Pollution in strategic multilateral exchange: taxing emissions or trading on permit markets?[1]

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In this paper, we introduce polluting emissions in the sequential bilateral oligopoly model with a productive sector of Julien and Tricou (2012), which extends the bilateral oligopoly model of Gabszewicz and Michel (1997). We define an equilibrium concept, namely the Stackelberg Cournot equilibrium with emissions. By modelling emissions as a negative externality, we show notably that the leader pollutes more (less) than her direct follower in the presence of strategic substituability (complementarity). Thus, we study two kinds of regulation to control the levels of emissions, namely three taxation mechanisms, and a permits market. Then, we compare the two kinds of policies, and we show that preferences matter, i.e., the effectiveness of economic policy also depends on preferences.

Key Words: Bilateral oligopoly, Stackelberg-Cournot competition, pollution
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1. INTRODUCTION

Positive and normative aspects of environmental pollution under strategic interactions have been mainly developed in partial equilibrium models (Montero, 2009, De Feo et al., 2013, Hintermann, 2017) or in two-sector models where one sector embodies one single price-taking consumer (Crettez et al., 2014). In these models, the supply side in the commodity market embodies big firms whose behavior is strategic whilst the demand side for the produced good is assumed to be perfectly competitive. The competitive side is represented either by a competitive fringe (Hahn, 1984, Sartzadakis 2004) or by an auctioneer whose task is to determine the market price (Malueg and Yates, 2009, Lange 2012, Haita, 2014, among others). In addition, the working and the effects of economic policies on emissions depend on some price-taking or partial equilibrium assumptions. In this paper, we consider a simple two-stage strategic market game in which the market demand is endogenous and *all* traders behave strategically to exploit the potential gains from trade. The objectives are twofold. First, we determine whether the sequential choice affects the pollution behavior. Second, we compare two kinds of regulation to control the levels of emissions, namely three taxation mechanisms and a permits market.

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To study polluting emissions in strategic market games, we extend the bilateral oligopoly model with a finite number of traders introduced by Gabszewicz and Michel (1997), and explored notably by Bloch and Ghosal (1997), Bloch and Ferrer (2001), and Dickson and Hartley (2008). The bilateral oligopoly model is a two-commodity version of the strategic market game models (Shapley and Shubik, 1977, Sahi and Yao, 1989, and Amir et al., 1990) in which each trader has corner endowment but wants to consume both commodities. There is a market price which aggregates the strategic supplies of all traders and allocates the amounts traded to each market participant. To simplify the analysis of the strategic equilibrium, and to perform some comparative statics exercises, we propose a simple model which has a unique non-trivial strategic equilibrium. Indeed, it enables to circumvent some technical difficulties associated with the existence of an equilibrium with trade in pure exchange models. Indeed, the existence of an active equilibrium, i.e., an equilibrium with trade, is not ensured in strategic market games (Cordella and Gabszewicz, 1998, Busetto and Codognato, 2005).

More specifically, we introduce polluting emissions in the sequential bilateral oligopoly model with a productive sector of Julien and Tricou (2012), which extends the bilateral oligopoly model of Gabszewicz and Michel (1997). Therefore, we consider a two-stage finite bilateral oligopoly market with a productive sector in which one leader and one follower produce one commodity with a linear polluting technology. The production activities generate polluting emissions, i.e., negative externalities on the utility of the strategic traders who belong to the other sector. The strategic traders compete on quantities, and all traders try to manipulate the market price through their supplies. The finite game is a two-stage game where the players are the traders, the strategies are the supplies, and the payoffs are the utility levels they reach in the market outcome. In this note, we compute the subgame perfect Nash equilibrium of the two-stage game, namely the Stackelberg-Cournot equilibrium (SCE henceforth) with emissions. Then, we compare the SCE with the Cournot equilibrium (CE) with emissions. Finally, we determine whether some taxation policies or a competitive permit market could limit the emissions.

Some studies have been undertaken on strategic market games. Godal (2011) considers various market oligopoly structures to deal with emissions trading from autarky to Pareto optimality. Pollution permits are notably treated as one commodity traded against commodity money. Dickson and MacKenzie (2018) consider the implications of strategic trade in pollution markets by using a sequential game. In a first stage, the permit market is developed as a bilateral oligopoly model where all firms behave strategically, and their roles as buyers or sellers of permits are determined endogenously. In a second stage, firms transact competitively in a product market. But if the firms behave strategically *a la* Cournot, the comparative statics is more difficult to handle as the market outcome depends on each firm's marginal cost in relation to those of its rivals. Therefore, the overall effect of strategic behavior in the product market is ambiguous. In this note, we consider strategic behavior in the output market, and we rather focus on the effects of pollution, and on some public policies which could limit these effects.

Our contribution to the literature is twofold. First, we propose a two-stage bilateral oligopoly model to analyze the effects of pollution in interrelated market. This two-stage specification captures some specific features such as heterogeneity in market power. The comparative statics exercices, i.e., the effects of a changes in taxes and in the price of permit, differ from those obtained in the one-stage game. One salient feature is the comparative statics that exhibits either complementarity or substituability between the leader and the follower strategies. Second, our model constitutes, to the best of our knowledge, the first strategic sequential model which compares two kinds of regulations: taxation mechanisms and permit market in interrelated markets. We notably show the performance of these two tools depends upon the preferences of traders. Therefore, our analysis complements the bilateral oligopoly models with taxation (Gabszewicz and Grazzini, 1999, Grazzini, 2006), and those with a permit market (Godal, 2011, Dickson and MacKenzie, 2018).

The remaining of the paper is organized as follows. Section 2 states the model. In Section 3, we compute the Stackelberg-Cournot equilibrium. Section 4 is devoted to the study of taxation mechanisms. In Section 5, we study the effects of a permits market. In Section 6 we conclude. An Appendix collects some useful computations.

2. THE MODEL

Consider an economy with two divisible homogeneous commodities labeled X and Y. Let p_X and p_Y be the corresponding unit prices. We assume commodity Y is the numeraire, so $p_Y = 1$. The economy embodies $n+2$ agents of two types: two agents of type I, who are consumers and producers, and n agents of type II, with $n \geqslant 2$, who are pure consumers. Type I agents are indexed by i, $i \in \{1, 2\}$, and type II agents are indexed by j, $j \in \{1, ..., n\}$.

The distribution of endowments among the two agents of type I and the n agents of type II satisfies respectively:

$$\omega_i = (0, 0),\ i = 1, 2 \tag{1}$$

$$\omega_j = (0, \frac{1}{n}),\ j = 1, ..., n. \tag{2}$$

Therefore, commodity X does not exist initially and must be produced. Thus, we assume, like in Gabszewicz and Michel (1997), that type I traders have inherited some technology which specifies how to produce some amount z_i of good X with some amount k_i of good Y. The production function $F_i(k_i)$ of agent i is given by:

$$z_i = F_i(k_i) = \frac{1}{\beta_i} k_i,\ \beta_i \geqslant 1,\ i = 1, 2. \tag{3}$$

In addition, following Sanin and Zanaj (2011), (2012), and Crettez et al. (2014), we assume that, when commodity Y is transformed to obtain commodity X, it acts like a polluting input. Thus, the polluting input generates a quantity of emissions e_i as follows:[5]

$$e_i = \frac{1}{\gamma} k_i,\ \gamma > 1,\ i = 1, 2. \tag{4}$$

Therefore, X is a polluting consumption good: from (3) and (4), $z_i = \frac{\gamma}{\beta_i} e_i$.

The preferences of agents are represented by the following utility functions:

$$u(x_i, y_i) = x_j^{\alpha} . y_j^{1-\alpha},\ \alpha \in (0, 1),\ i = 1, 2; \tag{5}$$

$$u(x_j, y_j, e_1, e_2) = x_j^{\alpha} . y_j^{1-\alpha} - \mu(e_1 + e_2),\ \alpha, \mu \in (0, 1),\ j = 1, ..., n, \tag{6}$$

[5]Stokey (1998) studies the limit on emissions via an index technology.

where x (resp. y) are the amount consumed of commodity X (resp. Y), and μ is the disutility of pollution as emissions display negative externalities on the individual welfare of agent $j \in \{1, ..., n\}$. The utility function (5) is continuous, twice continuously differentiable, strictly increasing, and strictly concave in (x, y) on $\mathbb{R}^2_{++}$.

The (symmetric) competitive equilibrium of this economy is given by $p^*_X = \beta$, $(e^*_i, q^*_i) = \left(\frac{1}{2}\frac{\alpha}{\gamma}, \frac{1}{2}\frac{\alpha}{\beta}\right)$, $(z^*_i, k^*_i) = \left(\frac{1}{2}\frac{\alpha}{\beta}, \frac{\alpha}{2}\right)$, $(x^*_i, y^*_i) = (0, 0)$, and $u^*_i = 0$, for each $i = 1, 2$; and by $b^*_j = \frac{\alpha}{n}$, $(x^*_j, y^*_j) = \left(\frac{\alpha}{\beta n}, \frac{1-\alpha}{n}\right)$, and $u^*_j = \left(\frac{\alpha}{\beta}\right)^\alpha (1-\alpha)^{1-\alpha}\frac{1}{n} - \mu\frac{\alpha}{\gamma}$, for all $j = 1, ..., n$.

To this exchange economy, we associate a two-stage non-cooperative strategic market game $\mathbf{\Gamma}$. The strategy sets of traders are given by:

$$Q_i = \{(q_i, b_i) \in \mathbb{R}^2_+ : q_i \leqslant z_i, b_i = 0\},\ i = 1, 2 \tag{7}$$

$$B_j = \{(q_j, b_j) \in \mathbb{R}^2_+ : q_j = 0, b_j \leqslant \frac{1}{n}\},\ j = 1, ..., n, \tag{8}$$

where q_i (resp. b_j) represents the pure strategies of trader i (resp. trader j). The strategy q_i represents the amount of commodity X trader i sells in exchange for commodity Y. Similarly, b_j is the pure strategy of trader j. A strategy profile is a vector $(\mathbf{q}; \mathbf{b}) = (q_1, q_2; b_1, b_2, ..., b_{N_2})$, with $(\mathbf{q}; \mathbf{b}) \in \prod_i Q_i \times \prod_j B_j$. Let $\mathbf{b}_{-j}$ denote the strategy profile of all traders of type II but j. It is worth noticing that, for each i, the level of emissions e_i, with $e_i \in \mathbb{R}_+$, for each $i = 1, 2$, is not a strategy like q_i, but merely a decision variable which depends upon the technology of emissions (4), and which is linked to the production technology (3). Therefore, the leader and her direct follower do not take into consideration the emissions made by her direct rival when making their choices.[6]

The finite game $\mathbf{\Gamma}$ is a two-stage game where the players are the traders, the strategies are the supplies, and the payoffs are the utility levels they reach in the market outcome. This game displays two stages of decisions and no discounting. We also assume the timing of positions is given. No trader makes a choice in two subgames. In addition, traders meet once and cannot make binding agreements. By precluding binding agreements, we consider each trader acts independently and without communication with any of the others. Thus, $\mathbf{\Gamma}$ is a two-stage game, which embodies one simultaneous move subgame between the followers. Finally, information is assumed to be complete and perfect. Information is perfect because any leader perfectly knows the behavior of all followers, and, each follower's information set is a single decision node.[7] In each decision node, any follower makes an optimal choice, so sequential rationality prevails. As sequential rationality is common knowledge, the game is solved by backward induction.

Any strategic trader i has two decisions to make: which quantity q_i of good X to sell strategically on the market; and, which quantity of good X to produce, which through (3) and (4) determines the level of emissions e_i. Thus, her income is equal to her profit $\Pi_i(e_i, q_i) := p_X q_i - \gamma_i e_i$, $i = 1, 2$. Any strategic trader j finances her purchase of commodity X by selling some amounts of commodity Y, i.e., $y_j = \frac{1}{p_X} b_j$, and her consumption of commodity Y is given by the surplus $\frac{1}{n} - b_j$. Traders behave strategically, and are aware of their influence on the price p_X.

[6] It will be shown that emissions may be written as functions of strategies.

[7] It is worth noticing that the entire (sub)game Γ is a sequential game with perfect information: every information set is a singleton, and every node initiates a subgame. In particular, followers perfectly know the optimal strategies of leaders.

