Recycling under environmental, climate and resource constraints

Etienne Lorang[*][†], Gilles Lafforgue[‡]

Abstract

This paper investigates the effect of climate change in an industrial sector with a recycling technology available. We build a centralized model with resource, waste and climate constraints and we analyze the optimal use of two resources: a virgin input and a recycled one. Both resources have different GHG emissions rates and therefore recycling is sometimes an opportunity two reduce the impact of consumption but can still affect the environment. We discuss the various optimal trajectories that can occur, depending on the choice of exogenous constants in our model and we show that two complementary parameters should be considered to improve the environmental efficiency: the emissions ceiling and the recovery rate.

---

1. Introduction

1.1. Context and motivation

The principal motivation for recycling has been the saving of extracted resources for a long time, and has been an important focus of economists in early studies on secondary materials economics, up until the 21st century and our current resources constraints [25]. To the opportunity of reducing resources constraints, we can add the mitigation of waste pollution and its increasing impacts on the environment and countries budgets. For these reasons, the concept of a circular economy, for which recycling is one of the cornerstones, is a solution for a more sustainable economical model, as formalized by William McDonough and Michael Braungart in their book *Cradle to Cradle* [5]. This concept generated a significant amount of "grey literature", through many non-governmental organization like *Institut de l'économie circulaire* and *Orée* in France or the Ellen McArthur Foundation on a more global scale. Besides, these grey literature and academic literature are especially concentrated in Europe and Asia, two areas with many implemented drivers in this field, both in academic domain and institutional initiatives [7].

The study of recycling can be extended when considering climate change as an additional externality. This extension of the problem leads to new arbitrations: recycling is in most cases a way of reducing the use of resources with a high carbon footprint [11] but is still the source of greenhouse gas emissions thus having an impact on climate change. It has already been highlighted that circularity and environmental issues are connected in an industrial sector, with for instance used tires [20]. In France, studies of Federec [11] and ADEME [1] quantified different impacts on GHG emissions, showing that industrial process are often highly carbon intensive compared to recycling industries.

With this in mind, we want to examine three different balances in an industrial sector: a material balance in order to examine the saving of natural resources and the reduction of waste accumulation, a carbon balance for the topic of climate change and an economic balance for the evolution of consumption. We see in fact environmental objectives of recycling going in three different directions: the saving of natural resources while its shortage could lead to economic difficulties ; the reduction of waste accumulation that is costly to manage for public and private entities and poses a threat to the environment ; the fight against climate change and especially the reduction of greenhouse gas emissions. The objective of this paper is to develop an approach embracing these issues simultaneously instead of detached problems.

---

[*] BETA, Université de Lorraine, INRA, Agro Paris Tech, 54000 Nancy, France

[†] Climate Economics Chair, Univ. Paris Dauphine, Paris, France

[‡] Toulouse Business School

1.2. Literature

Early studies from the 70s-80s already tackled resource scarcity. Smith [28] puts forward social costs linked to waste accumulation and stocks diminution. He focuses on the dynamics of waste when recycling is considered and showed that there is a trade off between private costs (labor, material) and social costs (waste accumulation, resource depletion). Dynamic models were able to draw the first economic guidelines motivating recycling. It soon added waste accumulation issues to the topic with various models to find the optimal level of pollution in an economy [24][13].

An important part of literature was later developed around the topic of green policies to promote recycling. Palmer, Sigman and Walls [22] use a static micro-economic model in order to analyze the effects of diverse economic incentives such as subsidies, waste tax and deposit-refunds. This approach gives many policy insights but only takes into account waste and recycling activities, but they have been completed with environmental effects associated with recycling and resource extraction[21][22][14]. Only Acuff and Kaffine [2] show that carbon emissions reduction is a strong incentive to increase recycling, and that green policies can be implemented with this goal. These articles add a significant contribution regarding public intervention linked to recycling activities. However this kind of static analysis omits the dynamic aspects of stocks mentioned above. A further analysis has to examine the arbitration between environmental externalities and resource depletion, extending the Acuff and Kaffine model [2] (initially being an extension of the Palmer and Walls model [22]) to a dynamic system.

A rare example of this type of work is the article of Huhtala [19], one of the few to analyze the optimal use of an exhaustible resource considering at the same time the issue of waste accumulation, resource depletion and pollutant emissions. She describes the best arbitration of labor between recycling and primary production, and designs a fitting tax-subsidy scheme to achieve it under a balanced budget. Pittel, Amigues and Kuhn [23] model a decentralized economy including a recycling activity and highlight the market failure resulting from the absence of a market for waste, despite their economical value. They provide an optimum by setting up a market for waste and subsidizing recycling activities. This dynamic approach is also the perspective of Di Vita [8][9] who assesses the possibility of an economic and welfare growth with a material constraint, thanks to investments in recycling. These articles share the use of optimal control theory, but propose different models to represent a circular economy. Boyce [4] chooses to specify a recycling stock separated from accumulated waste. He examines the dynamic of this stock when there is perfect substitution between virgin and recycled material. It enables him to describe economic arbitrations between the two material to manufacture a final good. With this Herfindhal analysis, he can depict economic paths and shows that recycling adds different sets of possible consumption paths, variations of the least-cost principle. In this paper we want to extend the study of recycling including the climate change constraint by extending Boyce model [4]. We investigate different economic paths and the influence of environmental and climate constraints on the use of an exhaustible resource.

1.3. Sketch of the model

This paper presents a model taking into account exhaustible resources constraints, waste accumulation and GHG emissions in the context of an industrial sector with a recycling branch. We do not pretend to calibrate policies to implement but we try to highlight crossed effects and eventual synergies between the fight against climate change, preservation of resources and the limitation of waste disposal. We choose to build a continuous centralized model maximizing utility of consumption of a manufactured good, using optimal control theory tools for an analytic study, supplemented by numerical simulations.

1.4. Outline

The remaining of this paper is organized as follows. Section 2 presents the model used, the optimal solution and describes the different possible scenarios of consumption. Section 3 studies the hypothesis of the model and gives some insights on the effects of a strong carbon policy and an efficient recycling technology. Section 4 concludes.

2. Model

2.1. Setup

We model an industrial sector producing a quantity $q(t)$ of a good from two different inputs at time t: an extracted one (exhaustible resource) and a recycled one (semi-renewable resource), of relative quantities $v(t)$ and $r(t)$. We assume that there is perfect substitution between the inputs, involving that $q(t) = v(t) + r(t)$.

