities of local governments who compete for workers or capital and generate spillovers (Oates (2001)). This last point was also studied between national and local governments when the latter transfers its tax burden on the former (Aronsson et al. (2006), Williams III (2012)). But among this emerging literature on environmental federalism, few papers focus on the disparities in wealth and access to clean goods. In a federation model, Garon and Seguin [2015], study the welfare effects of a revenue-neutral green tax reform that recycles green taxes revenues through labour taxes reduction. The authors assume that regions are unequally endowed with a non-renewable natural resource, that provides regional resource rents. This rent generates a motive for inefficiently relocating of labour to the resource-rich jurisdiction. they authors show that the green tax reform can mitigate this effect. Still, they use a partial equilibrium framework that does not account for sectoral changes.

Papers introducing environmental concern in Harris-Todaro models may represent a contribution. Harris and Todaro (1970) generalize a general equilibrium model of two-sectors introducing migration and difference in wealth between regions. This paper and related studies have provided a series of models that constitute the received theory of rural-urban migration. Workers are assumed to compare expected incomes in cities with agricultural wages and to migrate if the former exceeds the latter. Migration is the balancing force which equates the two expected incomes. Equilibrium is attained when they are equalized and there is no migration. Although there is an abundant literature about Harris-Todaro model, few studies consider the environmental problem faced by the developed countries. Wang (1990), building on this standard model, demonstrates that a raise in a green tax increases the agricultural wage and lowers urban unemployment by producing backward migration to the agricultural sector. Recently Daitoh (2003), in a model in which urban manufacturing production exerts a negative externality on consumers' utility function, derives the sufficient condition for a rise in the pollution tax rate on urban manufacturing to improve national income.

We intend to complement this short stream of literature to get a larger picture of the regional distributive and efficiency consequences of an environmental tax reform. Contrary to the main papers in Harris-Todaro tradition, we focus on developed countries in which we assume disparities between regions. To do so, we mainly focus on the paper by Daitoh (2003), to which we add two fundamental assumptions. First, households have a subsistence level of polluting goods that we allow to differ among regions (Jacobs and van der Ploeg (2017). Second, there is frictional unemployment in both sectors. In fact, our paper incorporates some features borrowed from papers that merge search generated unemployment literature (introduced by Pissarides (1998) within a two regions migration framework (see for example Sato (2004), Kuralbayeva (2018), **?**. In contrast to the previous studies in the Harris-Todaro framework, they show that migration toward city induces frictional urban unemployment that causes an inter-sector wage disparity. Because of the frictional externalities, the allocation of agents between regions / sectors can be sub-optimal. Thus, the original model of Harris-Todaro has been adapted in order to match the developed economies concern.

One of our key contributions is to combine frictional unemployment with non-homothetic preferences for the polluting good in a "Harris-Todaro" economy. We show that a difference in subsistence level of the polluting good among regions may exacerbate the sector-wage disparities due to frictions and this may generate spillovers. Moreover, these specifications allow us to work in an ideal framework in order to study the trade-off between efficiency (employment), inter-regional equity (due to perfect mobility) and environmental welfare of an environmental tax reform. Our paper then joins the Harris-Todaro literature with the traditional double dividend literature (see Bovenberg and De Mooij (1994), Goulder (1995), Bovenberg and Van Der Ploeg (1998)).

The remainder of the paper is organized as follows. We present our model in Section 2. Section 3 solves the general equilibrium, and analyses the effect of an increase in a green tax on wage disparities, unemployment and migrations. Section 3 presents simulation exercise and examine wages subsidies as a solution. Section 4 concludes.

2 The model

2.1 General setting

The framework is a general equilibrium model. We assume a closed economy made up of two regions (indexed by $i = 1, 2$, in the following). Each region is specialized in the production of one good, denoted X_i for region i.[3] We denote p_i the price of good X_i and treat good X_2 as a numeraire. p_1 stands therefore for the relative price of good (X_1/X_2).

There is a continuum of workers (or households) of exogenous size $\bar{L}$ in the economy. L_1 workers are living in region 1 and involved in the production of sector X_1, while:

$$L_2 = \bar{L} - L_1 \tag{1}$$

reside in region 2 and work in the corresponding sector X_2.

We assume structural unemployment in both regions caused by hiring costs. We use a search and matching model to formulate frictions on labour markets with individual worker-firm bargaining (Pissarides (1998)). In order to make the analysis as simple as possible, we adopt a static framework.[4]. We refer to l_i to specify employed workers in sector X_i. Consequently, each region consists of L_i agents who all live in this region and are involved in sector X_i. Among those, only

[3]Throughout the paper, we will refer to different regions.But this is just convenient terminology: the model is general, and could just as easily represent regions like urban/rural regions, or even just two sectors if consumers preferences are identical (see Section 3). Yet, it can be inconvenient for nations inside a Federal System as European Union: we do not assume different searching costs for migrants in our model (see for instance, Combes et al. (2016) for a theoretical framework of search and match with migrants discrimination).

[4]As Diamond (1982) showed, we can describe the essence of job search and recruiting externalities using a static model. For examples of static search and matching models, see Sato (2004), Keuschnigg and Ribi (2009)

l_i are employed workers, while $L_i - l_i$ workers remain unemployed. Still, labour is considered perfectly mobile between regions. Households may decide to move (to migrate) from one region to the other one and work in the corresponding sector. As a result, L_i and l_i are both endogenous variables.

We assume that beside regional goods X_1 and X_2, there is a third good in the economy, denoted by E. It can be considered as energy. This good enters both in the production process of firms as imperfect substitute of labour, and in the consumption basket of households. Accordingly, the production of each regional goods X_i requires the use of both labour input (l_i), and energy input (E_i), while households in each region consume the three goods of the economy. In order to make the distinction between production and consumption goods, we denote respectively $C_i^{X_1}$, $C_i^{X_2}$ and C_i^E the consumption of regional goods and of the polluting good by a household living in region i.

The use of energy by the firms and its consumption causes environmental damage. E is called the 'polluting' good. We assume, for simplicity, that the market for E does not exist and that supra-national government imposes a specific tax t_E on the use of E by firms and consumers (Copeland and Taylor (2004), Rapanos (2007), and Daitoh (2003)). The supra-national government uses the tax revenues to provide lump-sum transfers T to each household regardless its region.

Due to the large number of variables in this model, we provide in appendix A.1, an index of model variables to which the reader can refer.

2.2 Unemployment

Following Sato (2004), we assume that the production in sector X_i consists of F_i many small firms, each of which can employ only one worker. We assume that there are heterogeneities (or mismatches) in the labour market that make it costly for a worker or a firm to find a partner (Pissarides (1998)). Because, the model is static, initially all households are considered job searchers and all firms are assumed to have a vacant position. Thus, initially in each region i, a mass L_i of workers are searching a job in the region, while F_i firms are looking for a worker in order to start the production of good X_i. As in Pissarides (1998) and in Keuschnigg and Ribi (2009), we assume for simplicity "*one shot matching so that no other search opportunity is available*". The labour market heterogeneities of each region can be summarized in the matching function that gives the rate at which good matches are formed. In its simplest form, the matching function is defined as: $M_i = m_i(F_i, L_i)$, with positive first partial derivatives, negative second derivatives and constant returns to scale. The function implies that a firm looking for a worker finds one with a probability less than one, equals to $\frac{M_i}{F_i}$, even if there are enough jobs to satisfy all workers. Denoting $\theta_i = \frac{F_i}{L_i}$ the tightness ratio of the labour market, we can rewrite this probability as: $q_i(\theta_i) = \frac{M_i}{F_i} = m_i(1, 1/\theta_i)$. It represents the Poisson matching probability of a vacant job, i.e. the rate at which vacant jobs are filled. Then, among the F_i firms in region i, only $q_i(\theta_i)F_i$

firms find a worker and are operating. Symmetrically, the rate at which an unemployed worker finds a job is given by $\theta_i q_i(\theta_i) = \frac{M_i}{L_i}$. Then, for workers in region i, $\theta_i q_i(\theta_i) L_i = l_i$ are employed in sector X_i and $(L_i - l_i)$ are unemployed. Finally, due to our assumption that a firm can hire only one worker, the number of operating firms always equals the number of employed workers: $q_i(\theta_i)F_i = l_i = \theta_i q_i(\theta_i) L_i$.