Given an $n+2$-tuple of strategies $(\mathbf{q};\mathbf{b}) \in Q_1 \times Q_2 \times B_1 ... \times B_n$, the market price p_X, for which demand balances supply, that is, $\sum_{j=1}^{n} b_j = p_X \sum_{i=1}^{2} q_i$, obtains as:

$$p_X(\mathbf{q};\mathbf{b}) = \frac{\sum_{j=1}^{n} b_j}{\sum_{i=1}^{2} q_i}. \tag{9}$$

The final allocations assign the following bundles for each type of traders:

$$\forall i \in \{1,2\} \quad (x_i, y_i) = \left(\frac{\gamma}{\beta_i} e_i - q_i, \frac{\sum_{j=1}^{n} b_j}{q_1 + q_2} q_i - \gamma e_i \right); \tag{10}$$

$$\forall j \in \{1,...,n\} \quad (x_j, y_j) = \left(\frac{q_1 + q_2}{b_j + \sum_{-j \neq j} b_{-j}} b_j, \frac{1}{n} - b_j \right). \tag{11}$$

The corresponding utility levels may be written as payoffs:

$$\forall i \in \{1,2\} \quad \pi_i(e_i, q_i, q_{-i}; \mathbf{b}) = \left(\frac{\gamma}{\beta_i} e_i - q_i \right)^{\alpha} \left(\frac{\sum_{j=1}^{n} b_j}{q_1 + q_2} q_i - \gamma e_i \right)^{1-\alpha}; \tag{12}$$

$$\forall j \quad \pi_j(e_1, e_2; \mathbf{q}; b_j, \mathbf{b}_{-j}) = \left(\frac{q_1 + q_2}{b_j + \sum_{-j \neq j} b_{-j}} b_j \right)^{\alpha} \left(\frac{1}{n} - b_j \right)^{1-\alpha} - \mu(e_1 + e_2). \tag{13}$$

3. NON-COOPERATIVE EQUILIBRIA WITH EMISSIONS

3.1. SCE: definition and computation

We define the SCE as the noncooperative equilibrium outcome of the game Γ.

Definition 1. (SCE). A Stackelberg-Cournot equilibrium of $\mathbf{\Gamma}$ is a $(n+4)$-tuple $(\tilde{\mathbf{e}}, \tilde{\mathbf{q}}; \tilde{\mathbf{b}})$, which consists of a strategy profile $(\tilde{q}_1, \tilde{q}_2; \tilde{b}_1, ..., \tilde{b}_n)$ and an emission profile $(\tilde{e}_1, \tilde{e}_2)$ such that:

a. $\pi_2(\tilde{e}_2, \tilde{q}_1, \tilde{q}_2; \tilde{b}_1, ..., \tilde{b}_n))) \geqslant \pi_2(e_2, \tilde{q}_1, q_2; \tilde{b}_1, ..., \tilde{b}_n))$, for all $e_2 \in \mathbb{R}_+$, $q_2 \in Q_2$;

b. $\pi_j(\tilde{e}_1, \tilde{e}_2, \tilde{q}_1, \tilde{q}_2; \tilde{b}_1, ..., \tilde{b}_j, ..., \tilde{b}_n)) \geqslant \pi_j(\tilde{q}_1, \tilde{q}_2; \tilde{b}_1, ..., b_j, ..., \tilde{b}_n))$, for all $b_j \in B_j$;

c. $\pi_1(\tilde{e}_1, \tilde{q}_1, \tilde{q}_2(\tilde{q}_1); \tilde{\mathbf{b}}(\tilde{q}_1)) \geqslant \pi_1(e_1, q_1, q_2(q_1); \mathbf{b}(q_1))$, for all $\mathbf{b}(q_1) \in \prod_{j=1}^{n} B_j$, and all $q_2(q_1) \in Q_2$, for all $e_1 \in \mathbb{R}_+$, $q_1 \in Q_1$.

Proposition 1. *The interior SCE strategy profiles and emissions profiles of* Γ *are given by:*

$(\tilde{q}_1, \tilde{q}_2) = \left(\frac{\alpha}{4} \frac{\beta_2}{(\beta_1)^2} \frac{n-1}{n-\alpha}, \frac{\alpha}{2\beta_1} (1 - \frac{1}{2} \frac{\beta_2}{\beta_1}) \frac{n-1}{n-\alpha} \right)$;

$(\tilde{e}_1, \tilde{e}_2) = \left(\frac{\alpha(1+\alpha)}{4\gamma} \frac{\beta_2}{\beta_1} \frac{n-1}{n-\alpha}, \frac{\alpha}{2\gamma} \frac{2\alpha\beta_1 + (1-\alpha)\beta_2}{\beta_1} (1 - \frac{1}{2} \frac{\beta_2}{\beta_1}) \frac{n-1}{n-\alpha} \right)$;

$\tilde{b}_j = \frac{\alpha}{n} \frac{n-1}{n-\alpha}$, $j = 1, ..., n$.

Proof. See Appendix A. ∎

It is possible to check that $\tilde{q}_1 \gtreqless \tilde{q}_2$, whenever $\frac{\beta_2}{\beta_1} \lesseqgtr 1$: when the leader's marginal cost is lower, she has higher market share than her direct follower.

We now explore the issue of emissions in this strategic behavior quantity setting two stage game.

Remark 1. The emissions of the leader and the follower increase with their supplies as $\tilde{e}_1 = \frac{1+\alpha}{\gamma}\beta_1\tilde{q}_1$ and $\tilde{e}_2 = \frac{2\alpha\beta_1+(1-\alpha)\beta_2}{\gamma}\tilde{q}_2$.

Remark 2. The emissions are lower in the SCE than in the competitive equilibrium: we have $\tilde{e}_1 + \tilde{e}_2 = \frac{\alpha}{\gamma}\left[\frac{1+\alpha}{4}\frac{\beta_2}{\beta_1} + (1-\frac{1}{2}\frac{\beta_2}{\beta_1})\frac{2\alpha\beta_1+(1-\alpha)\beta_2}{2\beta_1}\right]\frac{n-1}{n-\alpha} < \frac{\alpha}{\gamma}$ as $\frac{1+\alpha}{4}\frac{\beta_2}{\beta_1} + (1-\frac{1}{2}\frac{\beta_2}{\beta_1})\frac{2\alpha\beta_1+(1-\alpha)\beta_2}{2\beta_1} < 1$ and $\frac{n-1}{n-\alpha} < 1$.

Proposition 2. *In a SCE, the leader's emissions are higher (resp. lower) than the follower's emissions when the strategies of trader of type I are substitutes (resp. complements).*

Proof. We have to show that $\tilde{e}_1 \geqslant \tilde{e}_2$ if $\beta_1 \leqslant \beta_2$. Assume $\beta_1 \leqslant \beta_2$, with $\beta_2 = \delta\beta_1$, $1 \leqslant \delta \leqslant 2$. By using the expression of $\tilde{e}_1$ and $\tilde{e}_2$ in Proposition 1, we define the difference:

$$\phi \equiv \tilde{e}_1 - \tilde{e}_2, \text{ with } \phi = \frac{1+\alpha}{\gamma}\beta_1\tilde{q}_1 - \frac{2\alpha\beta_1 + (1-\alpha)\delta\beta_1}{\gamma}\tilde{q}_2.$$

Some calculations yield:

$$\phi = \frac{1}{4}\frac{\alpha}{\gamma}\frac{n-1}{n-\alpha}\{1+\alpha - [2\alpha + (1-\alpha)\delta](2-\delta)\}.$$

Then, we have $\phi \geqslant 0$ as $1 \leqslant \delta \leqslant 2$ and $\alpha \in (0,1)$. Then, $\tilde{e}_1 \geqslant \tilde{e}_2$ if $\beta_1 \leqslant \beta_2$. ∎

The source of pollution comes from the production of commodity X. Less marginal cost means higher production and more emissions. As strategies are substitutes, the leader has higher market power. Therefore, when the leader has lower marginal cost, she pollutes more than her direct follower.

We determine now the SCE allocations, and the corresponding payoffs. From (9) and Proposition 1, the market price is given by $\tilde{p}_X = 2\beta_1$. Then, by using (10)-(11), the individual allocations are given by:

$$(\tilde{x}_1, \tilde{y}_1) = \frac{\alpha}{4}\frac{\beta_2}{\beta_1^2}\frac{n-1}{n-\alpha}(\alpha, (1-\alpha)\beta_1); \tag{14}$$

$$(\tilde{x}_2, \tilde{y}_2) = \frac{\alpha}{\beta_2}\left(1 - \frac{1}{2}\frac{\beta_2}{\beta_1}\right)^2\frac{n-1}{n-\alpha}(\alpha, (1-\alpha)\beta_2); \tag{15}$$

$$(\tilde{x}_j, \tilde{y}_j) = \left(\frac{1}{2\beta_1}\frac{\alpha}{n}\frac{n-1}{n-\alpha}, \frac{1-\alpha}{n-\alpha}\right). \tag{16}$$

Then, we deduce the corresponding payoffs for each trader $i \in \{1,2\}$:

$$\tilde{\pi}_1 = \frac{\alpha^{\alpha+1}}{4}\frac{\beta_2}{\beta_1}\left(\frac{1}{\beta_1}\right)^{\alpha}(1-\alpha)^{1-\alpha}\frac{n-1}{n-\alpha}; \tag{17}$$

$$\tilde{\pi}_2 = \alpha^{\alpha+1}\left(1 - \frac{1}{2}\frac{\beta_2}{\beta_1}\right)^2\left(\frac{1}{\beta_2}\right)^{\alpha}(1-\alpha)^{1-\alpha}\frac{n-1}{n-\alpha}; \tag{18}$$

and, for each trader $j \in \{1, ..., n\}$:

$$\tilde{\pi}_j = \frac{\left(\frac{\alpha(n-1)}{2\beta_1 n}\right)^{\alpha}(1-\alpha)^{1-\alpha}}{n-\alpha} - \mu\alpha\frac{\frac{\beta_2}{2} + \left(1 - \frac{1}{2}\frac{\beta_2}{\beta_1}\right)(2\alpha\beta_1 + (1-\alpha)\beta_2)}{2\gamma\beta_1}\frac{n-1}{n-\alpha}. \tag{19}$$

The SCE is inefficient: by using (14)-(16), we see that the marginal rates of substitution differ accross traders, i.e., $MRS^i_{X/Y} = \beta_i$, $i = 1, 2$, and $MRS^j_{X/Y} = \frac{2\beta_1}{n-1}$, $j = 1, ..., n$. The reason stems from the imperfect competitive behavior of traders. The market power of traders stems from the restriction of their supplies, which increases the relative price. In addition, it is easy to check that the leader's payoff is higher than her direct follower's payoff as $\tilde{\pi}_1 - \tilde{\pi}_2 = \frac{\alpha^{\alpha+1}}{4}(\frac{1}{\delta\beta_1})^{\alpha}(1-\alpha)^{1-\alpha}\frac{n-1}{n-\alpha}[\delta^{\alpha+1} - (2-\delta)^2] \geqslant 0$, as $\alpha \in (0,1)$, and $1 \leqslant \delta \leqslant 2$. Finally, there is no Pareto domination between the SCE and the competitive equilibrium (Julien and Tricou, 2012).

3.2. Comparison with the CE

To put forward the role played by the leader, let us now determine the Cournot equilibrium (CE) with emissions in which all traders interact simultaneously.

PROPOSITION 3. *The interior CE strategy profiles and emissions profiles are given by:*

$(\hat{q}_1, \hat{q}_2) = \left(\alpha\frac{\beta_2}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha}, \alpha\frac{\beta_1}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha}\right)$

$(\hat{e}_1, \hat{e}_2) = \left(\frac{\alpha}{\gamma}\frac{\beta_2(\beta_1+\alpha\beta_2)}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha}, \frac{\alpha}{\gamma}\frac{\beta_1(\beta_2+\alpha\beta_1)}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha}\right)$

$\hat{b}_j = \frac{\alpha}{n}\frac{n-1}{n-\alpha}$, $j = 1, ..., n$.

Proof. See Appendix B for a computation of the CE. ∎

It is easy to check that when $\beta_1 = \beta_2$, the SCE coincides with the symmetric CE (Julien and Tricou, 2012).

The next proposition compares the emissions in the SCE and in the CE.

PROPOSITION 4. *The level of emissions is higher (resp. lower) in the SCE than in the CE if the strategies of the leader and the follower are substitutes (resp. complements), i.e., if the leader has lower (resp. higher) marginal cost.*

Proof. In a SCE we have:

$$\tilde{e}_1 + \tilde{e}_2 = \frac{\alpha}{4\gamma}\frac{\beta_2}{(\beta_1)^2}[4\alpha(\beta_1)^2 + 3(1-\alpha)\beta_1\beta_2 - (1-\alpha)(\beta_2)^2].$$

In a CE, we have:

$$\hat{e}_1 + \hat{e}_2 = \frac{\alpha}{\gamma}\frac{1}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha}[\alpha(\beta_1)^2 + 2\beta_1\beta_2 + \alpha(\beta_2)^2].$$

Therefore, if strategies are substitutes, we have $\beta_1 \leqslant \beta_2$. Thus, by letting $\beta_2 = \delta\beta_1$, with $1 \leqslant \delta \leqslant 2$, we deduce:

$$(\tilde{e}_1 + \tilde{e}_2) - (\hat{e}_1 + \hat{e}_2) = \frac{\alpha(1-\alpha)}{4\gamma}\frac{(\beta_1)^2\beta_2}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha}\delta(\delta-1)(5-\delta) \geqslant 0.$$

∎

In case of strategic complementarity, the follower's supply of commodity X is higher than the leader's supply. Therefore the difference $(\tilde{e}_1 + \tilde{e}_2) - (\hat{e}_1 + \hat{e}_2)$ may be negative as trader 1 produces less when she behaves as a Cournot player (see Proposition 3).