Gross utility from consumption of the final good $q(t)$ is given by the function $u(q(t))$. Function $u(.)$ follows the Inada conditions[1] and standard hypothesis: of class C^2, increasing ($u' > 0$) and concave ($u'' < 0$). Here we do not differentiate utility drawn from the consumption of a good made out of virgin material and a good made out of recycled material, thus following the above hypothesis of perfect substitution in the production function[2]. We also add constant private marginal costs of production for both inputs, respectively c_v and c_r, constant over time. We assume that the marginal cost of production for the recycled input is superior to the private cost of extracting it, $c_r > c_v$. It follows the difference of maturity between the two technologies and can be observed as the recycling branch is usually not favored when social costs are not internalized.

Input flows $v(t)$ and $r(t)$ come from two different sources, a recycled stock and a virgin stock. The remaining virgin and recycled stocks at time t are given respectively by $V(t)$ and $R(t)$, and are exploited at the following rates:

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} \equiv \dot{V}(t) = -v(t),\ V(0) = V_0 > 0 \tag{1}$$

$$\frac{\mathrm{d}R(t)}{\mathrm{d}t} \equiv \dot{R}(t) = -r(t) + \beta q(t),\ R(0) = R_0 = 0 \tag{2}$$

We define two constant variables V_0 and R_0 that are the initial sizes of the stocks. V_0 is strictly positive for an arbitrage to happen during the trajectory of the economy. We suppose that recycling has never happened in our economy, meaning that the initial stock of recycled material is zero, $R_0 = 0$. As a matter of fact, β is an exogenous rate corresponding to the share of production collected after use, sorted and incorporated into the recycled stock. In this model, β is constant and $0 < \beta < 1$.

We introduce two environmental externalities that are emitted pollutants and waste accumulation. Both are modeled as stocks and their dynamics depend on the input flows of the model. $W(t)$ represents the stock of waste and $E(t)$ cumulative greenhouse gas emissions. Stocks dynamics are governed by the following equations:

$$\frac{\mathrm{d}W(t)}{\mathrm{d}t} \equiv \dot{W}(t) = (1-\beta)q(t) - \alpha W(t),\ W(0) = W_0 \geq 0 \tag{3}$$

$$\frac{\mathrm{d}E(t)}{\mathrm{d}t} \equiv \dot{E}(t) = \delta_v v(t) + \delta_r r(t),\ E(0) = E_0 \geq 0 \tag{4}$$

We define two constant variables W_0 and E_0 that are the initial sizes of theses pollution stocks. The waste stream $(1-\beta)q(t)$ corresponds to all that couldn't be collected and directed to the recycled stock. Moreover the waste stock is reduced by a bio-decomposition, or another natural resorption mechanism, at rate α, an exogenous constant such as $0 < \alpha < 1$. Input flows $v(t)$ and $r(t)$ contribute at rates δ_v and δ_r to GHG emissions.

For environmental externalities to be binding in our model, we define a damage function for waste accumulation and an emissions cap. The carbon budget $\bar{E}$ is the upper bound for the stock of GHG emissions. Beyond this, damage are supposed to be too high too be supported on a global scale. Emissions targets are set by international environmental efforts in order to curb global warming. As $E(t) \leq \bar{E}$, when the ceiling is reached, we cannot consume any more inputs, virgin or recycled, as both emit GHG. The cost of waste accumulation is represented by function $D(W)$, subtracted to utility at each instant. This function follows standard hypothesis: $D'(W) > 0$ and $D''(W) \geq 0$.

---

[1] $\lim_{q \to 0^+} u'(q) = +\infty$ and $\lim_{q \to \infty} u'(q) = 0$

[2] Consumers are confronted to a trade-off between a supposedly lower quality of recycled goods (sometimes not justified) and ecological awareness when purchasing goods.

2.2. Central planner optimization

In this model, the central planner maximizes the social welfare, evaluating trade-offs between consumption of and goods, resources exhaustion and pollution accumulation. We set an exogenous time-limit T to the model, corresponding to long-term climate objectives and its associated emissions ceiling, involving the constraint $E(T) \leq \bar{E}$. Social welfare is discounted at a constant rate ρ. Formally, we want to solve the following problem subject to all previously listed constraints added to the non-negativity of r and v, $v(t) \geq 0$ and $r(t) \geq 0$:

$$\max_{r,v} \int_0^T (u(v(t)+r(t)) - c_v v(t) - c_r r(t) - D(W(t)))\mathrm{e}^{-\rho t}\mathrm{d}t \tag{5}$$

This problem is solved by allocating the optimal amounts of inputs v and r at each instant. For that we use optimal control theory. First we write the current value Hamiltonian of the problem[3]:

$$\begin{aligned}\mathcal{H} = u(v+r) - c_v v - c_r r - D(W) + \lambda_V(-v) + \\ \lambda_R(\beta(r+v) - r) + \lambda_E(\delta_v v + \delta_r r) + \lambda_W((1-\beta)(r+v) - \alpha W)\end{aligned} \tag{6}$$

where λ_V, λ_R, λ_W and λ_E are the shadow prices of the virgin stock, the recycled stock, the waste stock and the emissions stock respectively. As resource stocks are public goods and emissions or waste are public bad, we can assume that $\{\lambda_V;\lambda_R\}$ are non-negative and $\{\lambda_E;\lambda_W\}$ are negative.

Adding the remaining constraints, the Lagrangian of the problem can be written:

$$\mathcal{L} = \mathcal{H} + \gamma_v v + \gamma_r r \tag{7}$$

where $\{\gamma_v;\gamma_r\}$ are the Lagrange multiplier associated to the positivity constraints on material flows.

The optimal solution for the social planner must satisfy the following first-order conditions:

$$u' = c_v + \lambda_V - \beta\lambda_R + (-\lambda_E)\delta_v + (-\lambda_W)(1-\beta) - \gamma_v \tag{8}$$

$$u' = c_r + (1-\beta)\lambda_R + (-\lambda_E)\delta_r + (-\lambda_W)(1-\beta) - \gamma_r \tag{9}$$

$$\dot{\lambda_V} = \rho\lambda_V \Leftrightarrow \lambda_V = \lambda_{V0}e^{\rho t} \tag{10}$$

$$\dot{\lambda_R} = \rho\lambda_R \Leftrightarrow \lambda_R = \lambda_{R0}e^{\rho t} \tag{11}$$

$$\dot{\lambda_W} = (\rho+\alpha)\lambda_W + D'(W) \tag{12}$$

$$\dot{\lambda_E} = \rho\lambda_E \Leftrightarrow \lambda_E = \lambda_{E0}e^{\rho t} \tag{13}$$

completed by the transversality conditions:

$$\lambda_V(T)V(T)e^{-\rho T} = 0 \tag{14}$$

$$\lambda_R(T)R(T)e^{-\rho T} = 0 \tag{15}$$

$$\lambda_W(T)W(T)e^{-\rho T} = 0 \tag{16}$$

$$\lambda_E(T)e^{-\rho T}(\bar{E} - E(T)) = 0 \tag{17}$$

and the complementary slackness conditions:

$$\gamma_v v = 0 \tag{18}$$

$$\gamma_r r = 0 \tag{19}$$

First order conditions (8) and (9) give the equality of the marginal utility of using each type of resource and a full marginal cost. As long as γ_v (respectively γ_r is strictly positive, it is not optimal to use the