We denote u_i the unemployment rate in region i. The standard Beveridge curve is defined as:

$$u_i = \left[\frac{L_i - \theta_i q_i(\theta_i) L_i}{L_i}\right] = 1 - \theta_i q_i(\theta_i) \tag{2}$$

In the remain of this paper, we will assume for simplicity that $q_i(\theta_i) = \frac{M_i}{F_i} = \mu_i \theta_i^{-\xi_i}$, with $0 < \xi_i = -\frac{\partial q(\theta_i)}{\partial \theta_i} * \frac{\theta_i}{q_i(\theta_i)} < 1$ that stands for the elasticity of the matching function and $\mu_i > 0$ the efficiency of the process.[5]

2.3 Household Behaviour

Consumption preferences

Each individual worker supplies one unit of labour in elastically and consumes the three goods $C_i^{X_1}$, $C_i^{X_2}$ and C_i^E (respectively the consumption of regional goods and of the polluting good). They are assumed imperfect substitutes in the following consumption utility function: $Q_i = \varrho_i\left(C_i^E, v(C_i^{X_1}, C_i^{X_2})\right)$. We assume functional separability between the polluting good and the regional goods in the joint utility function of consumption. This specification is similar to the one used by Copeland and Taylor [2004] and it allows us to solve the model analytically. Functional separation implicitly assumes that the price of the polluting good does not impact the ratio of regional goods prices through consumption demand effects.[6] Agents are all assumed risk neutral, meaning that ϱ_i and v are assumed to be linear in income. Moreover, we assume v homothetic (the aggregated demand of regional goods is independant of income distribution), but in contrast to the standard literature, we do not allow ϱ_i to be linearly homogeneous in C_i^E. In fact, usual quasi linear and homothetic preferences imply that the elasticity of substitution between polluting goods and other goods is constant and thus independent on individual revenues. However, polluting goods (in particular energy goods) are often considered as necessities (Deaton et al. (1980),Chung (1994),and Jacobs and van der Ploeg (2017)). Households generally face a minimum consumption of energy that meets their transports or heating needs. And the higher this minimum of consumption, the lower the substitution possibility between energy and regional goods, and the higher the energy tax

[5] Pissarides (1998), and Blanchard and Diamond (1989) have shown that a reasonable approximation of the matching function is a Cobb-Douglas function (with here parameter ξ, $1 - \xi$).

[6] Remember that t_E is also levied on energy inputs use. We will see latter that, through this way, t_E impacts p_1.

burden. This substitution elasticity may depend on regions: energy needs may differ across regions due to temperature differences or to a gap between the coverage of public transport network. In order to capture these regional disparities, we specify ϱ_i as Stone-Geary preferences.[7] Q_i can be written as:

$$Q_i = \varrho_i \ \left((C_i^E, v(C_i^{X_1}, C_i^{X_2})\right) = (C_i^E - \bar{E}_i)^\gamma \left(v(C_i^{X_1}, C_i^{X_2})\right)^{1-\gamma} \tag{3}$$

where $\bar{E}_i$ denotes the subsistence level for the polluting good, that differs between regions. The Stone-Geary utility function makes it possible to model a share of consumption that is not responsive to price changes ($\bar{E}_i$) and another share that can adapt instantaneously to price variations ($C_i^E - \bar{E}_i$).

As the environmental degradation acts as an externality, we assume that households ignore the adverse effect of their demand for polluting goods on the quality of the environment. Consequently, a household i chooses $C_i^{X_1}$, $C_i^{X_2}$ and C_i^E in order to maximize its consumption utility Q_i subject to its budget constraint: $p_1 C_i^{X_1} + C_i^{X_2} + t_E C_i^E = I_i$ (with I_i denoting the income of households i). From the first order conditions of the household maximization, we obtain the energy demand of households and their indirect utility of consumption:

$$C_i^E = \frac{\frac{\partial P_Q}{\partial t_E}}{P_Q}\left[I_i - t_E \bar{E}_i\right] + \bar{E}_i; \qquad Q_i^* = \frac{\left[I_i - t_E \bar{E}_i\right]}{P_Q}$$

where P_Q can be interpreted as the marginal price of consumption which is constant and does not differ between regions.[8] Consumers first have to purchase the subsistence level of the polluting good that costs $t_E \bar{E}_i$. Then, they decide how to allocate their leftover income ($I_i - t_E \bar{E}_i$) between polluting and non polluting goods, according to their respective preference parameter (γ, $1-\gamma$), similarly to the case of classical Cobb-Douglas preferences.

Income and welfare

Workers supply one unit of labour at wage w_i if employed in sector i. Both unemployed and employed workers receive the same amount of transfers T from the supra-national government. The reservation wage, for which a household is indifferent between being employed or unemployed,

[7] We could also just assumed that ϱ_i belongs to the class of Gorman Polar form. But this made it harder to read the paper without providing new insights (in particular the log-linearization results does not changed). We choose rather, to directly assume Stone-Geary preferences that are a special case of Gorman Polar utility functions. See Jacobs and van der Ploeg (2017) for more details on Gorman Polar form utility function.

[8] Although P_Q is equal to the inverse of the private marginal utility of income (*i.e.* the Lagrange multiplier associated with the budget constraint of household), P_Q does not correspond to the implicit price of aggregated consumption. Because the Stone-Geary utility function is non homogeneous, the price index P_i depends on income and varies across regions ($P_i Q_i^* = I_i => P_i = \left(\frac{I_i}{I_i - (t_E)\bar{E}_i}\right) P_Q$). The marginal price of consumption P_Q is still constant as we constrain incomes to be sufficiently high to purchase the subsistence level of polluting good ($I_i > t_E \bar{E}_i$).

is then driven to zero.[9] Because we consider a static framework of matching, the *ex ante* probability of being unemployed u_i in region i, is equal to the *ex post* unemployment rate (Sato (2004)). The expected indirect utility of workers can be represented by:

$$V_i = u_i * [Q_i^* (T)] + (1 - u_i) * Q_i^*(w_i + T) - \psi [E_{tot}] \tag{4}$$

where $-\psi [E_{tot}]$ denotes the dis-utility due to the environmental degradation, E_{tot} is the aggregated energy demand that is the source of global pollution.
Let's denote the relevant variables of region i, with the subscript e or u, depending on whether workers are employed or unemployed. The expected indirect utility of workers can be rewritten as:

$$V_i = u_i * [V_i^u] + (1 - u_i) * V_i^e - \psi [E_{tot}] \tag{5}$$

Migration

As in Harris and Todaro [1970] and many others studies, we assume that workers are perfectly mobile between sectors and regions, and that migration occurs so as to equate the expected indirect utility between regions. Then we obtain:

$$V_1 = V_2 \tag{6}$$

Using (2) and (3), this condition is reduced to:

$$\theta_1 q_1(\theta_1) * [w_1] + V_1^u = \theta_2 q_2(\theta_2) * [w_2] + V_2^u \tag{7}$$

We refer to this condition as the no-migration condition.

2.4 Firm's behavior

Technology

Firms need to post a vacancy in order to hire workers. For firms in region i, maintaining a vacancy costs c_i units of output. It can be interpreted as the fixed cost of labour recruitment which is represented in term of the good X_i. Analogously to Sato (2004), or Helsley and Strange (1990), before paying the cost of posting a vacancy, a firm is not sure to be matched with a worker. Due to frictions, a vacant job is matched to an unemployed worker with a probability $q_i(\theta_i) < 1$.[10] If the job of the firm is occupied, firms demand a polluting factor of production e_i at a price t_E and

[9] We could have introduced unemployment benefit and utility of leisure for unemployed worker that would have defined their reservation wage. But because, in our economy, global prices are equal between regions, there is no reason for different reservation wages between regions. Then, unemployment-benefit modelling becomes superfluous.

[10] Then $F_i = \frac{l_i}{q_i(\theta_i)} = \theta_i L_i$.

pay their unique worker at a wage w_i. Consequently, the amount of output per firm in sector i is: $x_i = f_i(e_i, 1)$ where f_i is concave and displays decreasing return to scale with respect to e_i, the demand of polluting good per firm. As in Bovenberg and De Mooij (1994), the aggregate production function amounts to $X_i = l_i x_i = f_i^*(l_i e_i, l_i) = f_i^*(E_i, l_i)$ where E_i stands for the aggregated amount of polluting input in region i. If the job of the firm remains unoccupied, the firm must still pay the vacancy cost c_i. Thus, the expected profit of each firm is equal to $q_i(\theta_i)(p_i x_i - w_i - t_E e_i)$-$c_i$. Each firm chooses its energy demand in order to maximize its profit subject to the production function $x_i = f_i(e_i, 1)$. The firm's polluting good demand (e_i^*) condition is therefore:

$$p_i \frac{\partial f_i(e_i^*, 1)}{\partial e_i} = t_E \tag{8}$$

Denoting α_i the elasticity of the production function x_i with respect to e_i, we can rewrite this condition as:

$$\alpha_i \frac{p_i x_i}{e_i^*} = t_E \tag{9}$$

Assuming free entry of firms, in the steady state, the expected profit of a firm is driven to zero: the expected profit from an occupied job equals the expected costs of filling a vacancy. This gives the following equation:

$$p_{Li} = w_i + \frac{c_i}{q_i(\theta_i)} \tag{10}$$

where $p_{L_i} = p_i x_i - t_E e_i^* = (1 - \alpha_i) p_i x_i$ denotes the productivity of labour in sector X_i.[11] Equation (7) represents the traditional job creation condition: the marginal cost of investing in a job vacancy (c_i) must correspond to the expected job rent $(q_i(\theta_i)(p_{Li} - w_i))$. In contrast to a competitive labour market where firms hire until marginal productivity equals the wage, the total cost of worker exceeds the wage by a recruitment cost.