In addition, it is worth noticing that a trader pollutes more when she behaves as a leader in a SCE than when she behaves à la Cournot in a CE, provided she has lower marginal cost than her direct rival, i.e., $\tilde{e}_1 \geqslant \hat{e}_1$, whenever $\beta_1 \leqslant \beta_2$. Indeed, $\tilde{q}_1 \geqslant \hat{q}_1$ when $\frac{\beta_2}{\beta_1} \geqslant 1$. In addition, $\tilde{x}_1 - \hat{x}_1 = \frac{\alpha^2}{4} \frac{\beta_2}{(\beta_1)^2} \frac{(\beta_1-\beta_2)^2}{(\beta_1+\beta_2)^2} \frac{n-1}{n-\alpha} \geqslant 0$. Then, we have $\tilde{e}_1 \geqslant \hat{e}_1$, whenever $\beta_1 \leqslant \beta_2$. But, for the follower the sign of $(\tilde{e}_2 - \hat{e}_2)$ is undetermined. Indeed, as $\frac{\beta_2}{\beta_1} \in (0, 2]$, we have $\tilde{q}_2 \leqslant \hat{q}_2$. But $\tilde{x}_2 - \hat{x}_2 = \frac{1}{4\delta\beta_1}(\frac{\alpha}{1+\delta})^2(1-\delta)(4+\delta-\delta^2)\frac{n-1}{n-\alpha} \leqslant 0$ as $\delta \in [1, 2]$. So, we cannot conclude that $\tilde{e}_2 \geqslant \hat{e}_2$.

Remark 3. Some computations yield $\tilde{\pi}_1 \geqslant \hat{\pi}_1$ and $\tilde{\pi}_2 \leqslant \hat{\pi}_2$, whenever $\beta_1 \leqslant \beta_2$.

The problem is now to determine whether the pollution could be decreased, either with a tax mechanism or with a permit market.

4. TAXATION MECHANISMS

We now introduce three fiscal policies, namely, an ad valorem taxation on emissions, a per unit taxation, and an ad valorem taxation on strategy. Such taxes have been introduced in the bilateral oligopoly model under Cournot competition by Gabszewicz and Grazzini (1999), and Grazzini (2006). To simplify, we assume the total tax product T is used to finance some exogenous government expenditure, namely G, with $0 < G < \infty$, subject to a balanced budget rule, i.e., $T = G$.

In the first case, consider that a tax $t_i \in (0,1)$, $i = 1, 2$, is levied on the emissions of the leader and the follower, with $T \equiv t_1\tilde{e}_1 + t_2\tilde{e}_2 = G$. Given an $n+2$-tuple of strategies $(\mathbf{q}; \mathbf{b}) \in Q_1 \times Q_2 \times B_1 ... \times B_n$, and a tax system $\mathbf{t} = (t_1, t_2)$, the resulting post tax allocation is given by $(x_i, y_i) = \left(\frac{\gamma}{\beta_i}(1-t_i)e_i - q_i, \frac{\sum_{j=1}^n b_j}{\sum_{i=1}^2 q_i} q_i - \gamma(1-t_i)e_i\right)$, for each $i = 1, 2$. Therefore, from (12), the payoffs in $\mathbf{\Gamma}$ with ad valorem taxation on emissions may be written:

$$\pi_i(e_i, q_i, q_{-i}; \mathbf{b}; \mathbf{t}) = \left(\frac{\gamma}{\beta_i}(1-t_i)e_i - q_i\right)^{\alpha} \left(\frac{\sum_{j=1}^n b_j}{\sum_{i=1}^2 q_i} q_i - \gamma(1-t_i)e_i\right)^{1-\alpha}, \ i = 1, 2. \tag{20}$$

In the second case, consider a tax $\nu_i \in (0,1)$, $i = 1, 2$, is levied on the supply q_i of commodities X, with $T \equiv \nu_1 q_1 + \nu_2 q_2 = G$. Given an $n+2$-tuple of strategies $(\mathbf{q}; \mathbf{b}) \in Q_1 \times Q_2 \times B_1 ... \times B_n$ and a tax system $\boldsymbol{\nu} = (\nu_1, \nu_2)$, the resulting post tax allocation is given by $(x_i, y_i) = \left(\frac{\gamma}{\beta_i}e_i - q_i, \left(\frac{\sum_{j=1}^n b_j}{\sum_{i=1}^2 q_i} - \nu_i\right) q_i - \gamma e_i\right)$, for each $i = 1, 2$. Therefore, from (12), the payoffs in $\mathbf{\Gamma}$ with per unit taxation may be written:

$$\pi_i(e_i, q_i, q_{-i}; \mathbf{b}; \boldsymbol{\nu}) = \left(\frac{\gamma}{\beta_i}e_i - q_i\right)^{\alpha} \left[\left(\frac{\sum_{j=1}^n b_j}{\sum_{i=1}^2 q_i} - \nu_i\right) q_i - \gamma e_i\right]^{1-\alpha}, \ i = 1, 2. \tag{21}$$

In the third case, consider a tax $\tau_i \in (0,1)$, $i = 1, 2$, is levied on strategy after exchange takes place, with $T \equiv \tau_1 q_1 + \tau_2 q_2 = G$. Given an $n+2$-tuple of strategies $(\mathbf{q}; \mathbf{b}) \in Q_1 \times Q_2 \times B_1 ... \times B_n$ and a tax system $\boldsymbol{\tau} = (\tau_1, \tau_2)$, the resulting post tax allocation is given by $(x_i, y_i) = \left(\frac{\gamma}{\beta_i}e_i - q_i, \frac{\sum_{j=1}^n b_j}{\sum_{k=1}^2 (1-\tau_k)q_k}(1-\tau_i)q_i - \gamma e_i\right)$, for each

$i = 1, 2$. Therefore, from (8) and (9), the payoffs in $\mathbf{\Gamma}$ with ad valorem taxation on trades may be written:

$$\pi_i(e_i, q_i, q_{-i}; \mathbf{b}; \boldsymbol{\tau}) = \left(\frac{\gamma}{\beta_i}e_i - q_i\right)^{\alpha} \left(\frac{\sum_{j=1}^{n} b_j(1-\tau_i)}{\sum_{k=1}^{2}(1-\tau_k)q_k}q_i - \gamma e_i\right)^{1-\alpha}, \ i = 1, 2. \tag{22}$$

Proposition 5. *Consider the three taxation mechanisms in* Γ*. The interior SCE strategy profiles and emissions profiles of* Γ *may be written as:*

$(\tilde{q}_1', \tilde{q}_2') = \left(\frac{\alpha}{4}\frac{\beta_2+\nu_2}{(\beta_1+\nu_1)^2}\frac{1-\tau_1}{1-\tau_2}\frac{n-1}{n-\alpha}, \frac{\alpha}{2}\frac{1}{\beta_1+\nu_1}\frac{1-\tau_1}{1-\tau_2}\left(1-\frac{1}{2}\frac{\beta_2+\nu_2}{\beta_1+\nu_1}\frac{1-\tau_1}{1-\tau_2}\right)\frac{n-1}{n-\alpha}\right);$

$(\tilde{e}_1', \tilde{e}_2') = \left(\frac{(1+\alpha)\beta_1+\alpha\nu_1}{\gamma(1-t_1)}\tilde{q}_1', \frac{\alpha[(2\beta_1+\nu_1)(1-\tau_2)-(1-\tau_1)\nu_2)]+(1-\alpha)\beta_2(1-\tau_1)}{\gamma(1-t_2)(1-\tau_1)}\tilde{q}_2'\right);$

$\tilde{b}_j' = \frac{\alpha}{n}\frac{n-1}{n-\alpha}$, $j = 1, ..., n$.

Proof. See Appendix C. ∎

Proposition 6. *Consider the three taxation mechanisms, namely ad valorem taxes on emissions* (t_1, t_2)*, per unit taxations* (ν_1, ν_2)*, and ad valorem taxation on trades* (τ_1, τ_2)*. Then, in a SCE with taxation, (i) the leader's emissions increase with emission taxes; (ii) the leader's emissions decrease (resp. increase) with the leader per unit tax (resp. with the follower per unit tax); and, (iii) the leader's emissions decrease (resp. increase) with the leader's ad valorem tax on trades (resp. with the follower's ad valorem tax on strategies). The same holds for the follower.*

Proof. See Appendix C. ∎

It is worth noticing that there is no differentiated tax which reduces all the emission levels simultaneously (see (C14)-(C24) in Appendix C). Result (ii) and (iii) in Proposition 4 may be explained as follows. The strategy of the leader are complements (resp. substitutes) when $\beta_1 > \beta_2$ (resp. $\beta_1 < \beta_2$). But as $\tilde{p}_X(\mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}) = 2\frac{\beta_1+\nu_1}{1-\tau_1}$ (see (11) in Appendix C), the market price faced by the follower increases with the taxes ν_1 and τ_1, which leads to an increase of the emissions of the follower. The same holds for the leader when increasing ν_2 and τ_2. Therefore, taxing emissions or taxing strategies via differentiated taxes may not be the appropriate tools to regulate the pollution caused by the leader and her direct follower. This means notably that heterogeneity in strategic behavior, which entails different taxes, has some policy implications.

We are able to state the following proposition.

Proposition 7. *Assume that, in an interior SCE, the objective assigned to the regulator is to determine the optimal tax such as to minimize the sum of emissions under a balanced budget rule. Then, in an interior SCE, a uniform per unit tax is the only taxation mechanism which reduces emissions of the leader (resp. follower) whenever* $\beta_1 \leqslant \beta_2 \leqslant 2\beta_1$ *(resp.* $\beta_1 \leqslant \beta_2 \leqslant 2\beta_1$ *and* $\alpha \leqslant \frac{1}{2}$*).*

Proof. Let $F : \mathbb{R}_+^2 \to \mathbb{R}_+$, with $F(e_1, e_2) = e_1 + e_2$, be the objective function, and $H : \mathbb{R}_+^2 \to \mathbb{R}_+$, with $H(e_1, e_2; G; \kappa_1, \kappa_2) = G - (\ \kappa_1 e_1 + \kappa_2 e_2)$, with $\boldsymbol{\kappa} = (\kappa_1, \kappa_2) >> \mathbf{0}$, where $\boldsymbol{\kappa} = \{\mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}\}$. The objective assigned to the regulator is to determine the optimal tax vector $\tilde{\boldsymbol{\kappa}}$, with $\tilde{\boldsymbol{\kappa}} = \{\tilde{\mathbf{t}}, \tilde{\boldsymbol{\nu}}, \tilde{\boldsymbol{\tau}}\}$, such as to minimize the sum of emissions under the balanced budget rule. Thus, this problem may be written:

$$\min\ e_1 + e_2 \text{ s.t. } \kappa_1 e_1 + \kappa_2 e_2 = G, \text{ where } 0 < G < \infty.$$

Let the Lagrangian be $\mathcal{L}:\mathbb{R}^2_+\times\mathbb{R}^*_+$, with $\mathcal{L}(e_1,e_2;\lambda):=e_1+e_2+\lambda[G-(\kappa_1 e_1+\kappa_2 e_2)]$. In an interior SCE with taxation, the optimality conditions lead to $\tilde{\kappa}_1=\tilde{\kappa}_2=\tilde{\kappa}$ and $\tilde{\kappa}(\tilde{e}_1+\tilde{e}_2)=G$. Consider now the SCE emissions for all taxation mechanisms. By using the expressions of $\tilde{e}_1(\mathbf{t},\boldsymbol{\nu},\boldsymbol{\tau})$ and $\tilde{e}_2(\mathbf{t},\boldsymbol{\nu},\boldsymbol{\tau})$ given in Proposition 4, and by letting $t_1=t_2=t$, $\nu_1=\nu_2=\nu$, and $\tau_1=\tau_2=\tau$, we deduce:

$$\tilde{e}_1(\mathbf{t},\boldsymbol{\nu},\boldsymbol{\tau})=\frac{\alpha}{4}\frac{(1+\alpha)\beta_1+\alpha\nu}{\gamma(1-t)}\frac{\beta_2+\nu}{(\beta_1+\nu)^2}\frac{n-1}{n-\alpha};$$

$$\tilde{e}_2(\mathbf{t},\boldsymbol{\nu},\boldsymbol{\tau})=\frac{\alpha[2\alpha\beta_1+\alpha\nu+(1-\alpha)\beta_2]\left(1-\frac{1}{2}\frac{\beta_2+\nu}{\beta_1+\nu}\right)\frac{n-1}{n-\alpha}}{2\gamma(1-t)(\beta_1+\nu)}.$$

Let $\nu_1=\nu_2=0$. Therefore, we have that:

$$\tilde{e}_1(t,t)=\frac{\alpha(1+\alpha)}{4\gamma(1-t)}\frac{\beta_2}{\beta_1}\frac{n-1}{n-\alpha};$$

$$\tilde{e}_2(t,t)=\frac{\alpha[2\alpha\beta_1+(1-\alpha)\beta_2]}{2\gamma(1-t)}\frac{\left(1-\frac{1}{2}\frac{\beta_2}{\beta_1}\right)}{\beta_1}\frac{n-1}{n-\alpha}.$$

We deduce $\frac{\partial\tilde{e}_i(t,t)}{\partial t}_{|t=\tilde{t}}>0$, $i=1,2$.