---

[3] although it is not specified, all control, state and co-state variables are functions of time

resource. If a particular input is used ($\gamma_v = 0$ and/or $\gamma_r = 0$ according to slackness conditions (18) and (19)), we see that the marginal utility of consumption of the good must be equal to: the private cost of the input; the social cost of reducing the stock of input; the social cost of replenishing the recycled stock (a negative cost, a stock being a good to society); the social cost of waste accumulation; the social cost of cumulative emissions. We can already determine the motivation of recycling as implemented in this resource model: relaxing the resource constraint by replenishing the recycled stock, reducing waste accumulation and potentially reducing GHG emissions if the virgin input is more polluting than the recycled one, $\delta_v > \delta_r$. However, the choice of using recycled inputs in production here only relaxes pressure on virgin stock and sometimes GHG emissions. Damage due to waste accumulation is not impacted, as the recycling process is an exogenous redirection of a share of the waste flow, and not directly a controlled flow from the waste stock.

Equations (14) to (17) rule the dynamics of the system. (14) and (15) show in particular the Hotelling rule, the social cost of the two resources growing at rate ρ. We define λ_{V0} and λ_{R0} the initial values of both social costs. If V is an exhaustible resource, R is however a semi-renewable resource, as the stock can be renewed at a certain extent depending on initial sizes of both stocks V and R. Finally, as waste and GHG emissions are public bads, their associated shadow costs are negative. We define λ_{E0} the initial value for the social cost of emissions.

From first-order conditions (8) and (9) we can infer that the optimum is one or more consecutive phases of production from one input or the other. Perfect substitution here does not allow interior solutions. Following the display of the model, we will be studying the switch from an input two the other and needed initial conditions.

2.3. Stocks dynamics

The exhaustion of the virgin and the recycled resources in (1) and (2) and initial sizes of the stocks gives us the evolution at each instant t:

$$V(t) = V_0 - \int_0^t v(s)\mathrm{d}s \tag{20}$$

$$R(t) = \beta(V_0 - V(t)) - (1-\beta)\int_0^t r(s)\mathrm{d}s \tag{21}$$

Besides, transversality conditions (14) and (15) give us a final state condition for the model. At final time T, stock V (resp. R) is either a scarce resource, meaning that it is exhausted, $V(T) = 0$ (resp. $R(T) = 0$), or it is an abundant resource and thus its social cost is zero, $\lambda_{V0} = 0$ (resp. $\lambda_{R0} = 0$).

The dynamic constraint on waste gives us the size of the waste stock at each instant:

$$W(t) = W_0 e^{-\alpha t} + (1-\beta)\int_0^t q(s)e^{-\alpha(t-s)}ds \tag{22}$$

We see here that the dynamic differs from the resources stocks as there is a self resorption factor α. Similarly, the first order condition on the waste stock (12) and the associated transversality condition (16) give the formulation of the shadow price of waste:

$$-\lambda_W(t) = \int_t^T D'(W)e^{-(\rho+\alpha)(s-t)}\mathrm{d}s \tag{23}$$

We see here that the initial value of waste social cost is independently given by conditions on waste, only taking into account the dynamic characteristics of the stock α and its cost for society $D(W)$. We see here that the shadow cost of waste accumulation ($-\lambda_W{>}0$) is equal to the discounted sum of marginal damage at rate $\rho + \alpha$, as waste is not only a flow but a stock that shrinks at rate α. The result obtained here is the same as result for CO_2 emissions in models considering emissions damages instead of a ceiling.

From the dynamic constraint (4) we get the evolution of the emissions stock:

$$E(t) = E_0 + \int_0^t (\delta_v v(s) + \delta_r r(s))\mathrm{d}s \leq \bar{E} \tag{24}$$

From transversality condition (17) we get a final condition on the shadow price of emissions. At final time T, the carbon budget is either exhausted, $E(T) = \bar{E}$, or its shadow cost is null, $\lambda_E(t) = 0$.

From equation (24), we know that when reaching threshold $\bar{E}$, production must stop: $v = r = 0$. The Inada condition $\lim_{q \to 0} u'(q) = +\infty$ involves that this cannot occur before the end of the program T. Therefore to be binding, the carbon budget must be saturated at the end of the program T:

$$E(T) = \bar{E} \tag{25}$$

We can assume this additional hypothesis as the focus of the paper is to take into account emissions constraints into the resource consumption system. Recycled and virgin inputs produce waste at the same rate $1 - \beta$, so GHG emissions will be the differentiating factor between the two when we are looking for a switch that is not induced by the exhaustion of a resource stock.

From dynamic constraints we can write the value of cumulative GHG emissions at each instant:

$$E(t) = E_0 + \frac{(1-\beta)\delta_v + \beta\delta_r}{1-\beta}(V_0 - V(t)) + \frac{\delta_r}{1-\beta}(-R(t)) \tag{26}$$

This expression links virgin and recycled resources stocks initially and at the end of the program to the design of a carbon policy, $\bar{E}$. For this reason, the choice of an emissions ceiling is one key element to analyze the switch of inputs.

Case 1. Total exhaustion of resources. In this case, both virgin and recycled resources are scarce and are exhausted at the end of the program. We have $V(T) = R(T) = 0$. Following (26), this hypothesis involves a very strong condition on the carbon budget, as it must be set at a precise level[4]:

$$\bar{E} = E_0 + \frac{(1-\beta)\delta_v + \beta\delta_r}{1-\beta} V_0 \tag{27}$$

This means that the exogenous decision of setting the carbon budget must meet the exact level to induce a constraint on both virgin and recycled resources, assuming that the emissions constraint is also binding. Technically, the whole emissions potential of the system is consumed. It leads to the highest possible carbon budget to be binding, and knowing other initial parameters of the model, this condition is met with a unique value of the carbon ceiling. This strong hypothesis is politically hard to meet as it would mean that the carbon budget is set following directly the resource situation, which is complex to design, and probably climatically inefficient. More realistically, the carbon budget is part of a global decision process where the characteristics of the sectors are not necessarily fully taken into account.