Wage determination

Once a suitably worker is found, a job rent (or a matching surplus) appears that corresponds to the sum of the expected search and hiring costs for the firm and the worker. For the firm, the matching surplus is the difference between the profit when it fills a vacancy and when it remains with vacancy: $(p_{L_i} - w_i - c_i) - (-c_i) = p_{L_i} - w_i$. For the worker, the matching surplus is the difference between its expected utility when employed and that when unemployed: $Q_i^*(w_i + T) - Q_i^*(T) = \frac{w_i}{P(t_E)}$. Wage needs to share this economic (local-monopoly) rent, in ad-

[11] Because we assume that each firm can hire only one worker, the productivity of labour is nothing else that the production of each firm $(p_i x_i)$ minus the part of energy in the production, i.e. $p_i \frac{\partial f_i(e_i^*,1)}{\partial e_i} e_i^* = t_E e_i^*$. Still, as in classical perfect labour market model, the productivity of labour equals the costs of hiring one more worker: $p_{Li} = w_i + \frac{c_i}{q_i(\theta_i)}$. This insures that expected profit of firms are zero: $q_i(\theta_i)(p_i x_i - w_i - t_E e_i) - c_i = 0 \iff (p_i x_i - t_E e_i) = w_i + \frac{c_i}{q_i(\theta_i)}$.

dition to compensating each side for its assets from forming the job. We assume a decentralized Nash-bargain, which imposes a particular splitting of the matching surplus between the two parties involved according to the relative bargaining power between them. As a result, w_i is determined by: $w_i = argmax \left\{ \left(\frac{w_i}{P_Q}\right)^{\beta_i} (p_{L_i} - w_i)^{1-\beta_i} \right\}$ with β_i the worker's bargaining power of sector i. Appendix A.II shows that the first-order condition for the maximization of the Nash product implies the following expression of the wage:

$$w_i = \frac{\beta_i}{1-\beta_i} * \left[\frac{c_i}{q_i(\theta_i)}\right] = \beta p_{Li} \tag{11}$$

If hiring costs are zero ($c_i = 0$), in equilibrium $w_i = 0$. Thus, positive hiring costs increase the gap between the utility of employment and that of unemployment. Similarly, a drop in the number of firms (i.e. a drop in θ_i) decreases the expected value of the firm's hiring costs ($\frac{c_i}{q_i(\theta_i)}$). This reduces the rent from the job match and decreases as well the wage. If the bargaining power of the worker equals one (*i-e* $\beta = 1$), then the wage equals the productivity of labour (similarly to competitive labour market), and labour demand does not depend at all of hiring costs.

2.5 The government budget constraint and the goods market equilibrium

To abstract from revenue-recycling approach, in line with Harberger (1962), and Daitoh (2003), we assume that the government transfers the tax revenue to consumers in a lump-sum fashion. In this context, the government revenue from pollution tax does not include any kind of redistribution that may affect firms or household location. The government budget constraint is described by:

$$T\bar{L} = t_E E_{tot} \tag{12}$$

To close the model, we need to determine the ratio of prices that depends on demand consumption. Because X_2 is assumed to be the numeraire, we only need to determine the price of X_1. Remember that we assume functional separability between the pollution good and regional goods in the joint utility function of consumption. Thus, the price of the polluting good does not impact directly the ratio of prices between both regional goods. Moreover $v(C_i^{X_1}, C_i^{X_2})$ is homothetic, i.e. exhibits constant returns to scale. Consequently, the ratio of the consumption of regional goods for each household is given by:

$$\frac{C_i^{X_2}}{C_i^{X_1}} = \rho\left(\frac{p_1}{p_2}\right)$$

where ρ is an increasing function of p_1. We can express $\frac{C_{tot}^{X_2}}{C_{tot}^{X_1}} = \rho(p_1)$ with $C_{tot}^{X_1}, C_{tot}^{X_2}$ respectively the aggregate private demand for good 1 and 2.

We can thus express, the inverse relative demand function (of regional goods) as $C^{-1} = \rho(\frac{C_{tot}^{X_2}}{C_{tot}^{X_1}})$

with ρ' the derivative of ρ positive and $C_{tot}^{X_1}, C_{tot}^{X_2}$ respectively the aggregate private demand for good 1 and 2. The goods market equilibrium requires that total demand equals total supply. It gives:

$$X_1 = x_1 l_1 = C_{tot}^{X_1} + \frac{l_1 c_1}{p_1 q_1(\theta_1)}; \qquad X_2 = l_2 x_2 = C_{tot}^{X_2} + \frac{l_2 c_2}{q_2(\theta_2)} \tag{13}$$

where $\frac{l_i c_i}{q_i(\theta_i)}$ represents the aggregated search costs. Dividing X_1 by X_2 and replacing $\frac{C_{tot}^{X_2}}{C_{tot}^{X_1}}$ by $\rho(p_1)$, we finally obtain:

$$\left(x_1 l_1 - \frac{l_1 c_1}{p_1 q_1(\theta_1)}\right) = \frac{1}{\rho(p_1)}\left(l_2 x_2 - \frac{l_2 c_2}{q_2(\theta_2)}\right) \tag{14}$$

Noting that $\frac{l_i c_i}{q_i(\theta_i)} = c_i F_i$, the goods market equilibrium is given by the following condition:

$$\frac{\rho(p_1)}{p_1}(p_1 l_1 x_1 - c_1 F_1) = l_2 x_2 - c_2 F_2 \tag{15}$$

3 General Equilibrium

The equilibrium of the model is defined as a tuple $(L_i^*, l_i^*, e_i^*, \theta_i^*, w_i^*, p_i^*)$ of 6*2 variables (for $i = 1, 2$) that satisfy the following conditions: the job creation conditions (3.8), the wage mark-up equations (3.9), the Beveridge curves (3.2), the firms' energy demands (3.6), the no-migration condition (3.5), the total labour endowment equation (3.1), the price equation (3.11) and the price normalization equation ($p_2 = 1$).

This section is divided into two parts. For a clear understanding of the mechanisms behind our model, in the first subsection, we compute the general equilibrium assuming that (i) the preferences of households do not differ between regions, (ii) preferences for regional goods are Cobb-Douglas, (iii) the output of firms take the form of $f_i(e_i, 1) = e_i^{\alpha_i}$ where α_i is supposed to be fixed. These assumptions are not realistic, but have the advantages to allow us to solve the model in level. Moreover, assuming that preferences do not differ between households allow us to refer to a well known situation: a general equilibrium model with a unique region and in which workers can choose freely in which sector to work. In the second subsection, these assumptions are released and the model is fully solved by log-linearization.

3.1 General Equilibrium in a particular case: same preferences for households

We assume a specific form for the utility function given by:

$$Q_i = \varrho_i\left((C_i^E, v(C_i^{X_1}, C_i^{X_2})\right) = (C_i^E - \bar{E})^{\gamma}\left(\left(C_i^{X_1}\right)^{\sigma}\left(C_i^{X_2}\right)^{1-\sigma}\right)^{1-\gamma} \tag{16}$$

The function v is a Cobb-Douglas and ϱ is a Stone-Geary utility function. Appendix A.III gives the solutions of the consumer problem. Before starting to describe the results of the general equilibrium, it is convenient to explain the specific case in which we are. We consider a particular case of our model in which prices, taxes, and the minimum consumption of energy are identical between regions. Then the indirect utility of unemployment workers does not differ between regions. In fact, the only difference for workers comes from wages and unemployment rates. This situation is similar to a model with a unique region in which workers can choose freely in which sector to work. If moreover labour markets are perfect, we already know from the literature, that free mobility between sectors requires the equality of wages.[12] In our framework, due to unemployment, this condition becomes $\theta_1 q_1(\theta_1) * [w_1] = \theta_2 q_2(\theta_2) * [w_2]$. This expression is the migration condition for $V_1^u = V_2^u$. This condition requires that the expected wage of workers (taking account the probability of unemployment) should be the same in both regions.

Comparatives statics

We start to consider that p_1 and L_1 are fixed (this implies L_2 fixed with equation 3.1). We want to analyze the impact of an uncompensated raise of green taxes on both labour markets. Equations (3.8), (3.9), (3.6), and the price normalization of p_2 yield the equilibrium demand for the polluting input: $e_i^* = \left[\frac{p_i \alpha_i}{t_E}\right]^{\frac{1}{1-\alpha_i}}$. Obviously, an increase of green taxes, decreases the polluting input. Remember that $p_{L_i} = (1-\alpha_i)\, p_i x_i$. Substituting this term into equation (3.9) gives :

$$p_{L_i}^* = (1-\alpha_i)\, p_i^{\frac{1}{1-\alpha_i}} \left[\frac{\alpha_i}{t_E}\right]^{\frac{\alpha_i}{1-\alpha_i}} \tag{17}$$

$$w_i^* = \beta_i p_{L_i}^* = \beta_i (1-\alpha_i)\, p_i^{\frac{1}{1-\alpha_i}} \left[\frac{\alpha_i}{t_E}\right]^{\frac{\alpha_i}{1-\alpha_i}} \tag{18}$$

$$q(\theta_i^*) = \frac{c_i}{(1-\beta_i) p_{L_i}} = \frac{c_i}{(1-\beta_i)(1-\alpha_i)} p_i^{-\frac{1}{1-\alpha_i}} \left[\frac{t_E}{\alpha_i}\right]^{\frac{\alpha_i}{1-\alpha_i}} \tag{19}$$

Intuitively, increasing the pollution tax rate, because it increases the energy factor price, lowers the productivity of labour (that is complementary to energy) (see equation (3.12)). Wages decrease according to equation (3.13). Still, this is not enough to overcome the raise of energy prices. The profit of an operating firm (with a job filled) decreases. Due to the free entry assumption, in the long run, the expected profit is always equivalent to the expected costs of opening a vacancy that is fixed. The zero profit condition leads thus to a decrease of F_i, the number of the firms in region i. With L_i fixed (by assumption in comparative statics), this leads to a decrease of the tightness of the labour markets θ_i^*. As a result, unemployment increases $(1 - \theta_i^* q_i(\theta_i^*))$. Thus, in a partial equilibrium, an increase of green tax lowers wage and energy input but increases unemployment.