Now, let $t_1=t_2=0$. Therefore, we have that:

$$\tilde{e}_1(\nu,\nu)=\alpha\frac{[(1+\alpha)\beta_1+\alpha\nu](\beta_2+\nu)}{4\gamma(\beta_1+\nu)^2}\frac{n-1}{n-\alpha};$$

$$\tilde{e}_2(\nu,\nu)=\alpha\frac{[\alpha(2\beta_1+\nu)+(1-\alpha)\beta_2]\left(1-\frac{1}{2}\frac{\beta_2+\nu}{\beta_1+\nu}\right)}{2\gamma(\beta_1+\nu)}\frac{n-1}{n-\alpha}.$$

Some computations lead to:

$$\frac{\partial\tilde{e}_1(\nu,\nu)}{\partial\nu}_{|\nu=\tilde{\nu}}=-\alpha\frac{(1+\alpha)\beta_1(\beta_2-\beta_1)+\beta_1\beta_2+[\alpha(1+\beta_2)+(1-\alpha)\beta_1-\tilde{\nu}]\tilde{\nu}}{4\gamma(\beta_1+\tilde{\nu})^3}\frac{n-1}{n-\alpha},$$

so we have $\frac{\partial\tilde{e}_1(\nu,\nu)}{\partial\nu}_{|\nu=\tilde{\nu}}<0$ if $\beta_2\geqslant\beta_1$. In addition, we get:

$$\frac{\partial\tilde{e}_2(\nu,\nu)}{\partial\nu}_{|\nu=\tilde{\nu}}=-\alpha\frac{3(1-2\alpha)(\beta_1+\tilde{\nu})\beta_2+2\beta_1\beta_2\left(1-(1-\alpha)\frac{\beta_2}{\beta_1}\right)+2\alpha\beta_1\tilde{\nu}}{4\gamma(\beta_1+\tilde{\nu})^3}\frac{n-1}{n-\alpha},$$

so we have $\frac{\partial\tilde{e}_2(\nu,\nu)}{\partial\nu}_{|\nu=\tilde{\nu}}<0$ if $\alpha\leqslant\frac{1}{2}$. ∎

The conditions under which the emissions decrease with the per unit tax differ for the leader and the follower. For the follower, the negative effect holds if commodity X is not strongly preferred to commodity Y. This result is rather different in the CE with taxation. Indeed, it is possible to show that the emissions of the leader and of the follower cannot decrease simultaneously (see Appendix D).

We compare now the individual welfares at an uniform per unit equilibrium tax. To this end, assume there exists some uniform per unit tax $\tilde{\nu}\in(0,1)$ such that $\tilde{\nu}\tilde{e}_1(\tilde{\nu},\tilde{\nu})+\tilde{\nu}\tilde{e}_2(\tilde{\nu},\tilde{\nu})=G$. Therefore, the price is given by $\tilde{p}_X(\tilde{\nu},\tilde{\nu})=2(\beta_1+\tilde{\nu})$

(see (C11) in Appendix C). The allocations of the leader and the follower are given by:

$$(\tilde{x}_1(\tilde{\nu},\tilde{\nu}),\tilde{y}_1(\tilde{\nu},\tilde{\nu})) = \frac{\alpha}{4\beta_1}\frac{\beta_2+\tilde{\nu}}{\beta_1+\tilde{\nu}}\frac{n-1}{n-\alpha}\left(\alpha,(1-\alpha)\beta_1\right); \tag{23}$$

$$(\tilde{x}_2(\tilde{\nu},\tilde{\nu}),\tilde{y}_2(\tilde{\nu},\tilde{\nu})) = \frac{\alpha}{2\beta_2}\frac{2\beta_1-\beta_2+\tilde{\nu}}{\beta_1+\tilde{\nu}}\left(1-\frac{1}{2}\frac{\beta_2+\tilde{\nu}}{\beta_1+\tilde{\nu}}\right)\frac{n-1}{n-\alpha}\left(\alpha,(1-\alpha)\beta_2\right); \tag{24}$$

and the allocation of trader $j \in [1,...,n\}$ is given by:

$$(\tilde{x}_j(\tilde{\nu},\tilde{\nu}),\tilde{y}_j(\tilde{\nu},\tilde{\nu})) = \left(\frac{1}{2(\beta_1+\tilde{\nu})}\frac{\alpha}{n}\frac{n-1}{n-\alpha},\frac{1-\alpha}{n-\alpha}\right). \tag{25}$$

Thus, the marginal rates of substitution are such that:

$$MRS^i_{X/Y} = \ \beta_i,\ i=1,2,\ MRS^j_{X/Y} = 2(\beta_1+\tilde{\nu})\frac{n}{n-1}, j=1,...,n. \tag{26}$$

Therefore, the taxation policy does not lead to a Pareto optimal allocation. The reason stems from the fact that the tax is not sufficiently strong enough to wipe out the inefficiency caused by strategic behavior (see Gabszewicz and Grazzini, 1999, Grazzini, 2006).

Finally, the next proposition considers the effect of an increase of the per unit tax on the payoffs of traders of type II.

PROPOSITION 8. *The payoffs of the traders of type II increase under per unit taxation whenever the negative effect of the per unit tax on emissions dominates the marginal decrease of indirect utility caused by the increase of the price.*

Proof. The payoff of trader $j \in \{1,...,n\}$ is given by:

$$\tilde{\pi}_j(\tilde{\nu},\tilde{\nu}) = \frac{\left(\frac{\alpha}{2(\beta_1+\tilde{\nu})}\frac{n-1}{n}\right)^{\alpha}(1-\alpha)^{1-\alpha}}{n-\alpha} - \mu\chi\frac{n-1}{n-\alpha},$$

where $\chi \equiv \frac{\alpha[(1+\alpha)\beta_1+\alpha\tilde{\nu}](\beta_2+\tilde{\nu})+[\alpha(2\beta_1+\tilde{\nu})+(1-\alpha)\beta_2](2\beta_1+\tilde{\nu}-\beta_2)}{4\gamma(\beta_1+\tilde{\nu})^2}$. Therefore, we have:

$$\frac{\partial\tilde{\pi}_j(\nu,\nu)}{\partial\nu}_{|\nu=\tilde{\nu}} = -\alpha\frac{(\frac{1}{\beta_1+\tilde{\nu}})^{\alpha+1}(\frac{\alpha(n-1)}{2n})^{\alpha}(1-\alpha)^{1-\alpha}}{n-\alpha} - \mu\left(\frac{\partial\tilde{e}_1(\nu)}{\partial\nu}_{|\nu=\tilde{\nu}}+\frac{\partial\tilde{e}_2(\nu)}{\partial\nu}_{|\nu=\tilde{\nu}}\right).$$

Therefore, we see that $\frac{\partial\tilde{\pi}_j(\nu,\nu)}{\partial\nu}_{|\nu=\tilde{\nu}} \geqslant 0$ if

$$\mu\left(\frac{\partial\tilde{e}_1(\nu,\nu)}{\partial\nu}_{|\nu=\tilde{\nu}}+\frac{\partial\tilde{e}_2(\nu,\nu)}{\partial\nu}_{|\nu=\tilde{\nu}}\right) \geqslant \alpha\frac{(\frac{1}{\beta_1+\tilde{\nu}})^{\alpha+1}(\frac{\alpha(n-1)}{2n})^{\alpha}(1-\alpha)^{1-\alpha}}{n-\alpha}.$$

This condition holds when $\alpha \leqslant \frac{1}{2}$. ∎

We now turn to the regulation of emissions with a permit market.

5. POLLUTION PERMITS MARKET

To control the pollution caused by production activities, we assume now there is a permits market. Each trader $i \in \{1, 2\}$ is initially endowed with an amount $\bar{e}_i$ of pollution permits, with $\bar{e}_1 + \bar{e}_2 = \tilde{e}_1 + \tilde{e}_2$. Therefore, the net purchases of emissions may be written as $r(\bar{e}_i - \tilde{e}_i)$, for any $i \in \{1, 2\}$. Let r be the permit price in terms of good Y. The price system is now given by $(p_X, 1, r)$. We assume perfect competition on the permits market, so the price of permits r is given (see notably Montero, 2009, and Schwartz and Stahn, 2013).[8]

We are able to state the following proposition.

PROPOSITION 9. *The interior SCE strategy profiles and emssions profiles of* Γ *with a permits market are given by:*

$(\tilde{q}_1(r), \tilde{q}_2(r)) = \left(\frac{\gamma}{\gamma+r}\tilde{q}_1, \frac{\gamma}{\gamma+r}\tilde{q}_2\right)$;

$(\tilde{e}_1(r), \tilde{e}_2(r)) = \left(\frac{\alpha}{\gamma+r}r\bar{e}_1 + \frac{\gamma}{\gamma+r}\tilde{e}_1, \frac{\alpha}{\gamma+r}r\bar{e}_2 + \frac{\gamma}{\gamma+r}\tilde{e}_2\right)$;

$\tilde{b}_j(r) = \frac{\alpha}{n}\frac{n-1}{n-\alpha}$, $j = 1, ..., n$.

Proof. See Appendix E. ∎

Remark 4. The excess supply of emissions, i.e., the quantity $(\tilde{e}_1(r) - \bar{e}_1)$, is such that $\tilde{e}_1(r) - \bar{e}_1 \gtreqless 0$ whenever $\bar{e}_1 \lesseqgtr \frac{\gamma}{\gamma+(1-\alpha)r}\tilde{e}_1$, with $\frac{\gamma}{\gamma+(1-\alpha)r} < 1$, where $(\tilde{e}_1(r) - \bar{e}_1) = \frac{\gamma}{\gamma+r}\tilde{e}_1 - \frac{\gamma+(1-\alpha)r}{\gamma+r}\bar{e}_1$, and $\tilde{e}_1 = \frac{\alpha(1+\alpha)}{4\gamma}\frac{\beta_2}{\beta_1}\frac{n-1}{n-\alpha}$.

PROPOSITION 10. *The emissions of the leader and the follower increase (resp. decrease) with the price of permits whenever* $\bar{e}_i > \frac{\tilde{e}_i}{\alpha}$ *(resp.* $\bar{e}_i < \frac{\tilde{e}_i}{\alpha}$*), for each* $i = 1, 2$.

Proof. Consider $\tilde{e}_1(r) = \frac{\alpha}{\gamma+r}r\bar{e}_1 + \frac{\gamma}{\gamma+r}\tilde{e}_1$. Some computations yield $\frac{\partial \tilde{e}_1(r)}{\partial r} = \frac{\alpha\gamma}{(\gamma+r)^2}\left(\bar{e}_1 - \frac{1+\alpha}{4\gamma}\frac{\beta_2}{\beta_1}\frac{n-1}{n-\alpha}\right)$. Then, $\frac{\partial \tilde{e}_1(r)}{\partial r} \gtrless 0$ whenever $\bar{e}_1 \gtrless \frac{1+\alpha}{4\gamma}\frac{\beta_2}{\beta_1}\frac{n-1}{n-\alpha} = \frac{\tilde{e}_1}{\alpha}$. The same calculation may be handled for $\frac{\partial \tilde{e}_2(r)}{\partial r}$. ∎

The preceding condition contained in Proposition 10 says that the increase of the level of emissions depends on the initial endowment of permits: emissions increase with the price of permits when the endowment of permits exceeds the emissions without the permits market. Indeed, by using Remark 4, the excess supply of emissions increases with the price of permits whenever $\bar{e}_i > \frac{\tilde{e}_i}{\alpha}$. In addition, when the preferences toward commodity X are low, i.e., $\alpha \to 0$, the amount of endowment $\bar{e}_1$ must be large. The same result holds in the CE case (see (F11) in Appendix F).

We determine now the SCE allocations, and the corresponding payoffs with a permit market. The market price is given by $\tilde{p}(r) = 2\beta_1\frac{\gamma+r}{\gamma}$, so the allocations are given by:

$$(\tilde{x}_1(r), \tilde{y}_1(r)) = \left(r\bar{e}_1 + \frac{\alpha}{4}\frac{\beta_2}{\beta_1}\frac{n-1}{n-\alpha}\right)\left(\frac{\alpha}{\beta_1}\frac{\gamma}{\gamma+r}, 1-\alpha\right); \tag{27}$$

$$(\tilde{x}_2(r), \tilde{y}_2(r)) = \left(r\bar{e}_2 + \alpha\left(1 - \frac{1}{2}\frac{\beta_2}{\beta_1}\right)^2\frac{n-1}{n-\alpha}\right)\left(\frac{\alpha}{\beta_2}\frac{\gamma}{\gamma+r}, 1-\alpha\right); \tag{28}$$

[8] There is a huge literature devoted to permits under strategic interactions (see Sartzadakis, 1997, Eshel, 2005, Kato, 2006, Von der Fehr, 1993, among others). Crettez et al. (2014) considers a two-sector Cournot-Walras model with pollution permits with Cournot behavior in one sector and competitive behavior in the other sector.

and the allocation of trader $j \in [1, ..., n\}$, which are given by:

$$(\tilde{x}_j(r), \tilde{y}_j(r)) = \left(\frac{1}{2\beta_1} \frac{\gamma}{\gamma + r} \frac{\alpha}{n} \frac{n-1}{n-\alpha}, \frac{1-\alpha}{n-\alpha} \right). \quad (29)$$

Therefore, the marginal rates of substitution are such that:

$$MRS^i_{X/Y} = \beta_i \frac{\gamma + r}{\gamma}, \; i = 1, 2, \; MRS^j_{X/Y} = 2\beta_1 \frac{\gamma + r}{\gamma} \frac{n}{n-1}, j = 1, ..., n. \quad (30)$$

Therefore, the permits market does not lead to a Pareto allocation. The reason stems once again from the strategic behavior of traders.