Case 2. No exhaustion at all. In this particular case, neither of the two resources are exhausted, meaning that $V(T) > 0$ and $R(T) > 0$. Thus there are no constraints on resources, their social value is always zero. We get:

$$\forall t \in [0;T], \lambda_V(t) = \lambda_R(t) = 0 \tag{28}$$

This case is not the focus of this paper as no physical constraint on resources appears here.

---

[4] A higher value is also compatible with a total exhaustion, but the carbon budget would not be exhausted.

Case 3. Recycled resource is scarce. In that case it means that the recycled resource is exhausted at time T: $R(T) = 0$. Moreover, the virgin stock is not exhausted at the end of the program. We can get the remaining stock by expressing (26) at T:

$$V(T) = V_0 - \frac{(1-\beta)(\bar{E} - E_0)}{(1-\beta)\delta_v + \beta\delta_r} \tag{29}$$

Moreover, the transversality condition we exposed above tells us that the social value of the recycled stock is zero during the program (no binding constraint) and the social value of the virgin resource is positive (binding scarcity constraint):

$$\forall t \in [0;T], \begin{cases} \lambda_V(t) = 0 \\ \lambda_R(t) > 0 \end{cases} \tag{30}$$

Case 4. Virgin resource is scarce. This case is symmetrical to the previous one. We get the conditions :

$$\forall t \in [0;T], \begin{cases} \lambda_V(t) > 0 \\ \lambda_R(t) = 0 \end{cases} \tag{31}$$

2.4. Possible paths for an industry

2.4.1. Trajectories

In order to get the optimal consumption path, we have to compare the full marginal costs (FMCs) of using each specific input (FMC_i with $i \in \{v;r\}$), given by (8) and (9):

$$FMC_v = c_v + \lambda_V - \beta\lambda_R + \delta_v(-\lambda_E) + (1-\beta)(-\lambda_W) \tag{32}$$
$$FMC_r = c_r + (1-\beta)\lambda_R + \delta_r(-\lambda_E) + (1-\beta)(-\lambda_W) \tag{33}$$

The dynamic evolution of social costs tells us the equality $FMCv = FMCr$ can be reached, if reached, at most at one instant. We notice that the waste stock does not come into play during the arbitration. In fact, products made out of recycled or virgin materials have the same impact on waste accumulation, as factor β does not change with the composition of the final good in our model. Finally, Herfindhal least-cost principle analysis of the input use [16] can be resumed to the comparison between $c_v + \lambda_V + \delta_v(-\lambda_E)$ and $c_r + \lambda_R + \delta_r(-\lambda_E)$[5].

We can define two different types of sub-trajectories, depending on the comparison between FMCs. Besides, there can be only one instant in $[0;T]$ with a production coming from both virgin and recycled inputs, and its existence is not guaranteed. Let us call this time of switch $\tilde{T}$, defined by the solution, if it exists, of the following equation:

$$\lambda_V(\tilde{T}) + (\delta_v - \delta_r)(-\lambda_E(\tilde{T})) = \lambda_R(\tilde{T}) + (c_r - c_v) \tag{34}$$

Each sub-trajectory is defined by a conditions on private and social costs, and a value for the FMC of the input used, coming from first order conditions (8) and (9). As we discuss it below, only two paths exist, due to the absence of interior solution for the use of inputs in our model.

$\mathcal{V}$ - Production from a virgin input. For this case, we have $\gamma_v = 0$ and $\gamma_r \neq 0$. It means the following inequality and first order condition:

$$u' = c_v + \lambda_V - \beta\lambda_R + \delta_v(-\lambda_E) + (1-\beta)(-\lambda_W) \tag{35}$$
$$\lambda_V + (\delta_v - \delta_r)(-\lambda_E) < (c_r - c_v) + \lambda_R \tag{36}$$

---

[5] Note that we supposed $\bar{E}$ is reached and not all resources are exhausted, meaning that at least $\lambda_V = 0$ or $\lambda_R = 0$.

$\mathcal{R}$ - Production from a recycled input. With the same reasoning we have $\gamma_r = 0$ and $\gamma_v \neq 0$ thus the following expressions:

$$u' = c_r + (1-\beta)\lambda_R + \delta_r(-\lambda_E) + (1-\beta)(-\lambda_W) \tag{37}$$

$$\lambda_R + (c_r - c_v) < \lambda_V + (\delta_v - \delta_r)(-\lambda_E) \tag{38}$$

From equation (34), we know that the optimal trajectory can include a switch from one path to the other, depending on the existence of $\tilde{T}$. Moreover, the solution can give a value of $\tilde{T}$ outside of bounds $[0;T]$ of our model, making the switch non relevant.

2.4.2. Optimal paths

In this part, systems giving the solutions for endogenous variables λ_V, λ_R, λ_W, λ_E and $\tilde{T}$ are given in Appendix. Note that for consumption paths with a single input, the definition of $\tilde{T}$ is not relevant ($\tilde{T} \notin [0;T]$). The discussion can be illustrated by resolutions of the model with appropriate parameters, all specified in Appendix.

As the recycled stock is initially empty, we can already dismiss optimal trajectories with an initial phase of consumption from R.

Virgin resource only. This path occurs when the FMC of using the recycled resource is always higher. As we took the hypothesis that there is a constraint on at least one resource stock, there must be the condition $V(T) = 0$ therefore $\lambda_V \neq 0$. This path must follow: full exhaustion of the virgin stock and full exhaustion of the carbon budget. From (36) we know that $\forall t \in [0;T]$, $\lambda_V + (\delta_v - \delta_r)(-\lambda_E) < (c_r - c_v)$. A trigger for this optimal trajectory could be very high recycling costs, for example in a not very mature industry. However combining both exhaustion hypothesis is unlikely to happen, as it would mean that the carbon budget is precisely set to meet total use of the virgin stock, $\bar{E} = \delta_v V_0$. For this reason, we drop one of these hypothesis on E or V.

The first situation that would achieve such an optimum is that resource stocks are not exhausted at all at T. This means that private costs difference is more important than emissions cost, $(\delta_v - \delta_r)(-\lambda_E) < (c_r - c_v)$. It concerns an industry with very high recycling costs and/or similar emissions rates, potentially combined with a carbon budget low enough to be exhausted. There are two situations, differentiated in growth rates of FMCs. Figure 1 shows the example of emissions rates that are close, but with slightly higher for the virgin input. The growth rate of FMC_v is higher but not enough to attain $\tilde{T}$ before T. For this reason, it does not compensate the private extra-cost for recycling, though the cost difference decreases with time. We get a similar result when emission rates for virgin inputs are slightly lower, but if so, FMCs do not converge, as seen on Figure 2. A practical example would be the paper and cardboard industry, where recycling is actually more carbon-intensive than producing with non-recycled wood pulp (see following section).