[12] See Copeland and Taylor [2004].

If the model assumed perfectly symmetry between regions, obviously prices would have not changed, and workers would have not been incited to move from one region to the other one.[13] In our particular case, preferences of households are assumed identical. But, still, the model exhibits asymmetry between production sectors: α_i the energy intensity, c_i the hiring cost of firms, and ξ_i the elasticity of the matching function differ between sectors. What are the impacts of energy taxes on p_1 and L_1 in this context?

General Equilibrium

L_1 and p_1 are considered now as endogenous variables and must satisfy the no-migration condition (3.5) and the goods market equilibrium condition (3.11). Equations (3.12), (3.13), and (3.14) give us immediately $p^*_{L_2}$, w^*_2, $q(\theta^*_2)$, because $p_2 = 1$. From the migration condition (8), we finally find θ^*_1 in function of θ^*_2: $\theta^*_1 = \frac{c_2\beta_2(1-\beta_1)}{c_1\beta_1(1-\beta_2)}\theta^*_2$. And replacing in equations (3.14), (3.12), and (3.13), we finally have:

$$p^*_1 = \left(\frac{t_E}{\alpha_1}\right)^{\varepsilon_1 - \frac{\alpha_2(1-\alpha_1)\xi_1}{(1-\alpha_2)\xi_2}} \left[\left(\frac{c_1}{\chi_1}\right)\left(\frac{\chi_2}{c_2}\right)^{\frac{\xi_1}{\xi_2}}\left(\frac{c_2\beta_2(1-\beta_1)}{c_1\beta_1(1-\beta_2)}\right)^{\xi_1}\right]^{(1-\alpha_1)} \tag{20}$$

where $\chi_i = \left(\mu_i(1-\beta_i)\,(1-\alpha_i)\,\alpha_i^{\frac{\alpha_i}{(1-\alpha_i)}}\right)$. Thus, $\frac{dp^*_1}{dt_E} > 0$ if and only if: $\frac{\alpha_1}{1-\alpha_1}\frac{1}{\xi_1} > \frac{1}{\xi_2}\frac{\alpha_2}{1-\alpha_2}$.

This result is really intuitive. Both production sectors use pollution. The relative price of goods depends then explicitly of their relative intensity. This is what we can call the *pollution-intensity effect*. The higher the intensity of sector 1 is, the more the green tax impacts sector 1 relatively to sector 2. Expected wages have to be equalized between sectors due to the no-migration condition. Thus, firms of region 1 must increase their price in order to insure the zero profit condition. Finally, the productivity of labour will not change except if $\xi_1 \neq \xi_2$.The same reasoning can be apply for the elasticity of the matching function, that characterizes frictions.

We can rewrite the migration condition, noting that $\theta_1 q(\theta_1) = \frac{l_1}{L_1}$ and substituting (10) into (11), we finally obtain:

$$\frac{L - L_1}{L_1} = \frac{L_2}{L_1} = \frac{(1-\alpha_1)}{(1-\alpha_2)}\frac{[1-(1-\alpha_1)(1-\beta_2)]}{[1-(1-\alpha_2)(1-\beta_1)]} * \frac{\sigma}{1-\sigma} \tag{21}$$

The previous equation shows us that the ratio of the number of households in each region does not depend on t_E. In this case, the green tax has no incidence at all on the reallocation of workers between sectors. This result might seem a bit surprising regarding to the previous models of Harris Todaro that deal with environmental issues. In reality, because we have $V^u_2 = V^u_1$, we can

[13] Perfect asymmetry in this model is equivalent to consider a closed economy made up of one region. In this case, prices are fixed and there is no reason to migrate. See Henderson and Thisse (2004) for more details.

link our results to the theorem of Pissarides (1998). With Cobb-Douglas utility functions, there is no possibility for a reallocation between regions even if sectors present asymmetry. Here, the ratio of the demand on regional good ($\frac{\rho(p_1)}{p_1}$) is exogenous. Consequently, the adjustment of prices is enough to insure the no-migration condition and the goods market equilibrium. This theorem holds with the Stone-Geary utility function because our introduction of a Stone-Geary utility does not have any incidence on global price (weak separability). Moreover a change of labour market characteristics impacts the number of workers per sector but not through green taxes. Indeed, due to the perfect mobility of households, we always have: $\theta_1 q(\theta_1) w_1 = \theta_2 q(\theta_2) w_2$. The following proposition summarizes the above arguments.

Proposition 1: *If (i) preferences of households do not differ between regions, (ii) preferences for regional goods are Cobb-Douglas, and (iii) the elasticity of energy input to taxes is fixed, then an increase of the green tax rate:*

1. *decreases wages and increases unemployment in both sectors,*
2. *decreases the relative price of goods (p_1^*) if and only if the ratio of the elasticity of the production function with respect to energy over the elasticity of the matching function is higher in region 1 than in region 2, ($\frac{1}{\xi_1}\frac{\alpha_1}{1-\alpha_1} > \frac{1}{\xi_2}\frac{\alpha_2}{1-\alpha_2}$).*
3. *does not influence the reallocation of workers between regions.*

As discussed previously, the result (3) of proposition 1 depends on assumptions (i), (ii) and (iii). In the next subsection, we identify and disentangle the impacts of the release of these assumptions on the reallocation of workers between regions. We show that the release of one of these assumptions is enough to lead to a variation of L_i.

3.2 General case: log-linearization of our model

We consider the general consumption utility function :

$$Q_i = (C_i^E - \bar{E}_i)^\gamma \left(v(C_i^{X_1}, C_i^{X_2})\right)^{1-\gamma} \tag{22}$$

where $v(C_i^{X_1}, C_i^{X_2})$ is homothetic with constant return to scale. In contrast to the previous subsection, we allow $\bar{E}_i$ to differ between regions. The migration condition depends on the difference of this level and we obtain:

$$V_1 = V_2 \Leftrightarrow \theta_1 q_1(\theta_1) * [w_1] = \theta_2 q_2(\theta_2) * [w_2] + \triangle(t_E) \tag{23}$$

with $\triangle(t_E) = t_E\left(\bar{E}_1 - \bar{E}_2\right) > 0$. The assumption $\triangle(t_E) > 0$ implies that the two regions differ by their need of polluting good. We can assume for example that the public transport of region 2 provides full coverage and that is not the case in the other region. Thus, the need of polluting good in region 1 is higher ($\triangle(t_E) > 0$), in order to compensated the lack of public transport coverage. Switching from region 2 to region 1 implies to be sure that the expected wage in region 1 overcomes the cost of the additional polluting good consumption.

Because we assume non explicit utility function, the model is solved through log-linearization. Appendix A.IV gives the details of the computations. The tilde $\widetilde{(\cdot)}$ denotes percentage changes relative to initial values, i.e. $\tilde{l} = \frac{dl}{l}$. Exceptions to this definition are separately indicated. We find:

Table 1. The Log-linearization solutions of the model.

Energy input of region 2	$\widetilde{e_2} = -\varepsilon_2 \widetilde{t_E}$
Tightness ratio of region 2	$\tilde{\theta}_2 = -\left[\frac{\omega_2}{\xi_2}\right]\widetilde{t_E}$
Wage of region 2	$\widetilde{w_2} = -\omega_2 \widetilde{t_E}$
Tightness ratio of region 1	$\widetilde{\theta_1} = -\left[\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1}\frac{\omega_2}{\xi_2} - \frac{\triangle(t_E)}{\theta_1 q_1(\theta_1) w_1}\right]\widetilde{t_E}$
Wage of region 1	$\widetilde{w_1} = -\xi_1\left[\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1}\frac{\omega_2}{\xi_2} - \frac{\triangle(t_E)}{\theta_1 q_1(\theta_1) w_1}\right]\widetilde{t_E}$
Energy input of region 1	$\widetilde{e_1} = -\varepsilon_1(\alpha_1)\left[\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1}\frac{\omega_2 \xi_1}{\xi_2} - \frac{\xi_1 \triangle(t_E)}{\theta_1 q_1(\theta_1) w_1} + 1\right]\widetilde{t_E}$
Ratio of prices	$\widetilde{p_1} = \left(\alpha_1 - (1-\alpha_1)\xi_1\left[\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1}\frac{\omega_2}{\xi_2} - \frac{t_E \triangle(t_E)}{\theta_1 q_1(\theta_1) w_1}\right]\right)\widetilde{t_E}$

with $\omega_2 = \frac{\alpha_2}{1-\alpha_2} = \frac{t_E e_2}{p_{L_2}}$; $\varepsilon_i = -\frac{\partial e_i}{\partial t_E}\frac{t_E}{e_i} = -\left(\frac{\partial \partial f_i}{\partial e_i}\frac{e_i}{\partial f_i}\right)^{-1} > 0$; *and* $\alpha_i = \frac{\partial f_i}{\partial e_i}\frac{e_i}{f_i}$