Finally, the next proposition considers the effect of an increase of the price of permits on payoffs.

Proposition 11. *The payoffs of the traders of type II increase with the price of permits either when there is a large number of traders of type II or when the consumers strongly prefer commodity Y.*

Proof. For each $j \in \{1, ...n\}$ we have that:

$$\tilde{\pi}_j(r) = \frac{\left(\alpha \frac{1}{2\beta_1} \frac{n-1}{n} \frac{\gamma}{\gamma + r} \right)^{\alpha} (1-\alpha)^{1-\alpha}}{n - \alpha} - \mu \left[\frac{\alpha r}{\gamma + r} (\bar{e}_1 + \bar{e}_2) + \frac{\gamma}{\gamma + r} (\tilde{e}_1 + \bar{e}_2) \right].$$

From the market clearing condition on the permits market, we have $\tilde{e}_1 + \bar{e}_2 = \bar{e}_1 + \bar{e}_2$, so we have:

$$\tilde{\pi}_j(r) = \frac{\left(\alpha \frac{1}{2\beta_1} \frac{n-1}{n} \frac{\gamma}{\gamma + r} \right)^{\alpha} (1-\alpha)^{1-\alpha}}{n - \alpha} - \mu \frac{\alpha r + \gamma}{\gamma + r} (\bar{e}_1 + \bar{e}_2).$$

Then, we deduce:

$$\frac{\partial \tilde{\pi}_j(r)}{\partial r} = \frac{(1-\alpha)\gamma}{(\gamma + r)^2} \left[\mu(\bar{e}_1 + \bar{e}_2) - \alpha \frac{\left(\frac{\alpha}{1-\alpha} \frac{1}{2\beta_1} \frac{n-1}{n} \right)^{\alpha} \left(\frac{\gamma + r}{\gamma} \right)^{1-\alpha}}{n - \alpha} \right].$$

Therefore, we conclude:

$$\begin{aligned} \lim_{\alpha \to 0} \frac{\partial \tilde{\pi}_j(r)}{\partial r} &= \frac{\gamma}{(\gamma + r)^2} \mu(\bar{e}_1 + \bar{e}_2) > 0 \\ \text{and} \lim_{n \to +\infty} \frac{\partial \tilde{\pi}_j(r)}{\partial r} &= \frac{(1-\alpha)\gamma}{(\gamma + r)^2} \mu(\bar{e}_1 + \bar{e}_2) > 0. \end{aligned}$$

∎

Therefore, when the preferences for commodity X is low, a rise of the price of permits, and the corresponding decrease of the relative price, are dominated by the decrease of emissions. In addition, unlike the effect of the per unit tax on payoffs, commodity X may be no longer desirable for the payoff to increase. In addition, when there is a large number of traders who compete for selling commodity Y, i.e., this side of the market would become perfectly competitive, the increase of the price of permits is dominated by the decrease of emissions. This result is not specific to Stackelberg as it holds in Cournot competition (see Appendix F). Nevertheless, Proposition 11 holds as soon as the permit market is competitive: it does not hold when strategic behavior affects the price of permit (Dickson and MacKenzie, 2018).

6. CONCLUSION

The preceding model introduced pollution in a two-stage strategic market game. It could find an echo in international trade with resource specialization. Indeed, heterogeneous strategic agents whose market power differed, and who lived in two distinct countries would compete on quantity on the world market. The production activity of one country pollutes the other country through a negative externality on some traders. The problem is to determine whether emissions levels could be reduced via some mechanisms when all traders behave strategically.

Two kinds of regulation were envisaged to limit the emissions: taxation mechanisms and permits market. The main conclusions are as follows. First, if the objective assigned to the regulator is to determine the optimal tax such as to minimize the sum of emissions under a balanced budget rule, then, in an interior Stackelberg Cournot equilibrium, a uniform per unit tax is the only taxation mechanism which reduces the emissions of the leader (resp. follower) whenever the strategies of the leader and of follower are substitutes and the produced commodity is not strongly preferred to the other good. Second, the way the emissions of the leader and the follower increase with the price of permits depends on preferences. More specifically, the payoffs of the traders of type II (the polluted traders) increase with the price of permits either when there is a large number of traders of type II or when the consumers strongly prefer the nonproduced commodity.

The model was linear in the production and the polluting technologies. Nonlinearities in the production technology and dynamic/intertemporal features such as in the introduction of time in the production activity are left for further researches.

7. APPENDIX

7.1. Appendix A: computation of the SCE strategies

Let us solve the game Γ by backward induction. To this end, consider, in the second stage of the game, the behavior of both types of followers. The problems of follower of type I and of type II may be written:

$$(e_2, q_2) \in \arg\max \left(\frac{\gamma}{\beta_2}e_2 - q_2\right)^{\alpha}\left(\frac{\sum_{j=1}^{n} b_j}{q_1+q_2}q_2 - \gamma e_2\right)^{1-\alpha}; \tag{A1}$$

$$b_j \in \arg\max \left(\frac{q_1+q_2}{b_j + \sum_{-j\neq j} b_{-j}}b_j\right)^{\alpha}\left(\frac{1}{n} - b_j\right)^{1-\alpha} - \mu(e_1+e_2),\ j = 1, ..., n. \tag{A2}$$

The sufficient first-order conditions for an interior solution for the follower of type I, i.e., $\frac{\partial \pi_2(q_1,q_2;\mathbf{b})}{\partial q_2} = 0$ and $\frac{\partial \pi_2(q_1,q_2;\mathbf{b})}{\partial e_2} = 0$, may be written as:

$$\left[-\alpha\left(\frac{\sum_{j=1}^{n} b_j}{q_1+q_2}q_2 - \gamma e_2\right) + (1-\alpha)\frac{\sum_{j=1}^{n} b_j}{(q_1+q_2)^2}q_1\left(\frac{\gamma}{\beta_2}e_2 - q_2\right)\right]A = 0; \tag{A3}$$

$$\left[\alpha\frac{\gamma}{\beta_2}\left(\frac{\sum_{j=1}^{n} b_j}{q_1+q_2}q_2 - \gamma e_2\right) - (1-\alpha)\gamma\left(\frac{\gamma}{\beta_2}e_2 - q_2\right)\right]A = 0; \tag{A4}$$

where $A \equiv \left(\frac{\gamma}{\beta_2}e_2 - q_2\right)^{\alpha-1}\left(\frac{\sum_{j=1}^{n} b_j}{q_1+q_2}q_2 - \gamma e_2\right)^{-\alpha}$. For the followers of type II, we have $\frac{\partial \pi_j(e_1,e_2;\mathbf{q};b_j,\mathbf{b}_{-j})}{\partial b_j} = 0$, $j \in \{1,...,n\}$, which may be written as:

$$\left[\alpha\frac{\sum_{-j,-j\neq j} b_{-j}}{(b_j+\sum_{-j,-j\neq j} b_{-j})^2}\left(\frac{1}{n}-b_j\right)-(1-\alpha)\frac{b_j}{b_j+\sum_{-j,-j\neq j} b_{-j}}\right]B=0, \quad \text{(A5)}$$

where $B \equiv \left(\frac{q_1+q_2}{b_j+\sum_{-j,-j\neq j} b_{-j}}b_j\right)^{\alpha-1}\left(\frac{1}{n}-b_j\right)^{-\alpha}$.

From (A3) and (A4), we deduce:

$$\frac{\sum_{j=1}^{n} b_j}{(q_1+q_2)^2}q_1 = \beta_2. \quad \text{(A6)}$$

The solution to (A6) and to (A5) yield the optimal decision mappings:

$$q_2(q_1;b_1,...,b_n) = -q_1 + \sqrt{\frac{1}{\beta_2}\sum_{j=1}^{n} b_j q_1}; \quad \text{(A7)}$$

$$b_j(q_1,q_2;\mathbf{b}_{-j}) = \frac{\alpha}{n}\frac{n-1}{n-\alpha},\ j \in \{1,...,n\}, \quad \text{(A8)}$$

where we assume $b_j = b_{-j}$, for all $j \neq -j$.

Therefore, in the first stage of the game, the problem of the leader may be written:

$$(\tilde{e}_1,\tilde{q}_1) \in \arg\max \ \left(\frac{\gamma}{\beta_1}e_1 - q_1\right)^{\alpha}\left(\sqrt{\alpha\beta_2\frac{n-1}{n-\alpha}q_1} - \gamma e_1\right)^{1-\alpha}. \quad \text{(A9)}$$

The sufficient first-order conditions (the function $\sqrt{\alpha\beta_2\frac{n-1}{n-\alpha}q_1}$ is strictly concave in q_1), namely $\frac{\partial \pi_1(q_1,q_2(q_1);\mathbf{b})}{\partial q_1} = 0$ and $\frac{\partial \pi_1(q_1,q_2(q_1);\mathbf{b})}{\partial e_1} = 0$ may be written:

$$\left[-\alpha\left(\sqrt{\beta_2\alpha\frac{n-1}{n-\alpha}q_1}-\gamma e_1\right)+\frac{1-\alpha}{2}\sqrt{\alpha\beta_2\frac{n-1}{n-\alpha}}q_1^{-\frac{1}{2}}\left(\frac{\gamma}{\beta_1}e_1-q_1\right)\right]A'=0; \quad \text{(A10)}$$

$$\left[\alpha\frac{\gamma}{\beta_1}\left(\sqrt{\alpha\beta_2\frac{n-1}{n-\alpha}q_1}-\gamma e_1\right)-(1-\alpha)\gamma\left(\frac{\gamma}{\beta_1}e_1-q_1\right)\right]A'=0; \quad \text{(A11)}$$

where $A' \equiv \left(\frac{\gamma_1}{\beta_1}e_1 - q_1\right)^{\alpha-1}\left(\sqrt{\alpha\beta_2\frac{n-1}{n-\alpha}q_1} - \gamma_1 e_1\right)^{-\alpha}$.

By considering the terms in brackets in (A3) an (A4), and by equalizing and cancelling, leads to:

$$\frac{1}{2}\sqrt{\alpha\beta_2\frac{n-1}{n-\alpha}}q_1^{-\frac{1}{2}} = \gamma.$$

The solution of (A12) yields the $\tilde{q}_1 = \frac{\alpha}{4}\frac{\beta_2}{(\beta_1)^2}\frac{n-1}{n-\alpha}$. From (A7) and (A8), we deduce $\tilde{q}_2 = \frac{\alpha}{2\beta_1}\frac{n-1}{n-\alpha}(1-\frac{1}{2}\frac{\beta_2}{\beta_1})$, and $\tilde{b}_j = \frac{\alpha}{n}\frac{n-1}{n-\alpha}$, $j = 1,...,n$. Then, we deduce the market price:

$$\tilde{p}_X = 2\beta_1. \tag{A12}$$

From (A11), we deduce $\tilde{e}_1 = \frac{\alpha(1+\alpha)}{4\gamma}\frac{\beta_2}{\beta_1}\frac{n-1}{n-\alpha}$, and from (A4), we deduce $\tilde{e}_2 = \frac{\alpha}{\gamma}(1-\frac{1}{2}\frac{\beta_2}{\beta_1})(\frac{2\alpha\beta_1+(1-\alpha)\beta_2}{2\beta_1})\frac{n-1}{n-\alpha}$, which are the magnitudes given in Proposition 1. Then, we deduce (14)-(19).

Finally, by using (A7), it is easy to check that $\frac{\partial q_2(.)}{\partial q_1} = -1 + \frac{\beta_2}{\beta_1} \lesseqqgtr 0$ if $\beta_1 \lesseqqgtr \beta_2$. So, the game diplays strategic substituability (resp. complementarity) between the leader and her direct follower when $\beta_1 < \beta_2$ (resp. $\beta_1 > \beta_2$).

7.2. Appendix B: computation of the CE

Consider now the CE in which all traders behave in a simultaneous move game. The problems of all traders may be written:

$$(\hat{e}_i, \hat{q}_i) \in \arg\max \left(\frac{\gamma}{\beta_i}e_i - q_i\right)^{\alpha}\left(\frac{\sum_{j=1}^{n} b_j}{q_1+q_2}q_i - \gamma e_i\right)^{1-\alpha}, \ i=1,2; \tag{B1}$$

$$\hat{b}_j \in \arg\max \left(\frac{q_1+q_2}{b_j+\sum_{-j\neq j} b_{-j}}b_j\right)^{\alpha}\left(\frac{1}{n}-b_j\right)^{1-\alpha} - \mu(e_1+e_2), \ j=1,...,n. \tag{B2}$$

The sufficient first-order conditions for an interior solution are given by (A5) for $j \in \{1,...,n\}$, and by (B3)-(B4) for $i \in \{1,2\}$, with:

$$\left[-\alpha\left(\frac{\sum_{j=1}^{n} b_j}{q_i+q_{-i}}q_i - \gamma e_i\right) + (1-\alpha)\frac{\sum_{j=1}^{n} b_j}{(q_i+q_{-i})^2}q_{-i}\left(\frac{\gamma}{\beta_i}e_i - q_i\right)\right]C = 0; \tag{B3}$$

$$\left[\alpha\frac{\gamma}{\beta_1}\left(\frac{\sum_{j=1}^{n} b_j}{q_i+q_{-i}}q_i - \gamma e_i\right) - (1-\alpha)\gamma\left(\frac{\gamma}{\beta_i}e_i - q_i\right)\right]C = 0, \ i=1,2, \tag{B4}$$

where $C \equiv \left(\frac{\gamma}{\beta_i}e_i - q_i\right)^{\alpha-1}\left(\frac{\sum_{j=1}^{n} b_j}{q_i+q_{-i}}q_i - \gamma e_i\right)^{-\alpha}$.