The second situation is when the carbon ceiling is not reached at T. At every instant, the private cost of producing the recycled input is too high, so never using it as an input is optimal: $\lambda_V + c_v < c_r$. This can also be combined with the non-exhaustion of the virgin resource, meaning that the optimal path of consumption is influenced neither by GHG emissions nor by resource scarcity. In this case, not only recycling is not useful to mitigate the effect of climate change, but also the carbon policy has no impact on conventional production.

For the following solutions, the GHG cap is supposed to be attained.

Virgin then recycled resource. This scenario is the most intuitive when investigating the optimal use of resources. However, environmental effects are not always causing the switch of input, especially for the case of virgin resource exhaustion. In this situation, the first phase is due to inequality $\lambda_V + (\delta_v - \delta_r)(-\lambda_E) < (c_r - c_v)$, and the change of sign (the switch) is caused not only by the emission cap but also by the lack of virgin input. In this case, environmental concerns are not necessarily very limiting, unlike the exhaustion of the stock, $V(\tilde{T}) = 0$.

The most interesting case would be the exhaustion of the recycled resource. In this case, the optimal planner sets up a first phase with a production from virgin resource, caused by important private costs for

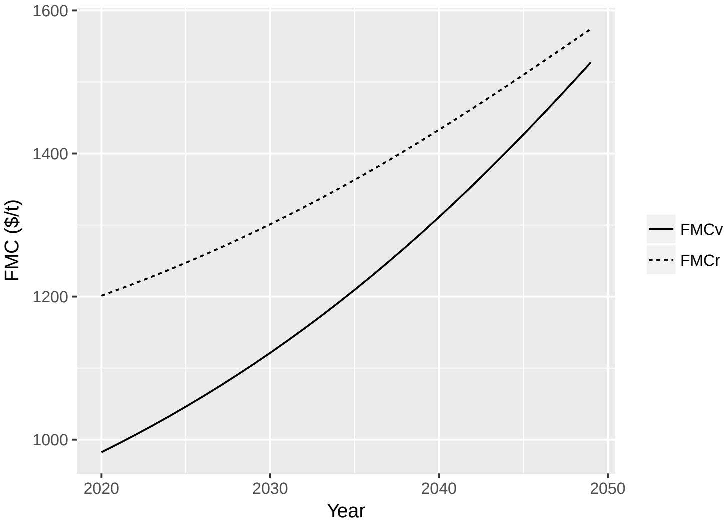

Figure 1: FMCs through time with $\delta_v/\delta_r = 1.33$

the recycling technology: $(\delta_v - \delta_r)(-\lambda_E) < (c_r - c_v) + \lambda_R$. After this phase, the social cost of emissions increases faster than the social cost of the recycled input, $(\delta_v - \delta_r)(-\lambda_{E0}) > \lambda_{R0}$, inducing an instant of switch when both inputs have the same full marginal costs: $(\delta_v - \delta_r)(-\lambda_E(\tilde{T})) = (c_r - c_v) + \lambda_R(\tilde{T})$. We can note that the virgin resource has to be the most polluting, regarding GHG emissions. Finally, the second phase is the exhaustion of the recycled resource, with lower emissions allowing the carbon budget to be respected.

The succession of these two phases are intuitively expected when dealing with a highly pollutant industry with important private costs for recycling, like the example of Figure 3, a numerical solution of the model. It is especially the case for manufacturing metals or plastic materials. With high private costs, the choice of input leans toward virgin stocks, while these industries produce large amounts of GHG emissions. Carbon budgets, if high enough, are useful to force the optimal path toward recycled inputs.

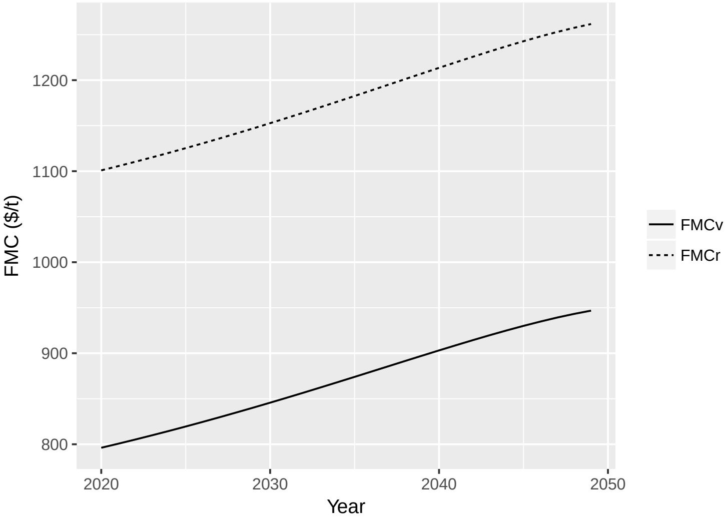

Figure 2: FMCs through time with $\delta_v/\delta_r = 0.97$

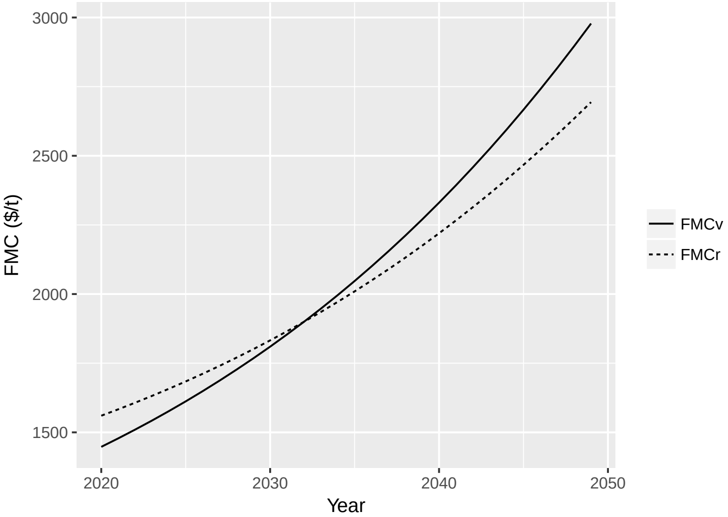

Figure 3: FMCs through time with $\delta_v/\delta_r = 3.33$

3. Discussion

3.1. Hypothesis of the model

3.1.1. Perfect substitution

Perfect substitution is a strong hypothesis here used in several models [4][17][18]. However it is known that for some raw materials, recycling lowers its quality, sometimes because different materials are mixed in the collecting process, producing a hybrid material with lower properties. Loss of quality is in fact an important topic for the academic research on circular economy, especially when the efficiency of a process is studied [12]. It is for instance the case in the paper and cardboard industry, when each recycling loop lowers the quality of the pulp and only allows a limited amount of cycles depending on the needed quality (usually 7 and 8 cycles are technically possible, but fiber is rather used 3.5 times on average in Europe [10]). There is also a common mistrust towards material coming from a waste flow, even for the same level of quality. To a certain extent, it explains the price structure of recycled material for which there is a systematic discount compared to the price of the virgin product, regardless of the cost structure. However, the manufacturer can also have a preference for recycled materials, for marketing reasons or a general pro-environment trend in the industry, leading to specific choices in favor of recycled material [27]. These two opposite motivations for the producer justify the choice of perfect substitutes. Moreover, technological progress in recycling tends to reach the same quality for virgin and recycled inputs [26].