We need to compute the variation of total workers between regions to complete the general equilibrium solutions. We want to compare our results with the results of the first subsection. Remember the denotation of the main assumptions in proposition 1: (i) preferences of households do not differ between regions; (ii) preferences for regional goods are Cobb-Douglas; (iii) the elasticity of energy input to taxes is fixed (α_i). We try to disentangle the effect of a release of each assumption on the reallocation of workers. Log-linearizing the equation (3.11) gives us:

$$\widetilde{L_1} - \widetilde{L_2} = \left(\underbrace{(\widetilde{\theta}_2 - \widetilde{\theta}_1)}_{=0\, under\, (i)} + \underbrace{(1 - \xi_{p_1})\,\widetilde{p_1}}_{=0\, under\, (ii)} \right) + \underbrace{\left(\frac{\alpha_2}{(1-\alpha_2)(1-(1-\beta_2)(1-\alpha_2))}\widetilde{\alpha_2} - \frac{\alpha_1}{(1-\alpha_1)(1-(1-\beta_1)(1-\alpha_1))}\widetilde{\alpha_1} \right)}_{=0\, under\, (iii)} \tag{24}$$

with $\widetilde{\alpha_i} = (1-(1-\alpha_i)\varepsilon_i)\widetilde{t_E}$,[14] and $\xi_{p_1} = \frac{\partial \rho(p_1)}{\partial p_1} * \frac{p_1}{\rho(p_1)}$ that stands for the relative-price elasticity of relative demand for the regional goods.

Thus, the release of one of the three assumptions is enough to lead to a reallocation of workers between regions/sectors.

The impact of assumption (iii) is very intuitive. Assumption (iii) means that elasticity of energy input is fixed. Thus ($\widetilde{\alpha_2} = \widetilde{\alpha_1} = 0$). Due to the assumption of decreasing return to scale, $\widetilde{\alpha_i}$ is always negative or nul. Thus, releasing the assumption of constant elasticity, and assuming that $\widetilde{\alpha_1} > \widetilde{\alpha_2}$, lead to a higher impact of energy tax in sector 1 than in sector 2. The production is more affected in sector1 than in sector 2. This lead to a reallocation of workers from region 1 to region 2 with no ambiguity ($\widetilde{L_2} > \widetilde{L_1} \Leftrightarrow d(\frac{L_2}{L_1}) > 0$).

With (ii), $\frac{\rho(p_1)}{p_1}$ is fixed and then $\xi_{p_1} = 1$ (by definition of the Cobb-Douglas function). Releasing this assumption, an increase of the relative price p_1 will have two distinctive impacts on the reallocation of workers. First, a raise in p_1 increases the relative productivity of labour and thus the relative wage of region 1 to region 2. This will incite workers from region 2 to move to region 1 until search and match frictions equalize expected wages. Finally, region 1 will present a higher wage but a higher unemployment rate. In the other side, an increase of p_1 decreases the relative demand for the good X_1 and thus decreases the demand of firms in region 1 at the elasticity ξ_{p_1}. Because each firm hires one worker, this second effect tends to lower the demand of workers in region 1. If the first effect dominates the second one, that is if ($1 > \xi_{p_1}$), then an increase of the relative price pushes workers to move from region 2 to region 1. Note that under (i) and (iii), $\widetilde{p_1} = \left((\alpha_1) - (1-\alpha_1)\left[\frac{\xi_1 \omega_2}{\xi_2}\right]\right)\widetilde{t_E}$. It gives $\widetilde{p_1} > 0$ if an only if $\frac{1}{\xi_1}\frac{t_E e_1}{p_{L_1}} = \frac{1}{\xi_1}\frac{\alpha_1}{(1-\alpha_1)} > \frac{1}{\xi_2}\frac{t_E e_2}{p_{L_2}} = \frac{1}{\xi_2}\frac{\alpha_2}{(1-\alpha_2)}$. It means that when the ratio of input prices in sector 1 is more intensive in pollution, an increase of green tax raises the relative price (proposition 1). Then, assuming than sector 1 is more intensive in polluting good, an increase in green taxes raises p_1 and raises L_1 if ($1 > \xi_{p_1}$). In this situation, workers reallocate from the less-intensive polluting industry to the more- polluting intensive industry. This result is similar to the one in Daitoh (2003). Green tax may increase the total labour in the more intensive energy sector, if the relative-price elasticity of relative demand for the good intensive in pollution (ξ_{p_1}) is small.

[14] $\alpha_i = \frac{\partial f_i}{\partial e_i}\frac{e_i}{f_i}$

The release of assumption (i) is new in this context. If $\triangle(t_E) = 0$, the migration condition is given by $\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1} = 1$. From the equations of the tightness ratio of the labour market (Table 1), we find immediately that $\widetilde{\theta_2} = \widetilde{\theta_1}$. The no-migration condition imposes similar variation of the labour tightness. It is no more the case as soon as we introduce a migration decision that depends on the green tax. We obtain a relative variation of the ratio of θ_i as follow:

$$\widetilde{\theta_2} - \widetilde{\theta_1} = -\frac{1}{\theta_1 q_1(\theta_1) w_1} \left[(\triangle(t_E)) \frac{\omega_2}{\xi_2} + \triangle(t_E) \right] \widetilde{t_E} < 0 \qquad if \quad \triangle(t_E) > 0. \tag{25}$$

In contrast to assumption (i) and (ii), the non-homothetic utility function ϱ impacts directly the formation of wages and unemployment. If $\triangle(t_E)$ is initially positive, the initial expected wage of region 1 is higher than the one in region 2.

Noting that $\widetilde{\theta_2} - \widetilde{\theta_1} = \widetilde{\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1}}$, equation (3.18) shows that an increase of green taxes raises the pre-existing expected wage gap between regions. Expected wages in region 1 have to overcome the additional surplus of migration cost, inducing by the raise of green taxes. Thus, it becomes relatively more costly for firms in region 1 to hire workers than for the firm 2. And finally, firms and thus workers are reallocated from sector 1 to sector 2 ($\widetilde{L_1} - \widetilde{L_2} < 0$). The difference in the subsistence level of the polluting good consumption implies necessarily a wage disparity that amplifies frictions in the region where it is the highest one. Here, if high wages gap is initially due to high labour market frictions, raising green taxes may lead to inefficient reallocation of workers and contribute to generate negative spillovers. The difference in subsistence level of production should be interpreted as an additional labour distortion. In other words, the no-migration condition, driven by wage disparities, causes an additional misallocation of labour between the two regions. Finally we have:

$$\begin{aligned} \widetilde{L_1} - \widetilde{L_2} = & -\frac{(\triangle(t_E))}{\theta_1 q_1(\theta_1) w_1} \left[\frac{\alpha_2 + (1-\alpha_2)\xi_2}{(1-\alpha_2)\xi_2} \right] [1 - (1-\xi_{p_1})\xi_1(1-\alpha_1)] \widetilde{t_E} \\ & + (1-\xi_{p_1}) \left((\alpha_1) - (1-\alpha_1) \left[\frac{\xi_1 \omega_2}{\xi_2} \right] \right) \widetilde{t_E} \\ & + \left[\frac{\alpha_2(1-\varepsilon_2(1-\alpha_2))}{(1-\alpha_2)(1-(1-\beta_2)(1-\alpha_2))} - \frac{\alpha_1(1-\varepsilon_1(1-\alpha_1))}{(1-\alpha_1)(1-(1-\beta_1)(1-\alpha_1))} \right] \widetilde{t_E} \end{aligned} \tag{26}$$

Thus, if $\triangle(t_E) > 0$ and $1 - (1-\xi_{p_1})\xi_1(1-\alpha_1) > 0$, the presence of a subsistence level of pollution always tends to reallocate workers from region 1 to region 2, independently of initial frictions in region 2. The two following propositions summarize the above arguments

Proposition 2 : *In a more general framework, assuming that sector 1 is the sector intensive in pollution, a raise of the green tax has an ambiguous impact on the reallocation of workers between regions. Workers will tend to move from region 1 to 2 the higher:*

- *the relative elasticity of the polluting good to the tax between region 1 and region 2 ($\widetilde{\alpha_1} > \widetilde{\alpha_2}$) is;*
- *the relative-price elasticity of relative demand for the regional goods (ξ_{p_1}) is;*
- *the relative subsistence level of the polluting between region 1 and 2 ($\triangle(t_E)$) is.*

Because we assume that the revenue of green taxes is recycled in the lump-sum fashion,[15] and because in this framework green taxes increase unemployment in both sectors, there is no possibility for obtaining a decrease of the general level of unemployment with an environmental policy. Still, we are able to predict if the reallocation of workers among regions, induced by green taxes, generates negative or positive spillovers on employment.