The solutions to these equations are the optimal decision mappings, which are given by:

$$q_1(q_2; b_1,...,b_n) = -q_2 + \sqrt{\frac{1}{\beta_1}\sum_{j=1}^{n} b_j q_2}; \tag{B5}$$

$$q_2(q_1; b_1,...,b_n) = -q_1 + \sqrt{\frac{1}{\beta_2}\sum_{j=1}^{n} b_j q_1}; \tag{B6}$$

$$b_j(q_1, q_2; \mathbf{b}_{-j}) = \frac{\alpha}{n}\frac{n-1}{n-\alpha}, \ j \in \{1,...,n\}, \tag{B7}$$

where we assume $b_j = b_{-j}$, for all $j \neq -j$.

The solutions to (B5)-(B7) are given by:

$$(\hat{q}_1, \hat{q}_2) = \left(\frac{\alpha\beta_2}{(\beta_1+\beta_2)^2} \frac{n-1}{n-\alpha}, \frac{\alpha\beta_1}{(\beta_1+\beta_2)^2} \frac{n-1}{n-\alpha} \right); \tag{B8}$$

$$\hat{b}_j = \frac{\alpha}{n} \frac{n-1}{n-\alpha}, j = 1, ..., n. \tag{B9}$$

Therefore, the equilibrium relative market price is given by:

$$\hat{p}_X = \beta_1 + \beta_2. \tag{B10}$$

Then, we deduce the emissions:

$$(\hat{e}_1, \hat{e}_2) = \left(\frac{\alpha}{\gamma} \frac{\beta_2(\beta_1 + \alpha\beta_2)}{(\beta_1+\beta_2)^2} \frac{n-1}{n-\alpha}, \frac{\alpha}{\gamma} \frac{\beta_1(\beta_2 + \alpha\beta_1)}{(\beta_1+\beta_2)^2} \frac{n-1}{n-\alpha} \right). \tag{B11}$$

The allocations are then:

$$(\hat{x}_1, \hat{y}_1) = \left(\frac{1}{\beta_1} \left(\frac{\alpha\beta_2}{\beta_1+\beta_2} \right)^2 \frac{n-1}{n-\alpha}, \alpha(1-\alpha) \left(\frac{\beta_2}{\beta_1+\beta_2} \right)^2 \frac{n-1}{n-\alpha} \right); \tag{B12}$$

$$(\hat{x}_2, \hat{y}_2) = \left(\frac{1}{\beta_2} \left(\frac{\alpha\beta_1}{\beta_1+\beta_2} \right)^2 \frac{n-1}{n-\alpha}, \alpha(1-\alpha) \left(\frac{\beta_1}{\beta_1+\beta_2} \right)^2 \frac{n-1}{n-\alpha} \right); \tag{B13}$$

$$(\hat{x}_j, \hat{y}_j) = \left(\frac{\alpha}{\beta_1+\beta_2} \frac{1}{n} \frac{n-1}{n-\alpha}, \frac{1-\alpha}{n-\alpha} \right), j = 1, ..., n. \tag{B14}$$

Therefore, the CE payoffs of traders $i = 1, 2$ are given by:

$$\hat{\pi}_1 = \alpha^{\alpha+1} \left(\frac{\beta_2}{\beta_1+\beta_2} \right)^2 \left(\frac{1}{\beta_1} \right)^\alpha (1-\alpha)^{1-\alpha} \frac{n-1}{n-\alpha}; \tag{B15}$$

$$\hat{\pi}_2 = \alpha^{\alpha+1} \left(\frac{\beta_1}{\beta_1+\beta_2} \right)^2 \left(\frac{1}{\beta_2} \right)^\alpha (1-\alpha)^{1-\alpha} \frac{n-1}{n-\alpha}; \tag{B16}$$

and, the CE payoffs of traders $j \in \{1, ..., n\}$ are given by:

$$\hat{\pi}_j = \frac{\left(\frac{\alpha}{\beta_1+\beta_2} \frac{n-1}{n} \right)^\alpha (1-\alpha)^{1-\alpha}}{n-\alpha} - \mu\alpha \frac{\beta_2(\beta_1+\alpha\beta_2) + \beta_1(\beta_2+\alpha\beta_1)}{\gamma(\beta_1+\beta_2)^2} \frac{n-1}{n-\alpha}. \tag{B17}$$

7.3. Appendix C: the SCE with taxations

In this Appendix we determine the SCE emissions by encompassing the three taxation mechanisms. To this end, let $(\mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}) \equiv (t_1, t_2, \nu_1, \nu_2, \tau_1, \tau_2)$, and let us rewrite the payoffs (21)-(23) as:

$$\pi_i(.) = \left(\frac{\gamma}{\beta_i}(1-t_i)e_i - q_i \right)^\alpha \left(\left(\frac{\sum_{j=1}^n b_j(1-\tau_i)}{\sum_{i=1}^2 (1-\tau_i)q_i} - \nu_i \right) q_i - \gamma(1-t_i)e_i \right)^{1-\alpha}, \tag{C1}$$

where $\pi_i(.) \equiv \pi_i(e_i, q_i, \mathbf{q}_{-i}; \mathbf{b}; \mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau})$, $i = 1, 2$.

Consider the followers. Differentiating the above expression with respect to q_2 and e_2 leads to the sufficient first-order conditions:

$$\frac{\partial \pi_2(.)}{\partial q_2} = \{-\alpha \left[\left(\frac{\sum_{j=1}^n b_j(1-\tau_2)}{\sum_{i=1}^2 (1-\tau_i) q_i} - \nu_2 \right) q_2 - \gamma(1-t_2)e_2 \right] +$$

$$(1-\alpha) \left[\frac{\sum_{j=1}^n b_j (1-\tau_1)(1-\tau_2) q_1}{(\sum_{i=1}^2 (1-\tau_i) q_i)^2} - \nu_2 \right] \left(\frac{\gamma}{\beta_2}(1-t_2)e_2 - q_2 \right) \} E = 0; \quad \text{(C2)}$$

$$\frac{\partial \pi_2(.)}{\partial e_2} = \{\alpha \frac{\gamma}{\beta_2}(1-t_2) \left[\left(\frac{\sum_{j=1}^n b_j(1-\tau_2)}{\sum_{i=1}^2 (1-\tau_i) q_i} - \nu_2 \right) q_2 - \gamma(1-t_2)e_2 \right] -$$

$$(1-\alpha)\gamma(1-t_2) \left(\frac{\gamma}{\beta_2}(1-t_2)e_2 - q_2 \right) \} E = 0, \quad \text{(C3)}$$

where $E \equiv \left(\frac{\gamma}{\beta_2}(1-t_2)e_2 - q_2 \right)^{\alpha - 1} \left(\left(\frac{\sum_{j=1}^n b_j(1-\tau_2)}{\sum_{i=1}^2 (1-\tau_i) q_i} - \nu_2 \right) q_2 - \gamma(1-t_2)e_2 \right)^{-\alpha}$.

The optimal decision mappings of all followers are given by:

$$q_2(q_1; \mathbf{b}; \mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}) = -\frac{1-\tau_1}{1-\tau_2} q_1 + \frac{1}{1-\tau_2} \sqrt{\frac{(1-\tau_1)(1-\tau_2)}{\beta_2 + \nu_2} \sum_{j=1}^n b_j q_1}; \quad \text{(C4)}$$

$$b_j(q_1, q_2; \mathbf{b}_{-j}; \mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}) = \frac{\alpha}{n} \frac{n-1}{n-\alpha}, \ j \in \{1, ..., n\}. \quad \text{(C5)}$$

Therefore, in the first stage of the game, the problem of the leader may be written:

$$\max \left(\frac{\gamma}{\beta_1}(1-t_1)e_1 - q_1 \right)^{\alpha} \left(\sqrt{\frac{(1-\tau_1)(\beta_2 + \nu_2) \sum_{j=1}^n b_j}{1-\tau_2} q_1} - \nu_1 q_1 - \gamma(1-t_1)e_1 \right)^{1-\alpha}. \quad \text{(C6)}$$

The first-order conditions, i.e., $\frac{\partial \pi_1(q_1, q_2(q_1); \mathbf{b}; \mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau})}{\partial q_1} = 0$ and $\frac{\partial \pi_1(q_1, q_2(q_1); \mathbf{b}; \mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau})}{\partial e_1} = 0$, may be written:

$$\{-\alpha \left[\sqrt{\frac{1-\tau_1}{1-\tau_2}(\beta_2 + \tau_2) \sum_{j=1}^n b_j q_1} - \nu_1 q_1 - \gamma(1-t_1)e_1 \right] +$$

$$(1-\alpha) \left[\frac{1}{2} \sqrt{\alpha \frac{n-1}{n-\alpha} \frac{1-\tau_1}{1-\tau_2}(\beta_2 + \tau_2)} q_1^{-\frac{1}{2}} - \nu_1 \right] \left(\frac{\gamma}{\beta_1}(1-t_1)e_1 - q_1 \right) \} F = 0; \quad \text{(C7)}$$

$$\{\alpha \frac{\gamma}{\beta_1}(1-t_1) \left[\sqrt{\frac{1-\tau_1}{1-\tau_2}(\beta_2 + \tau_2) \sum_{j=1}^n b_j q_1} - \nu_1 q_1 - \gamma(1-t_1)e_1 \right] -$$

$$(1-\alpha)\gamma(1-t_1) \left(\frac{\gamma}{\beta_1}(1-t_1)e_1 - q_1 \right) \} F = 0, \quad \text{(C8)}$$

where $F \equiv \left(\frac{\gamma}{\beta_1}(1-t_1)e_1 - q_1\right)^{\alpha-1}\left(\sqrt{\frac{1-\tau_1}{1-\tau_2}(\beta_2+\tau_2)\sum_{j=1}^{n} b_j q_1} - \nu_1 q_1 - \gamma(1-t_1)e_1\right)^{-\alpha}$.

The solution to (C7)-(C8) is given by:

$$\tilde{q}_1(\mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}) = \frac{\alpha}{4}\frac{\beta_2+\nu_2}{(\beta_1+\nu_1)^2}\frac{1-\tau_1}{1-\tau_2}\frac{n-1}{n-\alpha}; \tag{C9}$$

Then, from (C4), we deduce:

$$\tilde{q}_2(\mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}) = \frac{\alpha}{2}\frac{1}{\beta_1+\nu_1}\frac{1-\tau_1}{1-\tau_2}\left(1 - \frac{1}{2}\frac{\beta_2+\nu_2}{\beta_1+\nu_1}\frac{1-\tau_1}{1-\tau_2}\right)\frac{n-1}{n-\alpha}, \tag{C10}$$

which by letting $(\tilde{q}_1', \tilde{q}_2') \equiv (\tilde{q}_1(\mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}), \tilde{q}_2(\mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}))$ yields the values in Proposition (4).

The market price may be written:

$$\tilde{p}_X(\mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}) = 2\frac{\beta_1+\nu_1}{1-\tau_1}. \tag{C11}$$

By using the first-order conditions, we deduce the emissions:

$$\tilde{e}_1(\mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}) = \frac{(1+\alpha)\beta_1+\alpha\nu_1}{\gamma(1-t_1)}\tilde{q}_1(\mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}) \tag{C12}$$

$$\tilde{e}_2(\mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}) = \frac{\alpha[(2(\beta_1+\nu_1)(1-\tau_2)-(1-\tau_1)\nu_2)]+(1-\alpha)\beta_2(1-\tau_1)}{\gamma(1-t_2)(1-\tau_1)}\tilde{q}_2(\mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}), \tag{C13}$$

which by letting $(\tilde{e}_1', \tilde{e}_2') \equiv (\tilde{e}_1(\mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}), \tilde{e}_2(\mathbf{t}, \boldsymbol{\nu}, \boldsymbol{\tau}))$ yields the values in Proposition (4).

We now consider the thee taxation mechanisms. Therefore, (21) holds when $\tau = \nu = 0$; (22) holds when $t = \tau = 0$; and (23) holds when $t = \nu = 0$.