3.1.2. Recovery rates

Parameter β in the model can be called it "recycling rate" although it does not exactly fit the definition. A more accurate definition would be a "recovery rate". The instantaneous "recycling rate" is not meaningful here, as it would be the share of input that has been sorted, processed and then re-introduced into production. Values for factor β are very diverse and depend on the industrial sector under scrutiny and the geographical scope, as seen in Table 1. Here, final waste and recycled products are definitely separated after consumption.

Material	Paper & Cardboard[a]	Plastics[b]	Aluminum[b]	Glass[b]
Recovery rate	82%	31%	79%	60%

Table 1: Example of recovery rates for France

[a] Values from Copacel 2016 [6].
[b] Values from ADEME 2014 [3]

After the first sorting step, we do not consider here the possibility of extracting waste destined to be buried or incinerated and redirect it to the recycling stock. The usual setup is a direct flow coming from the waste accumulation and influencing its dynamics [23] [19], this modeling being less common [4]. Defining a specific recycled stock puts a constraint on the recycled flow as we cannot indefinitely dip into the waste stream. We also separate two activities that are part of recycling: the decision to redirect part of the waste flow (exogenous here), and then the decision of using this input as a substitute to virgin resources.The coherence of this hypothesis depends on the industry at stake. While there is an important difference between waste and recycling flow for household solid waste in Europe, the frontier is more vague for construction waste.

3.1.3. Emissions rates

For the variety of materials this model could concern, there is an important dispersion of values for emissions rates, depending on industries and geographical scope. For instance, Table 2 shows that the aluminum industry is very carbon-intensive, and this value is even higher for countries with a fossil-fuel based electricity production. Gautam et al. work on its LCA shows emissions that go from 6 000 to more than 40 000kg CO_2e/t [15]. The case of the paper industry stands out, as the value for the recycling process is higher than the virgin one. According to the previous analysis, our model does not allow the use of the virgin input, unless there is also a constraint on the virgin resource. However, this result must raise awareness regarding the possible effect a strong carbon policy could have on the recycling industry.

Material	Paper	PET Plastics	Aluminum	Glass[a]
δ_v(kg CO$_2$e/t)	297	3 270	9 827	923
δ_r(kg CO$_2$e/t)	317	202	513	409

Table 2: Examples of emissions rates in France

Note: Values obtained from Federec[11] and [a] from ADEME, "http://www.bilans-ges.ademe.fr"

3.2. Strengthening the carbon policy

When we apply comparative dynamics to observe how the optimal trajectory reacts to a strong carbon policy, we expect to see an earlier switch of inputs. For this purpose we use standard parameters exposed in Appendix, with a ratio $\delta_v/\delta_r = 3.33$, and we analyze the effect of changing the carbon budget allocated to the sector. Starting year being 2020, we can see that lowering the carbon budget from 20 to 2 GtCO$_2$e, a 90% reduction, only leads to a switch happening 4 years earlier, a 27% reduction of the phase with virgin input production.

Carbon budget $\bar{E}$ (MtCO$_2$e)	2	4	6	8	10	12	14	16	18	20
Switching year	2031	2031	2031	2032	2032	2033	2033	2033	2034	2035

Table 3: Switching time with various carbon budgets, $\delta_v/\delta_r = 3.33$

Moreover, the evolution of the optimal path is also causing a reduction of the consumption. An early switch to recycled materials cannot support on its own a more stringent carbon policy, as the recycling technology emits GHG too. Figure 4 shows numerical solutions of total flows with different carbon budgets, highlighting the reduction of production due to a lower emissions cap. It results in a sobriety effort in the economy in order to follow the optimal path.

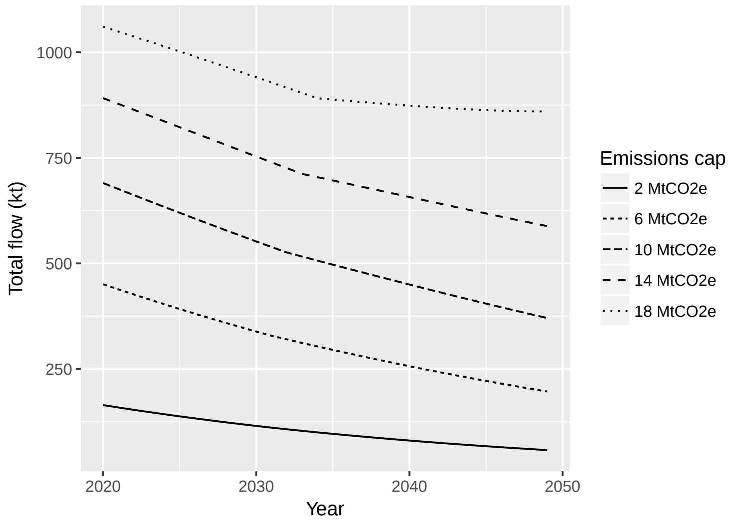

Figure 4: Material flow with various carbon budgets, $\delta_v/\delta_r = 3.33$

3.3. Increasing recycling efficiency

Noting the decrease of production when strengthening the carbon policy, we look at the recycling efficiency in order to find another lever to complete the emissions ceiling. In our model, the only parameter we can adjust in order to test the effect of the recycling efficiency is the recovery rate of the sector. With fixed parameters, we observe the effect of a variation in parameter β. Table 4 shows a wide variation in switching times when increasing β. By browsing a wide range of possible rate, we also browse almost the entire time range of the model (from 2020 to 2049). This result strongly encourage the effort toward more efficient recycling technologies in order to achieve a fast transition.

Recovery rate β	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Switching year	2045	2041	2038	2035	2032	2030	2027	2025	2022

Table 4: Switching time with various recovery rates, $\delta_v/\delta_r = 3.33$

Besides, the effect of increasing the recovery rate on the material flow is positive (Figure 5), unlike the negative effect observed with a change in climate policies. Those results encourage complementary actions on both climate policies and the efficiency of the recycling process.