Proposition 3: *We assume that sector 1 is the relatively polluting intensive sector. Then an increase of green taxes will always lead to an inefficiency reallocation of workers that contributes to increase the global level of unemployment if and only if :*

$$\begin{aligned}
&\left[\frac{(-\triangle(t_E))}{\theta_1 q_1(\theta_1) w_1}\frac{\omega_2}{\xi_2}\right] + [(1-\xi_{p_1})\xi_1(-\triangle(t_E))] \\
&\qquad > (1-\xi_{p_1})\left(-(\xi_1+1)\omega_1 - 1\right) \\
&\qquad +\left(\frac{\alpha_2}{(1-\alpha_2)(1-(1-\beta_2)(1-\alpha_2))}\widetilde{\alpha_2} - \frac{\alpha_1}{(1-\alpha_1)(1-(1-\beta_1)(1-\alpha_1))}\widetilde{\alpha_1}\right)
\end{aligned} \tag{27}$$

If $\triangle(t_E) < 0$, there is a migration cost for workers in sector 1 to move to sector 2. Raising green taxes exacerbate this cost. Basically, workers will be stocked in region 1 and less able to move. This tends in favor of a reallocation of workers from sector 2 to 1. The impact on the relative price p_1 is ambiguous. In one side, because sector 1 is more intensive in pollution, p_1 is susceptible to raise. On the other side, the expected wages of region 2 raises due to the increase of the migration cost and tends to lower p_1. If the first effect overcome the second, assuming $\xi_{p_1} < 1$ and $(\widetilde{\alpha_1} - \widetilde{\alpha_2})$ small, then green taxes will lead to a reallocation of workers from sector 2 to 1. Yet, the unemployment tax within this region is already the highest one. The reallocation is then inefficient in term of employment.

We have shown that if we relax the standard assumptions, green taxes may impact the reallocation of workers between regions. Moreover, assuming a subsistence level of polluting consumption, green tax may induce a misleading reallocation of workers and may generate a negative spillovers.

[15]This assumption will be removed in the next section.

How to remove this inefficiency? What are the instruments needed? The next section tries to give some elements of answers.

4 First-best and second best allocation of agents

We have shown in the preceding section, that the general equilibrium of our economies suffers from search and environmental externalities. The first subsection focus on the optimal control problem and the implementation of the first-best allocation. The second sub-section deals with wages taxes and subsidies that the government can set in order to be as close as possible to the first best. We derive numerical results in order to compare the first best and the second-best policy for different scenario.

4.1 First best allocation

We assume the social planner able to impose the number of workers are in both of sectors, and the number of firms. However, the planner is assumed to take the wage and price equations as given and to be unable to directly alter payments to individuals.

We define the optimal allocation of agents as a tuple (L_i^*, F_i^*,w_i^*, p_i^*) that maximizes with respect to L_i and F_i:

$$\underset{L_1,F_1,F_2}{max} W_{tot} = \sum \left[(L_i - l_i) * V_i^u + l_i * V_i^e\right] - \bar{L}\psi \left[E_{tot}\right] \tag{28}$$

Under the wage equations $w_i^* = \left[\frac{\beta}{1-\beta}\frac{c_1}{q_1(\theta_1^*)}\right]$, and the price equation (3.11). Substituting these equations into the W, $\Pi = F_2\pi_2 + F_1\pi_1$ and $\frac{l_i}{q(\theta_i)} = F_i$ into (3.12) gives us:

$$(l_1w_1 + l_2w_2 - t_E\bar{L}) + \pi_{tot} + T\bar{L} = (p_1l_1x_1 - c_1F_1) + (l_2x_2 - c_2F_2) \tag{29}$$

$$\begin{aligned} \underset{L_1,F_1,F_2}{max} W_{tot} = & \frac{1}{P_Q}\left[(p_1l_1x_1 - c_1F_1) + (l_2x_2 - c_2F_2)\right]\left(1 - \psi\bar{L}P_Q'(t_E)\right) \\ & - \psi\bar{L}\left[l_1(t_E) + l_2(t_E) + l_2x_2\frac{\alpha_2}{t_E} + l_1x_1\frac{\alpha_1}{t_E}\right] \\ s.t \qquad & \frac{\rho(p_1)}{p_1}(p_1l_1x_1 - c_1F_1) = l_2x_2 - c_2F_2 \end{aligned} \tag{30}$$

The Lagrangian associated to this maximizing problem is the following one:

$$
\begin{aligned}
L =& \frac{1}{P_{Q(t_E)}} \left[(p_1 l_1 x_1 - c_1 F_1) + (l_2 x_2 - c_2 F_2)\right] (1 - \psi \bar{L} P'_Q(t_E)) \\
& - \psi \bar{L} \left[l_1(t_E) + l_2(t_E) + l_2 x_2 \frac{\alpha_2}{t_E} + l_1 x_1 \frac{\alpha_1}{t_E} \right] - \lambda(\frac{\rho(p_1)}{p_1} (p_1 l_1 x_1 - c_1 F_1) - l_2 x_2 - c_2 F_2))
\end{aligned}
\tag{31}
$$

The first-order conditions for maximizing W with respect to L_1 and F_1,F_2 are in the Appendix A.V. Solving these equations together gives the optimal number of workers and firms (L_i^* and F_i^*):

Corollary *If $\psi = 0$, the equilibrium does not reach the optimal allocation of agents, except if $\xi_2 = (1-\alpha_2)(1-\beta_2)$; $\xi_1 = (1-\alpha_1)(1-\beta_1)$ and $\alpha_1 = c_2 \frac{\alpha_2}{c_1}$, this means that the initial equilibrium already exhibits distortions even without environmental externalities.*

Interpretation: Remember that there are two central market failures in the matching model : congestion externalities and appropriability problems. The congestion externalities are as follows. Workers fail to internalize the fact that should they look for a job, they generate extra jobs at a rate lower than their own probability to find a job. This externality leads to too much worker search, i.e. too much unemployment. The appropriability problems come from the process of wage bargaining, when workers and firms are engaged in a process to share the surplus of accepting a job. Workers only appropriate a fraction of the private value of the jobs they find. Hence the value of looking for a job (*i.e.* the opportunity cost of working) is underestimated. This is the appropriability problem which leads to too little worker search, *i.e.* too little unemployment. Under the Hosios Condition (Hosios (1990)), the low-skilled employment equilibrium is optimal: the appropriability and congestion problems exactly balance each other. With our constant returns to scale assumption on the matching function, Here the Hosios condition is satisfied if the workers' share in the surplus of a match (β_i) times $(1-\varepsilon_i)$) is equal to the elasticity of the matching function (ξ). In this case, we show that the equilibrium reaches the optimal allocation under the Hosios condition. What is interesting here, it is that there exists a case where even if the sector the most intensive in energy presents the highest employment rate, a green tax can lead to an allocation that is closer to the optimal allocation if in its sector $(1-\varepsilon_i)(1-\beta_i) < \xi_i$. It means that the initial number of firms in this sector is initially to high.

4.2 The Second best policies: an empirical illustration

The second sub-section deals with wages taxes and subsidies that the government can set in order to be as close as possible to the first best. Due to the complexity of the model, we are not able to derive analytically results that gives good intuition. We propose then numerical results in order to compare the second-best policy for different scenario.
We assume v as a standard CES. Thus Q can be written as:

$$Q_i = \left((C_i^E, v(C_i^{X_1}, C_i^{X_2})\right) = (C_i^E - \bar{E}_i)^\gamma \left(\left(\left(\delta C_i^{X_1}\right)^\sigma \left((1-\delta)C_i^{X_2}\right)^\sigma\right)^{\frac{1}{\sigma}}\right)^{1-\gamma}$$

These last assumptions are enough to insure migration due to variations of price and the minimum of consumption. Yet for simplicity we assume $f_i(e_i, 1) = e_i^{\alpha_i}$. The government introduces s_i, a wage subvention in sectors in order to allow reallocation.
We start in a configuration where region 1 is the more intensive in pollution $\alpha_1 > \alpha_2$, the expected wage of region 1 is lower than in the region 2 due to high level of unemployment ($\beta_1 > \beta_2$) and the variation of $\bar{E}_1 < \bar{E}_2$. We want to identify the optimal wage subsidies and transfers that result from the adoption of a given environmental tax.
Table 2 presents the value of parameters of this model.

Table 2. Parameters of the model.

Energy input intensity parameters	$\alpha_1 = 0.6$; $\alpha_2 = 0.4$
labour market parameters	$\mu_1 = \mu_2 = 0.37$; $\beta_1 = 0.6 > \beta_2 = 0.5$;
labour market parameters	$\xi_1 = \xi_2 = 0.5$; $c_i = 0.1 w_i$
Consumption preferences parameter	$\gamma = 0.1$;$\sigma_1 = 0.5$;$\sigma_1 = 0.6$; $\delta = 0.5$
Minimum of polluting good	$\bar{E}_1 = 0$;$\bar{E}_2 = -0.2 * E_2$
The green tax rate	$t_E = 0.1$

We calibrate $\mu_1 = \mu_2 = 0.37$ in order to have a unemployment rate at 7% in region 2 and 14.2% in region 1. Thus for the baseline scenario (see the second row of the next table), it gives a total unemployment rate at 10%. Moreover in the baseline scenario, we fixed the environmental damage weight ($\psi(E_{tot})$) in order to have an optimal environmental tax rate at 0.01. Table 3 presents the value of optimal wages subventions/taxes, and green taxes. Value of the total employment, and the total welfare are also reported.