We consider now the effects of taxation on emissions in the SCE. Therefore, let $\nu_1 = \nu_2 = \tau_1 = \tau_2 = 0$. Then, from (C12)-(C13), we deduce:

$$\tilde{e}_1(t_1, t_2) = \frac{\alpha(1+\alpha)}{4\gamma}\frac{\beta_2}{\beta_1}\frac{1}{1-t_1}\frac{n-1}{n-\alpha}; \tag{C14}$$

$$\tilde{e}_2(t_1, t_2) = \frac{\alpha}{2\gamma}\frac{2\alpha\beta_1+(1-\alpha)\beta_2}{\beta_1}\left(1-\frac{1}{2}\frac{\beta_2}{\beta_1}\right)\frac{1}{1-t_2}\frac{n-1}{n-\alpha}. \tag{C15}$$

Then, we get:

$$\frac{\partial\tilde{e}_i(t_1, t_2)}{\partial t_i} > 0,\ i = 1, 2. \tag{C16}$$

Now, let $t_1 = t_2 = \tau_1 = \tau_2 = 0$. Then, from (C12)-(C13), we deduce:

$$\tilde{e}_1(\nu_1, \nu_2) = \frac{\alpha}{4\gamma}\frac{[(1+\alpha)\beta_1+\alpha\nu_1](\beta_2+\nu_2)}{(\beta_1+\nu_1)^2}\frac{n-1}{n-\alpha}; \tag{C17}$$

$$\tilde{e}_2(\nu_1, \nu_2) = \frac{\alpha}{2\gamma}\frac{\alpha[(2(\beta_1+\nu_1)-\nu_2)]+(1-\alpha)\beta_2}{\beta_1+\nu_1}\left(1-\frac{1}{2}\frac{\beta_2+\nu_2}{\beta_1+\nu_1}\right)\frac{n-1}{n-\alpha}. \tag{C18}$$

Then, some computations yield:

$$\frac{\partial \tilde{e}_1(\nu_1,\nu_2)}{\partial \nu_1} < 0, \text{ and } \frac{\partial \tilde{e}_1(\nu_1,\nu_2)}{\partial \nu_2} > 0; \tag{C19}$$

$$\frac{\partial \tilde{e}_2(\nu_1,\nu_2)}{\partial \nu_2} < 0, \text{ and } \frac{\partial \tilde{e}_2(\nu_1,\nu_2)}{\partial \nu_1} > 0. \tag{C20}$$

Finally, let $t_1 = t_2 = \nu_1 = \nu_2 = 0$. Then, we have:

$$\tilde{e}_1(\tau_1,\tau_2) = \frac{\alpha(1+\alpha)}{4\gamma}\frac{\beta_2}{\beta_1}\frac{1-\tau_1}{1-\tau_2}\frac{n-1}{n-\alpha}; \tag{C21}$$

$$\tilde{e}_2(\tau_1,\tau_2) = \frac{\alpha}{2\gamma}\frac{1}{\beta_1}\frac{2\alpha\beta_1(1-\tau_2)+(1-\alpha)\beta_2}{1-\tau_2}\left(1-\frac{1}{2}\frac{\beta_2}{\beta_1}\frac{1-\tau_1}{1-\tau_2}\right)\frac{n-1}{n-\alpha}. \tag{C22}$$

Then, some computations yield:

$$\frac{\partial \tilde{e}_1(\tau_1,\tau_2)}{\partial \tau_1} < 0, \text{ and } \frac{\partial \tilde{e}_1(\tau_1,\tau_2)}{\partial \tau_2} > 0; \tag{C23}$$

$$\frac{\partial \tilde{e}_2(\tau_1,\tau_2)}{\partial \tau_2} < 0, \text{ and } \frac{\partial \tilde{e}_2(\tau_1,\tau_2)}{\partial \tau_1} > 0. \tag{C24}$$

7.4. Appendix D: the CE with taxations

In this Appendix we determine the SCE emissions by encompassing the three taxation mechanisms. To this end, consider the payoffs given by (C1). The sufficient first-order conditions for an interior solution are given by (C3), and by trader for trader $i \in \{1,2\}$:

$$\frac{\partial \pi_i(.)}{\partial q_i} = \{-\alpha\left[\left(\frac{\sum_{j=1}^n b_j(1-\tau_i)}{(1-\tau_i)q_i+(1-\tau_{-i})q_{-i}}-\nu_i\right)q_i-\gamma(1-t_i)e_i\right]+$$

$$(1-\alpha)\left[\frac{\sum_{j=1}^n b_j(1-\tau_1)(1-\tau_2)q_{-i}}{[(1-\tau_i)q_i+(1-\tau_{-i})q_{-i}]^2}-\nu_i\right]\left(\frac{\gamma}{\beta_i}(1-t_i)e_i-q_i\right)\}E=0; \tag{D1}$$

$$\frac{\partial \pi_i(.)}{\partial e_i} = \{\alpha\frac{\gamma}{\beta_i}(1-t_i)\left[\left(\frac{\sum_{j=1}^n b_j(1-\tau_i)}{(1-\tau_i)q_i+(1-\tau_{-i})q_{-i}}-\nu_i\right)q_i-\gamma(1-t_i)e_i\right]-$$

$$(1-\alpha)\gamma(1-t_i)\left(\frac{\gamma}{\beta_i}(1-t_i)e_i-q_i\right)\}E=0, \tag{D2}$$

where $E \equiv \left(\frac{\gamma}{\beta_i}(1-t_i)e_i-q_i\right)^{\alpha-1}\left(\left(\frac{\sum_{j=1}^n b_j(1-\tau_i)}{(1-\tau_i)q_i+(1-\tau_{-i})q_{-i}}-\nu_i\right)q_i-\gamma(1-t_i)e_i\right)^{-\alpha}$.

The unique solution to these equations are given by:

$$q_1(q_2;\mathbf{b};\mathbf{t},\boldsymbol{\nu},\boldsymbol{\tau}) = -\frac{1-\tau_2}{1-\tau_1}q_2+\frac{1}{1-\tau_1}\sqrt{\frac{(1-\tau_1)(1-\tau_2)}{\beta_1+\nu_1}\sum_{j=1}^n b_j q_2}; \tag{D3}$$

$$q_2(q_1;\mathbf{b};\mathbf{t},\boldsymbol{\nu},\boldsymbol{\tau}) = -\frac{1-\tau_1}{1-\tau_2}q_1 + \frac{1}{1-\tau_2}\sqrt{\frac{(1-\tau_1)(1-\tau_2)}{\beta_2+\nu_2}\sum\nolimits_{j=1}^{n} b_j q_1}; \tag{D4}$$

$$b_j(q_1, q_2; \mathbf{b}_{-j}; \mathbf{t},\boldsymbol{\nu},\boldsymbol{\tau}) = \frac{\alpha}{n}\frac{n-1}{n-\alpha},\ j \in \{1, ..., n\}, \tag{D5}$$

where we assume $b_j = b_{-j}$, for all $j \neq -j$.

The solution is given by:

$$\hat{q}_1(\mathbf{t},\boldsymbol{\nu},\boldsymbol{\tau}) = \alpha\frac{(\beta_2+\nu_2)(1-\tau_1)(1-\tau_2)}{[(1-\tau_1)(\beta_2+\nu_2)+(1-\tau_2)(\beta_1+\nu_1)]^2}\frac{n-1}{n-\alpha}; \tag{D6}$$

$$\hat{q}_2(\mathbf{t},\boldsymbol{\nu},\boldsymbol{\tau}) = \alpha\frac{(\beta_1+\nu_1)(1-\tau_1)(1-\tau_2)}{[(1-\tau_1)(\beta_2+\nu_2)+(1-\tau_2)(\beta_1+\nu_1)]^2}\frac{n-1}{n-\alpha}; \tag{D7}$$

$$\hat{b}_j(\mathbf{t},\boldsymbol{\nu},\boldsymbol{\tau}) = \frac{\alpha}{n}\frac{n-1}{n-\alpha}, j = 1, ..., n. \tag{D8}$$

Therefore, the equilibrium relative market price is given by:

$$\hat{p}_X(\mathbf{t},\boldsymbol{\nu},\boldsymbol{\tau}) = \frac{(1-\tau_1)(\beta_2+\nu_2)+(1-\tau_2)(\beta_1+\nu_1)}{(1-\tau_1)(1-\tau_2)}. \tag{D9}$$

By using the first-order conditions, we deduce the emissions:

$$\hat{e}_1(\mathbf{t},\boldsymbol{\nu},\boldsymbol{\tau}) = \frac{\alpha(1-\tau_1)(\beta_2+\nu_2)+\beta_1(1-\tau_2)}{\gamma(1-t_1)(1-\tau_2)}\hat{q}_1(\mathbf{t},\boldsymbol{\nu},\boldsymbol{\tau}); \tag{D10}$$

$$\hat{e}_2(\mathbf{t},\boldsymbol{\nu},\boldsymbol{\tau}) = \frac{\alpha(1-\tau_2)(\beta_1+\nu_1)+\beta_2(1-\tau_1)}{\gamma(1-t_2)(1-\tau_1)}\hat{q}_2(\mathbf{t},\boldsymbol{\nu},\boldsymbol{\tau}). \tag{D11}$$

We consider now the effects of taxation on emissions in the CE. Therefore, let $\nu_1 = \nu_2 = \tau_1 = \tau_2 = 0$. Then, we have:

$$\hat{e}_1(t_1, t_2) = \frac{\alpha\beta_2+\beta_1}{\gamma(1-t_1)}\frac{\alpha\beta_2}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha}; \tag{D10}$$

$$\hat{e}_2(t_1, t_2) = \frac{\alpha\beta_1+\beta_2}{\gamma(1-t_2)}\frac{\alpha\beta_1}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha}. \tag{D11}$$

Then, we get:

$$\frac{\partial\hat{e}_i(t_1, t_2)}{\partial t_i} > 0,\ i = 1, 2. \tag{D12}$$

Now, let $t_1 = t_2 = \tau_1 = \tau_2 = 0$. Then, we have:

$$\hat{e}_1(\nu_1, \nu_2) = \alpha\frac{\alpha(\beta_2+\nu_2)+\beta_1}{\gamma}\frac{\beta_2+\nu_2}{(\beta_1+\nu_1+\beta_2+\nu_2)^2}\frac{n-1}{n-\alpha}; \tag{D13}$$

$$\hat{e}_2(\nu_1, \nu_2) = \alpha\frac{\alpha(\beta_1+\nu_1)+\beta_2}{\gamma}\frac{\beta_1+\nu_1}{(\beta_1+\nu_1+\beta_2+\nu_2)^2}\frac{n-1}{n-\alpha}. \tag{D14}$$

Then, we get:

$$\frac{\partial \hat{e}_1(\nu_1,\nu_2)}{\partial \nu_1} < 0, \text{ and } \frac{\partial \hat{e}_1(\nu_1,\nu_2)}{\partial \nu_2} > 0; \tag{D15}$$

and, as $\beta_i > 1$, $i = 1, 2$, we have:

$$\frac{\partial \hat{e}_2(\nu_1,\nu_2)}{\partial \nu_2} < 0, \text{ and } \frac{\partial \hat{e}_2(\nu_1,\nu_2)}{\partial \nu_1} > 0. \tag{D16}$$

Now, let $t_1 = t_2 = \nu_1 = \nu_2 = 0$. Then, we have:

$$\hat{e}_1(\tau_1,\tau_2) = \frac{\alpha(1-\tau_1)\beta_2 + \beta_1(1-\tau_2)}{\gamma(1-\tau_2)} \frac{\alpha\beta_2(1-\tau_1)(1-\tau_2)}{[(1-\tau_1)\beta_2 + (1-\tau_2)\beta_1]^2} \frac{n-1}{n-\alpha}; \tag{D17}$$

$$\hat{e}_2(\tau_1,\tau_2) = \frac{\alpha(1-\tau_2)\beta_1 + \beta_2(1-\tau_1)}{\gamma(1-\tau_1)} \frac{\alpha\beta_1(1-\tau_1)(1-\tau_2)}{[(1-\tau_1)\beta_2 + (1-\tau_2)\beta_1]^2} \frac{n-1}{n-\alpha}. \tag{D18}$$

Then, we get:

$$\frac{\partial \hat{e}_1(\tau_1,\tau_2)}{\partial \tau_1} < 0, \text{ and } \frac{\partial \hat{e}_1(\tau_1,\tau_2)}{\partial \tau_2} > 0; \tag{D19}$$

and, as $\rho \in [-\infty, 1]$, we have:

$$\frac{\partial \hat{e}_2(\tau_1,\tau_2)}{\partial \tau_2} < 0, \text{ and } \frac{\partial \hat{e}_2(\tau_1,\tau_2)}{\partial \tau_1} > 0. \tag{D20}$$

Finally, assume that $t_1 = t_2 = \tau_1 = \tau_2 = 0$, and $\tilde{\nu}_1 = \tilde{\nu}_2 = \tilde{\nu}$. Some tedious computations lead to:

$$\frac{\partial \hat{e}_1(\nu,\nu)}{\partial \nu}_{|\nu=\tilde{\nu}} = \frac{\alpha}{\gamma} \frac{2\alpha(\beta_1+\tilde{\nu})(\beta_2+\tilde{\nu}) + \beta_1(\beta_1-\beta_2)}{(\beta_1+\tilde{\nu}+\beta_2+\tilde{\nu})^3} \frac{n-1}{n-\alpha}; \tag{D21}$$

$$\frac{\partial \hat{e}_2(\nu,\nu)}{\partial \nu}_{|\nu=\tilde{\nu}} = \frac{\alpha}{\gamma} \frac{2\alpha(\beta_1+\tilde{\nu})(\beta_2+\tilde{\nu}) + \beta_2(\beta_2-\beta_1)}{(\beta_1+\tilde{\nu}+\beta_2+\tilde{\nu})^3} \frac{n-1}{n-\alpha}. \tag{D22}$$

If $\beta_1 < \beta_2$, then $\frac{\partial \hat{e}_1(\nu,\nu)}{\partial \nu}_{|\nu=\tilde{\nu}} < 0$, and $\frac{\partial \hat{e}_2(\nu,\nu)}{\partial \nu}_{|\nu=\tilde{\nu}} > 0$. And, if $\frac{\partial \hat{e}_1(\nu,\nu)}{\partial \nu}_{|\nu=\tilde{\nu}} < 0$, then $\alpha < \frac{\beta_1\beta_2-(\beta_1)^2}{2\alpha(\beta_1+\tilde{\nu})(\beta_2+\tilde{\nu})}$, so we must have $\beta_1 < \beta_2$ as $\alpha > 0$. But, then $\frac{\partial \hat{e}_2(\nu,\nu)}{\partial \nu}_{|\nu=\tilde{\nu}} > 0$. Therefore, we cannot have at the same time $\frac{\partial \hat{e}_1(\nu,\nu)}{\partial \nu}_{|\nu=\tilde{\nu}} < 0$ and $\frac{\partial \hat{e}_2(\nu,\nu)}{\partial \nu}_{|\nu=\tilde{\nu}} < 0$.