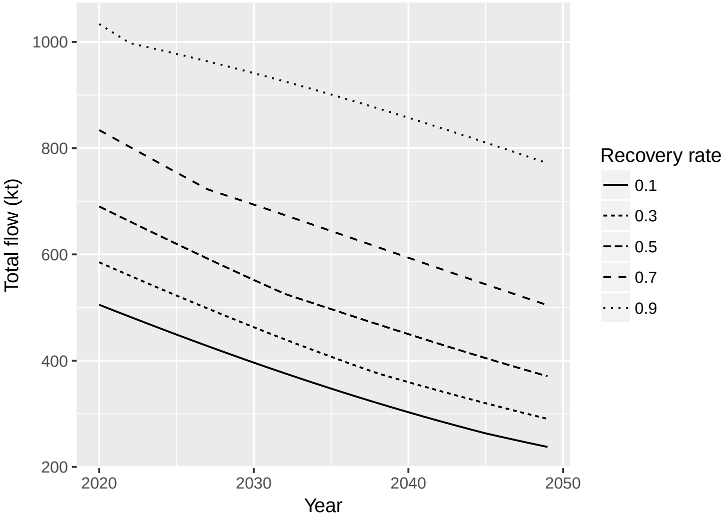

Figure 5: Material flow with various recovery rates, $\delta_v/\delta_r = 3.33$

We can observe the comparison of two scenarios with different parameters $\bar{E}$ and β, in order to assess this complementarity. Figure 6 shows this complementarity as we manage to get a material flow only slightly lower and an early switch of input, by lowering the carbon ceiling and increasing the recovery rate.

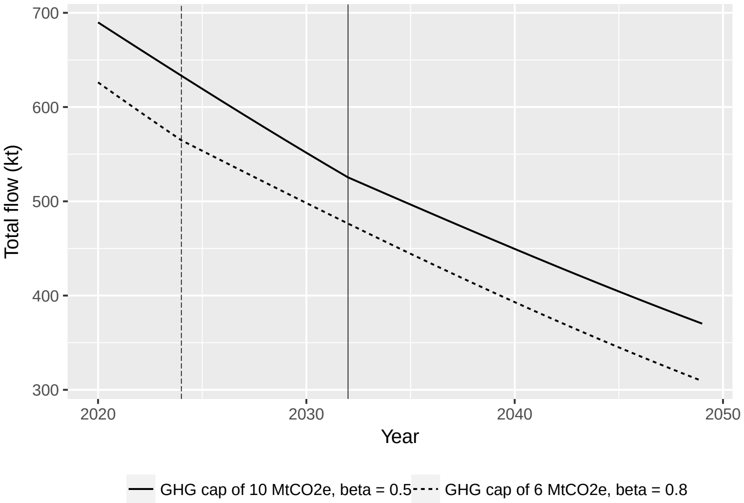

Figure 6: Material flow with different parameters, $\delta_v/\delta_r = 3.33$

4. Conclusion

We provided in this paper a discussion on optimal use of resource when climate change is an additional constraint for the social planner. Depending on initial characteristics of the industrial sector, the optimal path can be divided into phases of virgin and recycled production, with a potential switch of inputs if recycling becomes socially more profitable. Comparative dynamics are useful to highlight the effect of two exogenous parameters, the emissions ceiling and the recovery rate in the industry. As recycling still produces GHG emissions, strengthening the policy results in a decrease of production, that has to be compensated with an increase of the recovery rate.

Our model gives a better understanding of the complementarity between climate change objectives and resource issues but does not give empirical results for policy making. The following step of our analysis should be a more precise modeling of specific industrial sectors, in order to specify the need of public intervention. Besides, some sectors like paper and cardboard should do not present a preference for recycling in our model, whereas empirical data shows the opposite. For this reason, some hypothesis of the model, like perfect substitution, should be examined in detail in the context of a specific industry.

AppendixA. Total recycling capacity of the industry

Let N_0 be the total recycling capacity of the industry in the model. N_0 can be defined as the sum of the original recycling stock and the sum of all flows redirected into this stock during a program with full exhaustion:

$$N_0 = R_0 + \beta \int q(t)dt$$

By decomposing q into the two possible inputs, we get:

$$N_0 = R_0 + \beta \int v(t)dt + \beta \int r(t)dt = R_0 + \beta V_0 + \beta \int r(t)dt$$

We can also define this recycling capacity as the total recycled production: $N_0 = \int r(t)dt$

involving: $N_0 = R_0 + \beta V_0 + \beta N_0$

$$i.e.\ N_0 = \frac{\beta V_0 + R_0}{1-\beta} \tag{A.1}$$

AppendixB. Solutions of consumption paths

Here $W(t)$ is given by 22 and the inverse marginal utility.

Virgin resource only. When the virgin stock is exhausted and the emissions ceiling is not reached:

$$\begin{cases} \lambda_R = 0 \\ \int_0^T u'^{-1}(FMC_v)\mathrm{d}t = V_0 \\ \lambda_E = 0 \\ -\lambda_W = \int_t^T D'(W)e^{-(\rho+\alpha)(s-t)}\mathrm{d}s \end{cases} \tag{B.1}$$

with $FMC_v = c_v + \lambda_V + (1-\beta)(-\lambda_W)$

When the virgin stock is not exhausted and the emissions ceiling is reached:

$$\begin{cases} \lambda_R = 0 \\ \lambda_V = 0 \\ \delta_v \int_0^T u'^{-1}(FMC_v)\mathrm{d}t = \bar{E} - E_0 \\ -\lambda_W = \int_t^T D'(W)e^{-(\rho+\alpha)(s-t)}\mathrm{d}s \end{cases} \tag{B.2}$$

with $FMC_v = c_v + \delta_v(-\lambda_E) + (1-\beta)(-\lambda_W)$

Virgin then recycled resource. When the virgin stock is exhausted:

$$\begin{cases} \lambda_R = 0 \\ \int_0^{\tilde{T}} u'^{-1}(FMC_v)\mathrm{d}t = V_0 \\ \delta_v \int_0^{\tilde{T}} u'^{-1}(FMC_v)\mathrm{d}t + \delta_r \int_{\tilde{T}}^{T} u'^{-1}(FMC_r)\mathrm{d}t = \bar{E} - E_0 \\ \lambda_V(\tilde{T}) + (\delta_v - \delta_r)(-\lambda_E(\tilde{T})) = c_r - c_v \\ -\lambda_W = \int_t^T D'(W)e^{-(\rho+\alpha)(s-t)}\mathrm{d}s \end{cases} \tag{B.3}$$

with $FMC_r = c_r + \delta_r(-\lambda_E) + (1-\beta)(-\lambda_W)$ and $FMC_v = c_v + \lambda_V + \delta_v(-\lambda_E) + (1-\beta)(-\lambda_W)$