Table 3. optimal tax structure

	Optimal Allocation	baseline	Increases of environmental damages
t_1	-	0	-0.02
t_2	-	0	0.03
t_E	-	0.1	0.15
L_1	0.68	0.64	0.68
E_{tot}	0.22	0.27	0.25
$\frac{\bar{L}-(l_1+l_2)}{\bar{L}}$	0.7	0.1	0.11
T	3.65	3.54	4.23

As we can see, with the initial configuration, an increase of green taxes implies to subsidize wages in the region 1 and to tax wage in region 2, in order to internalize the spillovers. This can be counter-intuitive, we tax labour of the less-polluting intensive sector and we increase wage subsidies in the more polluting sector. But remember that in our model, there is a spillover effect due to energy minimum of consumption. In fact, substituting wages of region 1, will contribute to moderate the initial wage disparities due green taxes, and to increase employment in both sectors. Yet, in this case, higher employment rates are obtained at the expense of an increase of environment. This highlights the importance of the trade-off between environment and employment. Still, the minimum of consumption is lower in region 1. Consequently, the increase of pollution due the increase of employment seems to be small (0.25 vs 0.22 for the optimal allocation).

5 Conclusion

Based on the Harris-Todaro framework, our model contains several features that contribute to better understand the distribution of green taxes burden from the perspective of regional inequalities. In contrast to the previous studies in the Harris-Todaro framework, pollution is due to the use of a dirty input in the production processes of the two goods, that can also be consumed by households. Commodities tastes differ among regions and we assume non-homothetic preferences for the polluting good consumption. It allows us to represent the dirty good as a necessity. Finally, we introduced frictional unemployment in both sectors. Thus, we allow regions to differ with respect to three components: (i) the subsistence level of the dirty consumption of their residents, (ii) the pollution intensity of their production sector, and (iii) the level of frictions on their labour market.

We find that green taxes tend to decrease wages and increase unemployment in both sectors. Under non-homothety and/or non Cobb-Douglas utility function assumptions, a change of the relative price of goods is not enough to ensure the no-migration condition; a reallocation of workers between regions/sectors appears. Moreover, frictional unemployment and non-homothetic preferences bring about inter-region wages differential. Typically, non homothetic preferences introduce a cost of migration that depends on green taxes. Thus, an economy almost always exhibits distortions in the absence of the government intervention. Green taxes may exacerbate these distortions by generating spillovers, if the labour market is initially more frictional in the region where the subsistence level of the polluting good is the lowest one, and if the elasticity of the relative price is small. In consequence, the "natural" reallocation of workers in the long-run is inefficient and contributes to increase unemployment. The government needs to use other instruments in order to internalize these negative spillovers. Wages subsidies are explored as the solution to remove distortions. Simulations are done in order to compare the first best case, 'the optimal allocation of agents', with the second best situation. We find that, under some conditions on the minimum of dirty goods, it can be optimal to subsidize wages of the polluting intensive sectors, which underlines the trade-off between environment and employment. Still, we find that this instrument is not enough to overcome the total negative impact of green taxes on unemployment.

Appendix

Table A.1. Index of the variables of the model

Notation	Definition	Type
$\bar{L}$	total number of households in the economy.	exogenous
L_i	total number of households/workers in region i.	endogenous
l_i	total number of employed workers in region i.	endogenous
X_i	total production good of region i.	endogenous
x_i	the production good of region i per firm.	endogenous
E_i	aggregated energy input of region i.	endogenous
e_i	the energy input per operating firms on region i. $e_i = \frac{E_i}{l_i}$	endogenous
$C_i^{X_j}$	regional good X_j consumption of household living in region i.	endogenous
C_i^E	energy consumption of household living in region i.	endogenous
p_1	price of good X_1. It is also the relative price of regional goods.	endogenous
p_2	price of good X_2.	normalized to 1
t_E	energy tax, equivalent to the energy price.	exogenous
T	lump-sum transfers	endogenous
F_i	number of firms in region i.	endogenous
θ_i	tightness of the labour market of region i : $\theta_i = \frac{F_i}{L_i}$.	endogenous
$q_i(\theta_i)$	the probability for a firm in region i to find a worker.	endogenous
ξ_i	elasticity of the matching function: $0 < \xi_i = -\frac{\partial q(\theta_i)}{\partial \theta_i} * \frac{\theta_i}{q_i(\theta_i)} < 1$	exogenous
μ_i	the efficiency of the matching process.	exogenous
c_i	the cost of posting a vacancy for firm in the production sector i.	exogenous
I_i	income of household in region i.	endogenous
$P(t_E)$	marginal price of global consumption.	endogenous
w_i	wage of households working in sector X_i.	endogenous
β_i	bargaining power of workers in region i.	exogenous
α_i	the elasticity of the production function x_i with respect to e_i.	endogenous
ε_i	the elasticity of the production function x_i with respect to e_i.	endogenous
$\triangle(t_E)$	the difference of the energy minimum consumption between regions	endogenous

A.II : Wage bargaining

Wage of worker w_i is determined by: $w_L = argmax\left\{\left(Q_i^*\left(I_i^E\right) - Q(I_i^U)\right)^{\beta}\left(p_{Li} - w_L\right)^{1-\beta}\right\}$

where $Q * \left(I_i^E\right) = \left(\frac{w_i + T_i - t_E \bar{E}_i}{P_Q}\right)$ and $\left[Q^*(I_i^U)\right] = \left(\frac{T_i - t_E \bar{E}_i}{P_Q}\right)$

This is equivalent to $w_L = argmax\left\{\beta\left(lnQ * \left(I_i^E\right) - \left[Q^*(I_i^U)\right]\right) + (1-\beta)ln\left(p_{Li} - w_L\right)\right\}$. First order condition gives: $\beta\left[\frac{1}{P_Q\left[Q_i^{E*} - Q_i^{U*}\right]}\right] - (1-\beta)\left[\frac{1}{p_{L_i} - w_L}\right] = 0$. And with equation (7) we obtain:

$P_Q\left[Q_i^{E*} - Q_i^{U*}\right] = w_i = \frac{\beta}{1-\beta} * \left[p_{L_i} - w_{Li}\right] = \frac{\beta}{1-\beta} * \left[\frac{c_i}{q_i(\theta_i)}\right]$ (A.1)

A.III : The general equilibrium: The Cobb-Douglas example.

$$Q_i = q\left((C_i^E, v(C_i^{X_1}, C_i^{X_2})\right) = (C_i^E - \bar{E})^\gamma\left(\left(C_i^{X_1}\right)^\sigma\left(C_i^{X_2}\right)^{1-\sigma}\right)^{1-\gamma}$$

From the first-order conditions of the maximization of Q, we obtain the uncompensated demand for good $C_i^{X_1}$, $C_i^{X_2}$ and C_i^E and the indirect utility of consumption.

$$C_i^{E*} = \frac{\gamma}{t_E}\left[I_i - t_E\bar{E}_i\right] + \bar{E}_i$$

$$C_i^{X_1*} = (1-\gamma)\frac{1}{\sigma p_1}\left[I_i - t_E\bar{E}_i\right] = \frac{(1-\sigma)\sigma}{\sigma p_1}C_i^{X_2*}$$

$$Q_i^* = \frac{\left[I_i - t_E\bar{E}_i\right]}{P_Q}$$

where P_Q represents the marginal price of global consumption defined as : $\left(\frac{t_E}{\gamma}\right)^\gamma\left[\frac{(\sigma p_1)^\sigma(1-\sigma)^{1-\sigma}}{(1-\gamma)}\right]^{1-\gamma}$. Thus:

$$\frac{C_i^{X_1*}}{C_i^{X_2*}} = \frac{\sigma p_1}{(1-\sigma)}$$

A.IV : The general equilibrium: The log-linearization.

We start to solve the model with equation (5) for the region 2 (the price is numeraire). We have thus: $\frac{\partial f_2\left(e_2^*,1\right)}{\partial e_2} = t_E$. The log-linearization of this equation gives:

$$\frac{\partial\partial f_2\left(e_2^*,1\right)}{\partial e_2} * \frac{e_2}{\partial f_2\left(e_i^*,1\right)}\frac{\partial f_2\left(e_2^*,1\right)}{e_2}\widetilde{e_2} = dt_E$$

$$\frac{\partial\partial f_2\left(e_2^*,1\right)}{\partial e_2} * \frac{e_2}{\partial f_2\left(e_2^*,1\right)}\widetilde{e_2} = \widetilde{t_E}$$

We find:

$$\widetilde{e_2} = -\varepsilon_2\widetilde{t_E}$$

where$\varepsilon_2 = -\left(\frac{\partial\partial f_2\left(e_2^*,1\right)}{\partial e_2} * \frac{e_2}{\partial f_2\left(e_2^*,1\right)}\right)^{-1} = -\frac{\partial e_2}{\partial t_2} * \frac{t_2}{e_2}$¡0;

Using the definition of the productivity of labour ($p_{Li} = p_i x_i - t_E e_2$), we can find directly the relative variation of the productivity $\widetilde{p_{L_2}}$:

$$\widetilde{p_{L2}} = \left(\frac{1}{p_{L2}}\right)\left(\frac{\partial f_2\left(e_2^*,1\right)}{\partial e_2}\frac{e_2}{f_2\left(e_2^*,1\right)}f_2\left(e_2^*,1\right)\widetilde{e_2} - t_E e_2(\widetilde{e_2}+\widetilde{t_E})\right) \tag{32}$$