7.5. Appendix E: the SCE with a permits market

Consider, in the second stage of the game, the behavior of the follower of type I (the problem of each follower of type II is not modified), which may be written:

$$(e_2, q_2) \in \max \left(\frac{\gamma}{\beta_2} e_2 - q_2\right)^{\alpha} \left(\frac{\sum_{j=1}^n b_j}{q_1+q_2} q_2 - \gamma e_2 + r(\bar{e}_2 - e_2)\right)^{1-\alpha}. \tag{E1}$$

By following the same procedure as in Appendices A and C, the optimal decision mappings are given by (A8), i.e., $b_j = \frac{\alpha}{n}\frac{n-1}{n-\alpha}$, $j = 1, ..., n$, and by:

$$q_2(q_1; b_1, ..., b_n) = -q_1 + \sqrt{\frac{1}{\beta_2}\frac{\gamma}{\gamma + r}\sum_{j=1}^{n} b_j q_1}. \tag{E2}$$

Therefore, in the first stage of the game, the problem of the leader may be written:

$$(\tilde{e}_1(r), \tilde{q}_1(r)) \in \max \left(\frac{\gamma}{\beta_1}e_1 - q_1\right)^{\alpha} \left(\sqrt{\alpha\beta_2\frac{\gamma + r}{\gamma}\frac{n-1}{n-\alpha}q_1} - \gamma e_1 + r(\bar{e}_1 - e_1)\right)^{1-\alpha}. \tag{E3}$$

The sufficient first-order conditions (the function $\sqrt{\alpha\beta_2\frac{\gamma+r}{\gamma}\frac{n-1}{n-\alpha}q_1}$ is strictly concave in q_1), namely $\frac{\partial\pi_1(q_1,q_2(q_1);\mathbf{b})}{\partial q_1} = 0$ and $\frac{\partial\pi_1(q_1,q_2(q_1);\mathbf{b})}{\partial e_1} = 0$ may be written:

$$\{-\alpha[\sqrt{\alpha\beta_2\frac{n-1}{n-\alpha}q_1} - \gamma e_1 + r(\bar{e}_1 - e_1)] + \frac{(1-\alpha)\sqrt{\alpha\beta_2\frac{\gamma+r}{\gamma}\frac{n-1}{n-\alpha}}q_1^{-\frac{1}{2}}\left(\frac{\gamma}{\beta_1}e_1 - q_1\right)}{2}\}A'' = 0; \tag{E4}$$

$$[\alpha\gamma\frac{\sqrt{\alpha\beta_2\frac{\gamma+r}{\gamma}\frac{n-1}{n-\alpha}q_1} - \gamma e_1 + r(\bar{e}_1 - e_1)}{\beta_1} - (1-\alpha)(\gamma+r)\left(\frac{\gamma}{\beta_1}e_1 - q_1\right)]A'' = 0; \tag{E5}$$

where $A'' \equiv \left(\frac{\gamma}{\beta_1}e_1 - q_1\right)^{\alpha-1}\left(\sqrt{\alpha\beta_2\frac{\gamma+r}{\gamma}\frac{n-1}{n-\alpha}q_1} - \gamma e_1 + r(\bar{e}_1 - e_1)\right)^{-\alpha}$.

By considering the terms in brackets in (E4) and (E5), and by equalizing and cancelling, leads to:

$$\frac{1}{2}\sqrt{\alpha\beta_2\frac{\gamma + r}{\gamma}\frac{n-1}{n-\alpha}}q_1^{-\frac{1}{2}} = \beta_1\frac{\gamma + r}{\gamma}. \tag{E6}$$

The solution of (E6) yields the $\tilde{q}_1(r) = \alpha\frac{\beta_2}{4(\beta_1)^2}\frac{\gamma}{\gamma+r}\frac{n-1}{n-\alpha}$. From (E2), we deduce $\tilde{q}_2 = \frac{\alpha}{2\beta_1}\frac{n-1}{n-\alpha}(1 - \frac{1}{2}\frac{\beta_2}{\beta_1})$. We also have $\tilde{b}_j = \frac{\alpha}{n}\frac{n-1}{n-\alpha}$, $j = 1, ..., n$.. Therefore, the market price is given by:

$$\tilde{p}_X(r) = \frac{\gamma + r}{\gamma}\tilde{p}_X. \tag{E6}$$

From (E5), we deduce $\tilde{e}_1(r) = \frac{\alpha}{\gamma+r}r\bar{e}_1 + \frac{\alpha(1+\alpha)}{\gamma+r}\frac{\beta_2}{4\beta_1}\frac{n-1}{n-\alpha}$, and from (E4), we deduce $\tilde{e}_2(r) = \frac{\alpha}{\gamma+r}r\bar{e}_2 + \frac{\alpha}{\gamma+r}(1 - \frac{1}{2}\frac{\beta_2}{\beta_1})(\frac{2\alpha\beta_1+(1-\alpha)\beta_2}{2\beta_1})\frac{n-1}{n-\alpha}$, which are the magnitudes of Proposition 8. Finally, we deduce (27)-(29).

7.6. Appendix F: the CE with a permits market

The problems of all traders may be written:

$$(\hat{e}_i, \hat{q}_i) \in \max \left(\frac{\gamma}{\beta_i}e_i - q_i\right)^{\alpha}\left(\frac{\sum_{j=1}^{n} b_j}{q_1 + q_2}q_i - \gamma e_i + r(\bar{e}_i - e_i)\right)^{1-\alpha}, \; i = 1, 2; \tag{F1}$$

$$\hat{b}_j \in \max \left(\frac{q_1+q_2}{b_j + \sum_{-j \neq j} b_{-j}} b_j \right)^{\alpha} \left(\frac{1}{n} - b_j \right)^{1-\alpha} - \mu(e_1+e_2),\ j = 1,...,n. \quad \text{(F2)}$$

The sufficient first-order conditions for an interior solution are given by (A5) for $j \in \{1,...,n\}$, and by (F3)-(F4) for $i \in \{1,2\}$, with:

$$\{-\alpha[\frac{\sum_{j=1}^{n} b_j}{q_i+q_{-i}} q_i - \gamma e_i + r(\bar{e}_i - e_i)] + (1-\alpha)\frac{\sum_{j=1}^{n} b_j}{(q_i+q_{-i})^2} q_{-i} \left(\frac{\gamma}{\beta_i} e_i - q_i \right)\} C' = 0; \quad \text{(F3)}$$

$$\{\alpha \frac{\gamma}{\beta_1} [\frac{\sum_{j=1}^{n} b_j}{q_i+q_{-i}} q_i - \gamma e_i + r(\bar{e}_i - e_i)] - (1-\alpha)\gamma \left(\frac{\gamma}{\beta_i} e_i - q_i \right)\} C' = 0,\ i = 1,2, \quad \text{(F4)}$$

where $C' \equiv \left(\frac{\gamma}{\beta_i} e_i - q_i \right)^{\alpha-1} \left(\frac{\sum_{j=1}^{n} b_j}{q_i+q_{-i}} q_i - \gamma e_i + r(\bar{e}_i - e_i) \right)^{-\alpha}$.

The solutions to these equations are the optimal decision mappings, which are given by:

$$q_1(q_2; b_1, ..., b_n; r) = -q_2 + \sqrt{\frac{1}{\beta_1} \frac{\gamma}{\gamma+r} \sum_{j=1}^{n} b_j q_2}; \quad \text{(F5)}$$

$$q_2(q_1; b_1, ..., b_n; r) = -q_1 + \sqrt{\frac{1}{\beta_2} \frac{\gamma}{\gamma+r} \sum_{j=1}^{n} b_j q_1}; \quad \text{(F6)}$$

$$b_j(q_1, q_2; \mathbf{b}_{-j}; r) = \frac{\alpha}{n} \frac{n-1}{n-\alpha},\ j \in \{1,...,n\}, \quad \text{(F7)}$$

where we assume $b_j = b_{-j}$, for all $j \neq -j$.

The solutions to (F5)-(F7) are given by:

$$(\hat{q}_1(r), \hat{q}_2(r)) = \left(\frac{\alpha\beta_2}{(\beta_1+\beta_2)^2} \frac{\gamma}{\gamma+r} \frac{n-1}{n-\alpha}, \frac{\alpha\beta_1}{(\beta_1+\beta_2)^2} \frac{\gamma}{\gamma+r} \frac{n-1}{n-\alpha} \right); \quad \text{(F8)}$$

$$\hat{b}_j(r) = \frac{\alpha}{n} \frac{n-1}{n-\alpha}, j = 1, ..., n. \quad \text{(F9)}$$

Therefore, the equilibrium relative market price is given by:

$$\hat{p}_X(r) = (\beta_1+\beta_2)\frac{\gamma+r}{\gamma}. \quad \text{(F10)}$$

Then, we deduce the emissions:

$$(\hat{e}_1(r), \hat{e}_2(r)) = \left(\alpha \frac{r}{\gamma+r} \bar{e}_1 + \frac{\gamma}{\gamma+r} \hat{e}_1, \alpha \frac{r}{\gamma+r} \bar{e}_2 + \frac{\gamma}{\gamma+r} \hat{e}_2 \right). \quad \text{(F11)}$$

The allocations are then:

$$(\hat{x}_1(r), \hat{y}_1(r)) = \left(\alpha \frac{\gamma}{\beta_1} \frac{r}{\gamma+r} \bar{e}_1 + \frac{\gamma}{\gamma+r} \hat{x}_1, (1-\alpha) r \bar{e}_1 + \hat{y}_1 \right); \quad \text{(F12)}$$

$$(\hat{x}_2(r), \hat{y}_2(r)) = \left(\alpha \frac{\gamma}{\beta_2} \frac{r}{\gamma + r} \bar{e}_2 + \frac{\gamma}{\gamma + r} \hat{x}_2, (1-\alpha) r \bar{e}_2 + \hat{y}_2\right); \tag{F13}$$

$$(\hat{x}_j(r), \hat{y}_j(r)) = \left(\frac{\gamma}{\gamma + r} \frac{\alpha}{\beta_1 + \beta_2} \frac{1}{n} \frac{n-1}{n-\alpha}, \frac{1-\alpha}{n-\alpha}\right), \, j = 1, ..., n. \tag{F14}$$

Therefore, the CE payoffs for traders $i = 1, 2$ are given by:

$$\hat{\pi}_1(r) = \left(\alpha \frac{\gamma}{\beta_1} \frac{r}{\gamma + r} \bar{e}_1 + \frac{\gamma}{\gamma + r} \hat{x}_1\right)^{\alpha} \left((1-\alpha) r \bar{e}_1 + \hat{y}_1\right)^{1-\alpha}; \tag{F15}$$

$$\hat{\pi}_2(r) = \left(\alpha \frac{\gamma}{\beta_2} \frac{r}{\gamma + r} \bar{e}_2 + \frac{\gamma}{\gamma + r} \hat{x}_2\right)^{\alpha} \left((1-\alpha) r \bar{e}_2 + \hat{y}_2\right)^{1-\alpha}; \tag{F16}$$

and, those for traders $j \in \{1, ..., n\}$ are given by:

$$\hat{\pi}_j(r) = \frac{\left(\frac{\alpha}{\beta_1 + \beta_2} \frac{\gamma}{\gamma + r} \frac{n-1}{n}\right)^{\alpha} (1-\alpha)^{1-\alpha}}{n - \alpha} - \mu \frac{\gamma + \alpha r}{\gamma + r} (\bar{e}_1 + \bar{e}_2). \tag{F17}$$

Finally, we have that:

$$\frac{\partial \hat{\pi}_j(r)}{\partial r} = \frac{(1-\alpha)\gamma}{(\gamma + r)^2} \left\{ \mu(\bar{e}_1 + \bar{e}_2) - \alpha \frac{\left[\frac{\alpha}{(1-\alpha)} \frac{1}{\beta_1 + \beta_2} \frac{\gamma}{\gamma + r} \frac{n-1}{n}\right]^{\alpha}}{n - \alpha} \right\}. \tag{F18}$$

Therefore, we conclude:

$$\frac{\partial \hat{\pi}_j(r)}{\partial r} > 0 \text{ whenever } \alpha \to 0 \text{ or } n \to +\infty. \tag{F19}$$

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