When the recycled stock is exhausted:

$$\begin{cases} \lambda_V = 0 \\ \int_{\tilde{T}}^{T} u'^{-1}(FMC_r)\mathrm{d}t = N_0 \\ \delta_v \int_0^{\tilde{T}} u'^{-1}(FMC_v)\mathrm{d}t + \delta_r \int_{\tilde{T}}^{T} u'^{-1}(FMC_r)\mathrm{d}t = \bar{E} - E_0 \\ (\delta_v - \delta_r)(-\lambda_E(\tilde{T})) = \lambda_R(\tilde{T}) + (c_r - c_v) \\ -\lambda_W = \int_t^T D'(W)e^{-(\rho+\alpha)(s-t)}\mathrm{d}s \end{cases} \tag{B.4}$$

with N_0 the total recycling capacity of the economy, as defined in A.1, in the appendix above, $FMC_r = c_r + (1-\beta)\lambda_R + \delta_r(-\lambda_E) + (1-\beta)(-\lambda_W)$ and $FMC_v = c_v - \beta\lambda_R + \delta_v(-\lambda_E) + (1-\beta)(-\lambda_W)$

AppendixC. Standard calibration for the model

We use here a NLP solver in order to compute the optimal use of inputs. The simulation is set for 30 years and divided into discrete periods of one year. The details of the constant parameters are in Table C.5. For the utility function we use a simple logarithmic function with a scalar parameter A, thus $u(q) = A\log(q)$. In the simulation we use a linear damage function for waste accumulation, with a linear parameter c_w, thus $D(W) = c_w W$.

Parameter	Value
T	30 years
ρ	0.04
β	0.5
α	0.05
δ_r	300 kg CO_2e/t
δ_v	varies between simulations
c_v	500 \$/t
c_r	800 \$/t
c_w	30 \$/t
V_0	300 000 kt
R_0	0
W_0	0
E_0	0
$\bar{E}$	10 000 000 t CO_2e
A	10 G\$

Table C.5: Constant parameters

References

[1] ADEME - Site Bilans GES http://www.bilans-ges.ademe.fr/.

[2] K. Acuff and D. T. Kaffine. Greenhouse gas emissions, waste and recycling policy. *Journal of Environmental Economics and Management*, 65(1):74–86, 2013.

[3] ADEME and Bio by Deloitte. Bilan national du recyclage 2005-2014. Technical report, ADEME, 2017.

[4] J. R. Boyce. Recycling of non-renewable resources and the least-cost-first principle. Department of Economics, University of Calgary, Working Paper, 2012.

[5] M. Braungart and W. McDonough. *Cradle to Cradle: Remaking the Way We Make Things.* North Point Press, 2002.

[6] Copacel. Rapport annuel 2016. Technical report, Union Française des Industries des Cartons, Papiers et Celluloses, 2016.

[7] A. de Jesus and S. Mendonça. Lost in transition? drivers and barriers in the eco-innovation road to the circular economy. *Ecological Economics*, 145(C):75–89, 2018.

[8] G. Di Vita. Technological change, growth and waste recycling. *Energy Economics*, 23(5):549–567, September 2001.

[9] G. Di Vita. Exhaustible resources and secondary materials: A macroeconomic analysis. *Ecological Economics*, 63(1):138–148, June 2007.

[10] ERPC. European declaration on paper recycling. Technical report, European Recovered Paper Council, 2015.

[11] FEDEREC and ADEME. Evaluation environnementale du recyclage en france selon la méthodologie de l'analyse de cycle de vie. Technical report, FEDEREC and ADEME, 2017.

[12] F. Figge, A. S. Thorpe, P. Givry, L. Canning, and E. Franklin-Johnson. Longevity and circularity as indicators of eco-efficient resource use in the circular economy. *Ecological Economics*, 150:297–306, 2018.

[13] B. A. Forster. Optimal consumption planning in a polluted environment. *Economic Record*, 49(4):534–545, 1973.

[14] D. Fullerton and T. C. Kinnaman. Garbage, Recycling, and Illicit Burning or Dumping. *Journal of Environmental Economics and Management*, 29(1):78–91, July 1995.

[15] M. Gautam, B. Pandey, and M. Agrawal. Carbon footprint of aluminum production: Emissions and mitigation. In *Environmental Carbon Footprints*, pages 197–228. Elsevier, 2018.

[16] O. C. Herfindahl. Depletion and economic theory. *Extractive resources and taxation*, pages 63–90, 1967.

[17] A. Hollander and P. Lasserre. Monopoly and the preemption of competitive recycling. *International Journal of Industrial Organization*, 6(4):489–497, 1988.

[18] R. Hoogmartens, J. Eyckmans, and S. Van Passel. A hotelling model for the circular economy including recycling, substitution and waste accumulation. *Resources, Conservation and Recycling*, 128:98–109, 2018.

[19] A. Huhtala. Optimizing production technology choices: conventional production vs. recycling. *Resource and Energy Economics*, 21(1):1–18, January 1999.

[20] G. Lonca, R. Muggéo, H. Imbeault-Tétreault, S. Bernard, and M. Margni. Does material circularity rhyme with environmental efficiency? case studies on used tires. *Journal of Cleaner Production*, 183:422–435, 2018.

[21] K. Palmer, H. Sigman, and M. Walls. The Cost of Reducing Municipal Solid Waste. *Journal of Environmental Economics and Management*, 33(2):128–150, June 1997.

[22] K. Palmer and M. Walls. Optimal policies for solid waste disposal Taxes, subsidies, and standards. *Journal of Public Economics*, 65(2):193–205, August 1997.

[23] K. Pittel, J.-P. Amigues, and T. Kuhn. Recycling under a material balance constraint. *Resource and Energy Economics*, 32(3):379–394, August 2010.

[24] C. G. Plourde. A model of waste accumulation and disposal. *The Canadian Journal of Economics*, 5(1):119–125, 1972.

[25] U. N. E. Programme. Unep 2013 annual report, 2014.

[26] L. Rouge and G. Lafforgue. A dynamic model of recycling with endogenous technological breakthrough. FAERE Working Paper 2018.14, 2018.

[27] L. Shrum, T. M. Lowrey, and J. A. McCarty. Recycling as a marketing problem: A framework for strategy development. *Psychology & Marketing*, 11(4):393–416, 1994.

[28] V. L. Smith. Dynamics of Waste Accumulation: Disposal versus Recycling. *The Quarterly Journal of Economics*, 86(4):600–616, 1972.

This research is part of the Agriculture and Forestry research program by the Climate Economics Chair. The authors want to thank the Climate Economics Chair for financial support.

The BETA contributes to the Labex ARBRE ANR-11-LABX-0002-01.