And noting that : $\frac{\partial f_2\left(e_2^*,1\right)}{\partial e_2}\frac{e_2}{f_2\left(e_2^*,1\right)}f_2\left(e_2^*,1\right) = t_E e_2$ (equation 6), we finally have:

$$\widetilde{p_{L2}} = -\frac{t_E e_2}{p_{L2}}\left(\widetilde{t_E}\right) = -\omega_2\left(\widetilde{t_E}\right)$$

We know from equation (8), that $w_2 = \beta p_{L2}$ that gives immediately: $\widetilde{w_2} = \widetilde{p_{L2}}$. Moreover, with equation (8), we have $w_2 = \left(\frac{\beta}{1-\beta}\right)\frac{c_2}{q(\theta_2)}$. The log-linearization of the previous equation gives:

$$dw_2 = -\left(\frac{\beta}{1-\beta}\right)\frac{c_2}{q(\theta_2)}\frac{\partial q(\theta_2)}{\partial\theta_2} * \frac{\theta_2}{q(\theta_2)}\widetilde{\theta_2}$$

Using the definition of the matching elasticity ξ_2, we have:

$$\widetilde{\theta_2} = \frac{1}{\xi_2}\widetilde{w_2} = \frac{1}{\xi_2}\widetilde{p_{L2}} = -\frac{1}{\xi_2}\frac{t_E e_2}{p_{L2}}\left(\widetilde{t_E}\right)$$

The no migration condition gives us: $\theta_1 q_1(\theta_1) * [w_1] = \theta_2 q_2(\theta_2) * [w_2] + \triangle(t_E)$

Noting that $q_i(\theta_i) * [w_i] = q_i(\theta_i) * \left(\frac{\beta}{1-\beta}\right)\frac{c_i}{q(\theta_i)} = \left(\frac{\beta}{1-\beta}\right)c_i$, we can rewrite the previous equation as:

$$\theta_1 c_1 = \theta_2 c_2 + \triangle(t_E)$$

Log-linearizing this function gives:

$$\widetilde{\theta_1}\theta_1 c_1\left(\frac{\beta}{1-\beta}\right) = \left[\widetilde{\theta_2}\theta_2 c_2\left(\frac{\beta}{1-\beta}\right) - \triangle(t_E)\widetilde{t_E}\right]$$

Replacing $\widetilde{\theta_2}$ and dividing everything by $\theta_1 c_1\left(\frac{\beta}{1-\beta}\right) = \theta_1 q_1(\theta_1) w_1$, this gives:

$$\widetilde{\theta_1} = -\left[\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1}\frac{\omega_2}{\xi_2} - \frac{\triangle(t_E)}{\theta_1 q_1(\theta_1) w_1}\right]\widetilde{t_E}$$

As previously, we have: $\widetilde{w_1} = \widetilde{\theta_1}\xi_1$ and $\widetilde{w_1} = \widetilde{p_{L1}}$. We find:

$$\widetilde{w_1} = -\xi_1 \left[\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1} \frac{\omega_2}{\xi_2} - \frac{\triangle(t_E)}{\theta_1 q_1(\theta_1) w_1} \right] \widetilde{t_E}$$

Using the definition of the productivity of labour ($p_{L1} = p_1 x_1 - t_E e_1$), we can find directly the relative variation of the price $\widetilde{p_1}$ on function of $\widetilde{p_{L_1}}$:

$$\widetilde{p_{L1}} p_{L1} = \left(p_1 f_1 \left(e_1^*, 1\right) \widetilde{p_1} + p_1 \frac{\partial f_1 \left(e_1^*, 1\right)}{\partial e_1} \frac{e_1}{f_1 \left(e_1^*, 1\right)} f_1 \left(e_1^*, 1\right) \widetilde{e_1} + - t_E e_1 (\widetilde{e_1} + \widetilde{t_E}) \right) \tag{33}$$

And noting that $p_1 f_1 \left(e_1^*, 1\right) \frac{\partial f_1 \left(e_1^*, 1\right)}{\partial e_1} \frac{e_1}{f_1 \left(e_1^*, 1\right)} = t_E e_1$ (equation 6), we finally have:

$$\widetilde{p_{L1}} p_{L1} + \widetilde{t_E} t_E e_1 = \left(p_1 f_1 \left(e_1^*, 1\right) \widetilde{p_1} \right)$$

That gives:

$$\widetilde{p_1} = \left(\left(\frac{t_E e_1}{x_1 p_1}\right) - \frac{p_{L_1}}{x_1 p_1} \xi_1 \left[\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1} \frac{\omega_2}{\xi_2} - \frac{\triangle(t_E)}{\theta_1 q_1(\theta_1) w_1} \right] \right) \widetilde{t_E}$$

The price equilibrium on the market of good is given by equation (10):

$$\left(x_1 l_1 - \frac{l_1 c_1}{p_1 q_1(\theta_1)} \right) = \frac{1}{\rho(p_1)} \left(l_2 x_2 - \frac{l_2 c_2}{q_2(\theta_2)} \right)$$

Replacing $\frac{c_i}{q_i(\theta_i)}$ with the wage equation (8) and using the definition of the labour productivity gives us:

$$(l_1 w_1) = \frac{p_1}{\rho(p_1)} \left(\frac{(1-\alpha_1)\,\beta_1}{(1-\alpha_2)\,\beta_2} \frac{[(1-\alpha_2)(\beta_2)+\alpha_2]}{[(1-\alpha_1)(\beta_1)+\alpha_1]} \right) (l_2 w_2)$$

with $\alpha_1 = \frac{\partial f_i\left(e_i^*, 1\right)}{\partial e_i} \frac{e_i}{f_i\left(e_i^*, 1\right)}$

$$\frac{L_1}{L_2} = \frac{(\theta_2 q_2(\theta_2) w_2)}{(\theta_1 q_1(\theta_1) w_1)} \frac{p_1}{\rho(p_1)} \left(\frac{(1-\alpha_1)\,\beta_1}{(1-\alpha_2)\,\beta_2} \frac{[(1-\alpha_2)(\beta_2)+\alpha_2]}{[(1-\alpha_1)(\beta_1)+\alpha_1]} \right)$$

Noting that $\widetilde{\theta_2} - \widetilde{\theta_1} = \widetilde{\frac{\theta_2 q_2(\theta_2) w_2}{\theta_1 q_1(\theta_1) w_1}}$, the log-linearization of the last equation gives

$$\widetilde{L_1}-\widetilde{L_2}=\left(\underbrace{(\widetilde{\theta_2}-\widetilde{\theta_1})}_{=0\,under\,(i)}+\underbrace{(1-\xi_{p_1})\,\widetilde{p_1}}_{=0\,under\,(ii)}+\underbrace{(\widetilde{\alpha_2}-\widetilde{\alpha_1})}_{=0\,under\,(iii)}\right) \tag{34}$$

where $\xi_{p_1}=\frac{\partial\rho(p_1)}{\partial p_1}*\frac{p_1}{\rho(p_1)}$ is the relative-price elasticity of relative demand for the regional goods (see the proof in appendix C.III). Thus,

A.V: The optimal reallocation

$$\begin{aligned}\underset{L_1,f_1,f_2}{max}\,Wtot &= \frac{1}{P_{Z(t_E)}}\left[(p_1l_1x_1-c_1f_1)+(l_2x_2-c_2f_2)\right](1-\psi\bar{L}P'_{(t_E)})\\ &-\psi\bar{L}\left[l_1\varphi_1'(t_E)+l_2\varphi_2'(t_E)+l_2x_2\frac{\epsilon_2}{t_E}+l_1x_1\frac{\epsilon_1}{t_E}\right]\\ s.t\qquad &\frac{\rho(p_1)}{p_1}(p_1l_1x_1-c_1f_1)=l_2x_2-c_2f_2\end{aligned} \tag{35}$$

$\frac{\partial W}{\partial L_1}=A\left(l_ix_1\right)^{1-\sigma}\left(l_2x_2-c_1f_1-c_2f_2\right)^{\sigma}\left((1-\sigma)\frac{\xi_1}{L_1}+\frac{\sigma l_2x_2/L_2}{l_2x_2-c_1f_1-c_2f_2}\right)-\bar{L}\psi'\left[E_{tot}\right]\left(\frac{\epsilon_1-\epsilon_2}{t_E}\right)\frac{l_2}{L_2}x_2\frac{\sigma}{1-\sigma}=0$

$\frac{\partial W}{\partial f_1}=A\left(l_ix_1\right)^{1-\sigma}\left(l_2x_2-c_1f_1-c_2f_2\right)^{\sigma}\left((1-\sigma)\frac{(1-\xi_1)}{f_1}+\frac{\sigma c_1}{l_2x_2-c_1f_1-c_2f_2}\right)-\bar{L}\psi'\left[E_{tot}\right]\left(\frac{\epsilon_1}{t_E}\right)\frac{\sigma}{1-\sigma}=0$

$\frac{\partial W}{\partial f_2}=A\left(l_ix_1\right)^{1-\sigma}\left(l_2x_2-c_1f_1-c_2f_2\right)^{\sigma}\left[(1-\xi_2)\,x_2l_2+cf_2\right]-\bar{L}\psi'\left[E_{tot}\right]\left(\frac{\epsilon_2}{t_E}\right)\frac{\sigma}{1-\sigma}=0$

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